ISBN: 978-3-95450-232-5 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2022-WEPA78

PROTON-ELECTRON FOCUSING IN EIC RING ELECTRON COOLER*

S. Seletskiy†, A.V. Fedotov, D. Kayran, J. Kewisch
Brookhaven National Laboratory, Upton, NY, USA

Abstract

The Electron Ion Collider (EIC) requires a cooling of protons at the top energy. The Ring Electron Cooler (REC) is a suitable option for such a cooling. In this paper we consider an effect of a proton-electron space charge (SC) focusing on the quality of the electron beam in the REC. We show that, with properly adjusted parameters of the Ring Electron Cooler, the SC focusing in the REC cooling section does not significantly affect the cooler performance.

INTRODUCTION

A recent success of Low energy RHIC electron cooler (LEReC) [1-6], the first RF-based non-magnetized electron cooler, demonstrated that operational electron cooling [7] does not require e-beam magnetization and can be achieved with a bunched electron beam. While LEReC operated at $\gamma = 4.1$ and $\gamma = 4.9$, LEReC approach becomes especially attractive for high energy electron coolers because it significantly simplifies the engineering aspects of a cooler design. For example, to cool protons in the Electron Ion Collider [8] at $\gamma = 293$ one can use an electron storage ring with non-magnetized electron bunches [9].

In electron coolers electrons co-propagate with a hadron beam with the same average velocity in a straight section of the storage ring, called a cooling section (CS). A hadron interacts with electrons in a CS via Coulomb force, which introduces dynamical friction [10] acting on each hadron. Over many revolutions in the accelerator the average friction reduces both the transverse and the longitudinal velocity spread of hadrons, thus increasing the 6-D phase space density of the bunch.

In a non-magnetized cooler, the cooling rate scales as $1/\sigma_\theta^2$, where $\sigma_\theta = \sigma_{\theta x} = \sigma_{\theta y}$ is the electron beam angular spread [11]. Therefore, it is important to carefully evaluate all effects which can influence the e-beam emittance.

High intensity proton bunches in the cooling section produce an additional space charge focusing on the electron beam. This proton-electron (p-e) focusing can significantly affect beam dynamics in the REC [12].

In this paper we show that for the most recent parameters of the REC the effect of p-e focusing on the electron beam emittance is tolerable. The current REC parameters relevant for this study are given in Table 1.

BEAM DYNAMICS WITH p-e FOCUSING

General Considerations

Consider a Gaussian, circularly symmetric transverse density distributions for both bunches in the cooling section. The equation of motion of an individual electron in this case is [13]:

$$r'' = \frac{K_e}{r}\left[\left(1 - e^{-\frac{r^2}{2\sigma_e^2}}\right) - \frac{I_p}{I_e}\left(1 - e^{-\frac{r^2}{2\sigma_p^2}}\right)\right] - \kappa r \quad (1)$$

where generalized perveance $K_e = 2I_e/(I_A\gamma^3)$, Alfven current $I_A \approx 17$ kA, indexes "e" and "p" signify that respective parameter is given for an electron or a proton beam, I is a bunch's peak current, $\sigma = \sigma_x = \sigma_y$ is the rms transverse size of the bunch, and the term κr represents a magnetic focusing.

Table 1: Optimized REC Parameters

Parameter	Value
γ-factor	293
Number of protons per bunch	6.88e10
rms p-bunch length [cm]	6
rms p-bunch momentum spread	6.6e-4
p-bunch geometric emittance (x/y) [nm]	9.6/1.5
p-bunch CS β-functions (x/y) [m]	200/1200
Number of electrons per bunch	1.4e11
rms e-bunch length [cm]	6
rms e-bunch momentum spread	8.9e-4
e-bunch geometric emittance (x/y) [nm]	15/15
e-bunch CS β-functions (x/y) [m]	100/100
CS length [m]	170
REC circumference [m]	426
REC tunes (x/y)	59.92/59.85

Defocusing caused by the self SC of the e-bunches alleviates the p-e focusing in the cooling section. We are dropping the self SC part in (1), thus intentionally overestimating the effect of the p-e focusing on beam dynamics. Then, we can rewrite (1) as:

$$x'' = \frac{K_p}{x^2+y^2}\left(1 - e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right)x - \kappa x$$
$$y'' = \frac{K_p}{x^2+y^2}\left(1 - e^{-\frac{x^2+y^2}{2\sigma_p^2}}\right)y - \kappa y \quad (2)$$

where $K_p = K_e I_p / I_e$.

We introduce Courant-Snyder variables ($\xi_{h,v}$, $\zeta_{h,v}$) for horizontal and vertical motion of an electron. For example, for horizontal direction:

$$\xi_h = \frac{x}{\sqrt{\beta}}; \quad \zeta_h = \frac{x\alpha}{\sqrt{\beta}} + x'\sqrt{\beta} \quad (3)$$

* Work supported by the US Department of Energy under contract No. DE-SC0012704.
† seletskiy@bnl.gov

ISBN: 978-3-95450-232-5 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2022-WEPA78

We assume that the p-e focusing in the CS is weak enough to be represented simply by an instantaneous change in the electron angles on each turn in the storage ring. This assumption will be justified later.

Then, for a non-linear part in the change in ζ on single a pass through the CS we get [14]:

$$\delta\zeta_{h,v} = K_p L_{cs}\left(\frac{1-e^{-\frac{\beta_{cs}}{2\sigma_p^2}(\xi_h^2+\xi_v^2)}}{\xi_h^2+\xi_v^2} - \frac{\beta_{cs}}{2\sigma_p^2}\right)\xi_{h,v} \quad (4)$$

where L_{cs} is the length of the cooling section and β_{cs} is electrons' average Twiss β-function in the CS.

Finally, we can write equations of motion of an electron in (ξ, ζ) phase space:

$$\begin{gathered}\xi'_{h,v} = \zeta_{h,v} \\ \zeta'_{h,v} = -\xi_{h,v} + \delta\zeta_{h,v}\cdot\sum_n \delta_D\left(\varphi_{h,v} - 2\pi n Q_{h,v}\right)\end{gathered} \quad (5)$$

Here φ is a betatron phase, δ_D is the Dirac delta function, and $Q_{h,v}$ are respectively the horizontal and the vertical tunes of the REC.

Single Pass Effect

Proton-electron focusing introduces a nonlinear correlation between transverse coordinate and angle of an electron. Such a nonlinear focusing by itself does not necessarily lead to a growth of the beam emittance. Whether the motion in the presence of p-e focusing is stable for all betatron amplitudes can be found from beam tracking.

First, let us consider a single pass effect of the p-e focusing. Here we will assume that the nonlinearity leads to a simple emittance increase. Although, strictly speaking, such an assumption is not correct, it is useful for estimating a magnitude of the effect.

We will use action-angle variables (J, φ), where:

$$J = \frac{\xi^2+\zeta^2}{2};\ \xi = \sqrt{2J}\cos\varphi\,;\ \zeta = \sqrt{2J}\sin\varphi \quad (6)$$

Gaussian beam distribution in (J, φ) phase space is given by:

$$f_{J,\varphi} = \frac{1}{2\pi\varepsilon}e^{-J/\varepsilon};\ \langle J\rangle = \varepsilon = \int_0^\infty\int_0^{2\pi} J f_{J,\varphi}\, d\varphi\, dJ \quad (7)$$

From Eq. (6) we get:

$$\delta J = J\,\delta\zeta\cdot\sin\varphi + \frac{\delta\zeta^2}{2} \quad (8)$$

Then from Eqs. (4), (7) and (8) we get:

$$\begin{gathered}\frac{\delta\varepsilon}{\varepsilon} = \frac{L_{cs}^2 I_p^2}{2\pi\varepsilon^2 I_A^2\gamma^6}\,\Xi\left(\frac{\sigma_e}{\sigma_p}\right) \\ \Xi(d) = \int_0^\infty\int_0^{2\pi}\frac{e^{-j}\left(1-e^{-d^2 j\cos^2\varphi} - d^2 j\cos^2\varphi\right)^2}{j\cos^2\varphi}\,d\varphi\, dj\end{gathered} \quad (9)$$

We introduced a dimensionless variable $j = J/\varepsilon$ and dropped x-y coupling terms in Eq. (4) to simplify the estimative expression Eq. (9).

An "overlap" function Ξ depends only on the ratio of the two beams' sizes in the cooling section (Fig. 1).

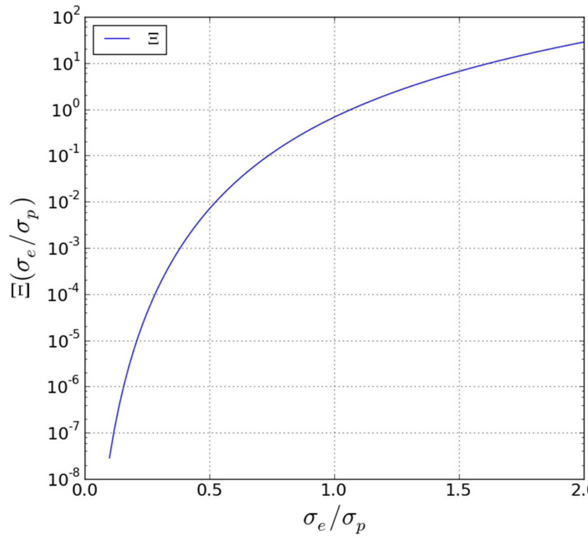

Figure 1: Overlap function $\Xi(\sigma_e/\sigma_p)$.

For parameters given in Table 1, the fractional emittance change over a single pass through the cooling section is $\frac{\delta\varepsilon}{\varepsilon} \approx 6\cdot 10^{-3}$. We conclude that the effect of the proton-electron kick is indeed small and our approach of treating it as a small nonlinear excitation is justified.

Beam Tracking

In Courant-Snyder coordinates one revolution in the storage ring is given by:

$$\begin{pmatrix}\xi_{1(h,v)}\\ \zeta_{1(h,v)}\end{pmatrix} = \begin{pmatrix}\cos 2\pi Q_{h,v} & \sin 2\pi Q_{h,v}\\ -\sin 2\pi Q_{h,v} & \cos 2\pi Q_{h,v}\end{pmatrix}\begin{pmatrix}\xi_{0(h,v)}\\ \zeta_{0(h,v)}\end{pmatrix} \quad (10)$$

The fastest way to simulate a turn-by-turn motion of an ensemble of electrons is to apply the transformation Eq. (10) to particles' coordinates obtained in a previous iteration and to add the respective kicks Eq. (4) to the result. Solving Eq. (5) numerically gives the same outcome but slows down the simulations.

To explore the effects of p-e focusing on the electron beam dynamics in the REC, we first performed a tracking of individual particles having initial action in the range from 0 to 6ε. These studies showed that for the chosen parameters an electron's motion is stable and that no noticeable emittance growth is expected from the p-e focusing.

Next, we performed a tracking of the ensemble of 10^4 electrons for $2\cdot 10^4$ turns. The tracking confirmed that in the presence of the proton-electron focusing in the cooling section the emittance stays essentially unchanged (Fig. 2).

ISBN: 978-3-95450-232-5 ISSN: 2673-7000 doi:10.18429/JACoW-NAPAC2022-WEPA78

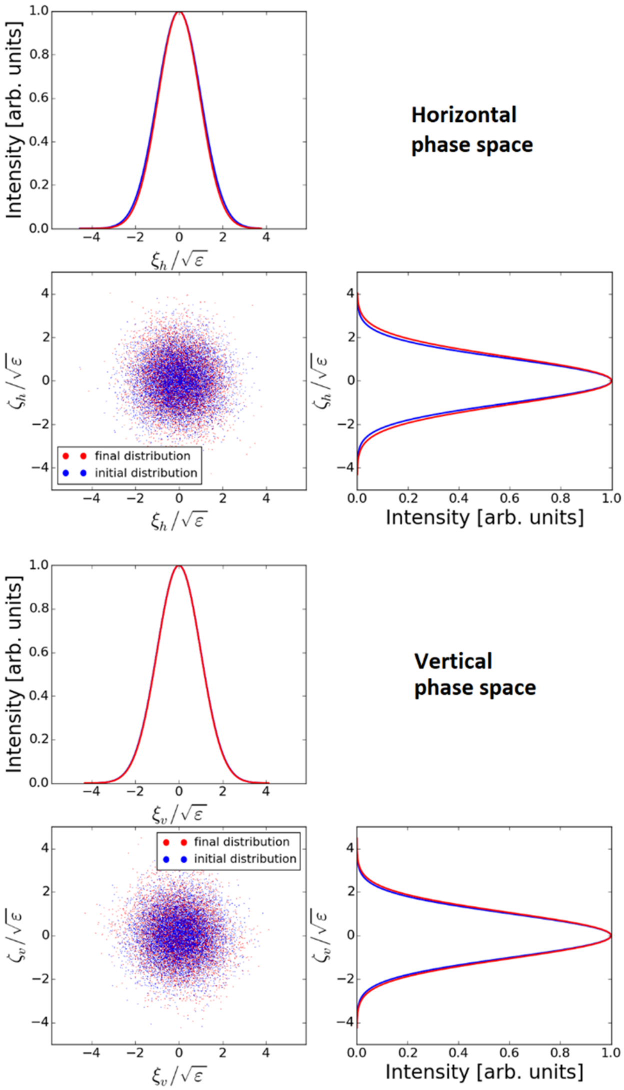

Figure 2: Initial (blue) distribution of electrons and the electrons' distribution after 20k turns in the REC (red) in the horizontal (top plots) and vertical (bottom plots) phase spaces. The tracking was performed in the presence of the space charge focusing from protons on electrons in the REC cooling section.

CONCLUSION

We considered the effect of proton-electron focusing in the cooling section of the Ring Electron Cooler.

It was shown that for the current parameters of the Ring Electron Cooler the proton-electron focusing can be treated as an instantaneous nonlinear angular kick obtained by electrons on every pass through the cooling section.

The tracking studies showed that in the presence of the proton-electron focusing the electrons' motion in the Ring Electron Cooler is stable and that the proton-electron focusing does not result in the emittance growth of the electron beam.

REFERENCES

[1] S. Seletskiy *et al.*, "Accurate setting of electron energy for demonstration of first hadron beam cooling with RF accelerated electron bunches", *Phys. Rev. Accel. Beams*, vol. 21, p. 111004, 2019. doi:10.1103/PhysRevAccelBeams.22.111004

[2] A. Fedotov *et al.*, "Experimental demonstration of hadron beam cooling using radio-frequency accelerated electron bunches", *Phys. Rev. Lett.*, vol. 124, p. 084801, 2020. doi:10.1103/PhysRevLett.124.084801

[3] X. Gu *et al.*, "Stable operation of a high voltage high current dc photoemission gun for the bunched beam electron cooler in RHIC", *Phys. Rev. Accel. Beams,* vol. 23, p. 013401, 2020. doi:10.1103/PhysRevAccelBeams.23.013401

[4] D. Kayran et al., "High brightness electron beams for linac based bunched beam electron cooling", *Phys. Rev. Accel. Beams*, vol. 23, p. 021003, 2020. doi:10.1103/PhysRevAccelBeams.23.021003

[5] H. Zhao *et al.*, "Cooling simulation and experimental benchmarking for an rf-based electron cooler", *Phys. Rev. Accel. Beams*, vol. 23, p. 074201, 2020. doi:10.1103/PhysRevAccelBeams.23.074201

[6] S. Seletskiy *et al.*, "Obtaining transverse cooling with non magnetized electron beam", *Phys. Rev. Accel. Beams*, vol. 23, p. 110101, 2020. doi:10.1103/PhysRevAccelBeams.23.110101

[7] G. I. Budker, "An effective method of damping particle oscillations in proton and antiproton storage rings", *At. Energy*, vol. 22, p. 346, 1967.

[8] J. Beebe-Wang (editor) *et al.*, "Electron Ion Collider conceptual design report", Brookhaven National Laboratory, 2021, www.bnl.gov/ec/files/EIC_CDR_Final.pdf

[9] H. Zhao, J. Kewisch, M. Blaskiewicz, and A. Fedotov, "Ring-based electron cooler for high energy beam cooling", *Phys. Rev. Accel. Beams*, vol. 24, p. 043501, 2021. doi:10.1103/PhysRevAccelBeams.24.043501

[10] S. Chandrasekhar, "Brownian motion, dynamical friction and stellar dynamics", *Rev. Mod. Phys*., vol. 21, p. 3, 1949. doi:10.1103/RevModPhys.21.383

[11] S. Seletskiy, A. Fedotov, "Useful formulas for non-magnetized electron cooling", Brookhaven National Laboratory, BNL-222963-2022-TECH, 2022.

[12] V. Lebedev *et al.*, "Conceptual design report: a ring-based electron cooling system for the EIC", *J. Instrum*., vol. 16, p. T01003, 2021. doi:10.1088/1748-0221/16/01/T01003

[13] S. Seletskiy, A. Fedotov, D. Kayran, "Beam dynamics in non-magnetized electron cooler with strong hadron-electron focusing", Brookhaven National Laboratory, BNL-216225-2020-TECH, 2020.

[14] S. Seletskiy, A. Fedotov, D. Kayran, J. Kewisch, "Proton-electron focusing in EIC Ring Cooler", Brookhaven National Laboratory, BNL-222987-2022-TECH (2022).