A QUANTITATIVE METHOD OF COUPLER CAVITY TUNING AND SIMULATION

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Abstract

Based on coupling-cavity chain equivalent circuit model, a quantitative method is developed to determining the coupling coefficient and frequency deviation of a coupler cavity. This method can simplify the tuning and simulation process of coupler cavity. As an example, we derived the analytical expression of the coupling coefficient and frequency deviation of a coupler cavity of periodic traveling wave accelerating tube. Compared with the method of R. L. Kyhl, it is consistent when coupling is correct. And much more, it can calculate the coupling coefficient and frequency deviation when coupler cavity is not matched.

1 INTRODUCTION

In accelerator tube microwave tuning, coupler matching is an important step. It decides how much microwave power can put into an accelerator tube and how pure of travelling wave field is in travelling wave accelerator.

In this paper, we describe a quantitative method of coupler matching, which is developed from method of R. L. Kyhl [1] and Marie Chanudet [2]. It is useful in coupler cavity tuning and simulation.
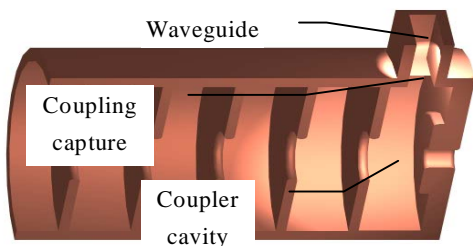

figure 1. structure of accelerating structure with a coupler

In the following section, we first describe the equivalent circuit model of monoperiodic travelling wave accelerating structure including coupler cavity. Based on this model, we derived an expression between measured reflection angle and coupler cavity frequency deviation Δf and coupling coefficient β. Then we expand this result to any travelling wave accelerating structure.

2 MONOPERIODIC STRUCTURE

For monoperiodic travelling accelerating structure, we repaired the equivalent circuit mode of Kyhl's. It can include the effect of coupler aperture. In this section, we assume that coupling coefficient between cavities satisfies $k \ll 1$. So we can neglect second and higher order of k.

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2.1 More integrated equivalent circuit model

In order to calculating the coupling status, we must include the effect of coupling capture of coupler cavity. In repaired circuit model, a transformer is represented the field transform function of coupling capture. And because the influence of coupling capture, the equivalent inductance and capacitance of coupler cavity is not long same as others. Figure 1 shows the equivalent circuit model of Kyhl's and repaired model.
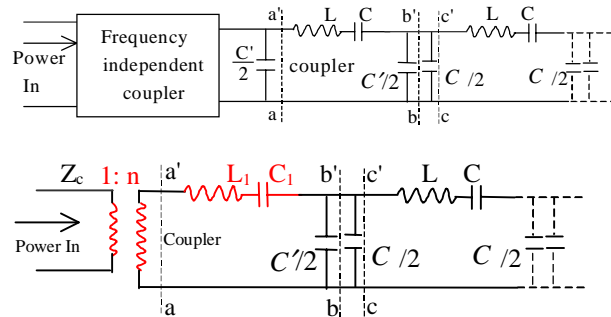

Figure 2. Equivalent circuit model of monoperiodic travelling wave structure including coupler cavity
up: Kyhl's model   down: repaired model

The equivalent circuit of accelerating structure is same in two models. So the following conclusion is same as reference [2]:

1. The characteristic impedance of π-series network is

$$Z_e^2(\omega_\theta) = -4X^2(\omega_\theta)tg^2\frac{\theta}{2}$$

Hereinto, θ is mode angle, and

$$X(\omega_\theta) = \frac{1}{j\omega_\theta C'}$$

2. the π/2 mode frequency is

$$\omega_{\frac{\pi}{2}} = \sqrt{\frac{1}{L\gamma}}$$

Hereinto, γ is the series capacitance of one cavity.

$$\frac{1}{\gamma} = \frac{1}{C} + \frac{2}{C'}$$

2.2 matching parameter of coupler cavity

Coupler matching requires impedance of accelerating structure equal to that of waveguide. At reference position a-a', the leftward impedance is

$$Z_{al} = n^2 Z_c$$

and the rightward impedance is

$$Z_{ar}(\omega_\theta) = j\omega_\theta L_1 + \frac{1}{j\omega_\theta C_1} + \frac{2X \cdot 2|X| tg\frac{\theta}{2}}{2X + 2|X| tg\frac{\theta}{2}} =$$

$$|X|\sin\theta + j|X|(\frac{4\delta}{k} + \cos\theta)$$

Hereinto,

$$\delta = \frac{\omega_c}{\omega_{\frac{\pi}{2}}} - 1, \quad \omega_c = \sqrt{\frac{1}{L_1 v_1}}, \quad \frac{1}{v_1} = \frac{1}{C_1} + \frac{1}{C'}$$

Matching in operation mode θ_0 requires that

$$Z_{al} = Z_{ar}$$

so we can get

$$\frac{n^2 Z_c}{|X(\omega_{\theta_0})|\sin\theta_0} = \beta_s = 1, \quad \frac{4\delta_0}{k} + \cos\theta_0 = 0$$

and the matching frequency of coupler cavity is

$$\omega_{c0} = \omega_{\pi/2}\left(1 - \frac{k}{4}\cos\theta_0\right) \approx \frac{(\omega_{\pi/2} + \omega_{\theta_0})}{2}$$

2.3 Measurement step

For tuning a coupler cavity, we must measurement the parameter at first. Figure 3 shows the measure step and corresponding circuits model.
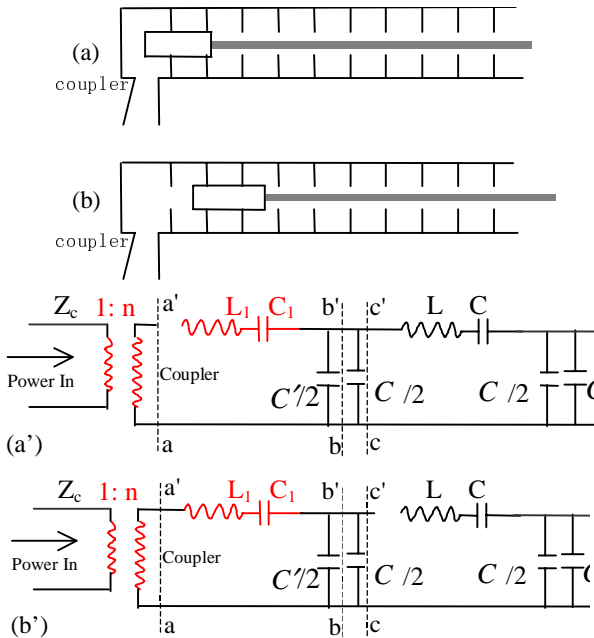

figure 3. Measurement progress and corresponding circuits model (a, a'): detuning first cavity (b, b'): detuning second cavity

Step 1. Detuning the first coupler cavity, and measuring the reflected angles of 2 frequencies near the $\pi/2$ mode frequency.

Step 2. Detuning the second cavity, then measuring the reflected angles of aforesaid 2 frequencies. According to the reflected angles, we can determine the matching status of coupler cavity.

2.4 calculation of $\Delta\omega_c$ and β

According to figure 3, we can find the equation about the reflect angle φ and any angle frequency ω

$$tg\frac{\varphi}{2} = \frac{n^2\omega z_c}{L_1(\omega^2 - \omega_c^2)}$$

If we measure two reflect angles φ_1, φ_2 under two frequencies ω_1 ω_2, we can get

$$\omega_c = \sqrt{\omega_1\omega_2}\sqrt{\frac{tg\frac{\varphi_2}{2}\omega_2 - tg\frac{\varphi_1}{2}\omega_1}{\omega_1 tg\frac{\varphi_2}{2} - \omega_2 tg\frac{\varphi_1}{2}}}$$

$$\beta = \frac{1}{\frac{k}{2}\omega_{\pi/2}\sin\theta_0} \cdot \frac{tg\frac{\varphi_1}{2}tg\frac{\varphi_2}{2}(\omega_1^2 - \omega_2^2)}{tg\frac{\varphi_2}{2}\omega_1 - tg\frac{\varphi_1}{2}\omega_2}$$

So we can get

$$\Delta\omega_c = \omega_c - \omega_{c0}$$

3 GENERALIZE TO NONUNIFORM STRUCTURE

According to the analyse of section 2, we can see that the tuning process is just calculating and measure the parameter of coupler cavity. Now we generalize it to nonuniform structure, and calculate and measure the frequency ω_c and external quality factor Q_{ec} of coupler cavity.

3.1 calculation of ω_{c0} and Q_{ec0}

For uniform structure, we can get the analyse equation of matching frequency ω_{c0} and external quality factor Q_{ec0} of coupler cavity based on section 2.

$$\omega_{c0} \approx \frac{\omega_{\pi/2} + \omega_{\theta_0}}{2}, \quad Q_{ce0} \approx \frac{2}{k\sin\theta_0}$$

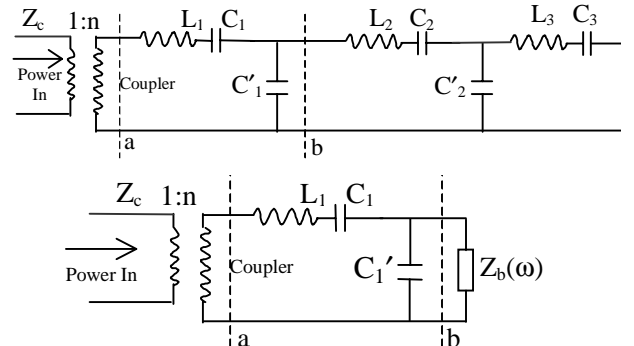For nonuniform structure, we also can get numerical result by means of numerical simulation.

Figure 4. Equivalent circuit model of cavity-chain

3.2 measure of ω_c and Q_{ec}

If the second cavity is detuned, the reflection Γ is

$$\Gamma = \frac{j\dfrac{\omega}{\omega_c Q_0} + 1 - \left(\dfrac{\omega}{\omega_c}\right)^2 - j\dfrac{\omega}{\omega_c Q_{ec}}}{j\dfrac{\omega}{\omega_c Q_0} + 1 - \left(\dfrac{\omega}{\omega_c}\right)^2 + j\dfrac{\omega}{\omega_c Q_{ec}}}$$

So we can find ω_c and Q_{ec} just taking two measurements.

When Q_0 approaching infinite, we can get ω_c and Q_{ec} by measure 2 reflect angles under 2 frequencies.

$$\omega_c = \sqrt{\omega_1 \omega_2}\sqrt{\frac{\omega_1 tg\dfrac{\phi_1}{2} - \omega_2 tg\dfrac{\phi_2}{2}}{\omega_2 tg\dfrac{\phi_1}{2} - \omega_1 tg\dfrac{\phi_2}{2}}}$$

$$Q_{ec} = \frac{\sqrt{\omega_1 \omega_2}}{tg\dfrac{\phi_1}{2} tg\dfrac{\phi_2}{2}} \frac{\sqrt{\left(\omega_2 tg\dfrac{\phi_1}{2} - \omega_2 tg\dfrac{\phi_1}{2}\right)\left(\omega_1 tg\dfrac{\phi_1}{2} - \omega_2 tg\dfrac{\phi_2}{2}\right)}}{\left|\omega_1^2 - \omega_2^2\right|}$$

3.3 whole design and tuning process

In this part we describe a general step to design and tune a coupler cavity applying this quantitative method.

1. Calculate matching frequency ω_{c0} and external quality factor Q_{ec0} of coupler cavity by designed or measured cavity-chain parameter.
2. Calculate the frequency ω_c and external quality factor Q_{ec} of model coupler cavity using electromagnetic analyse program, such as MAFIA.
3. Change the size of coupler cavity until $\omega_c = \omega_{c0}$, $Q_{ec} = Q_{ec0}$. Record the relationship between ω_c, Q_{ec} and coupler cavity size.
4. Machine actual coupler according to calculation result.
5. Measure frequency ω_c and external quality factor Q_{ec} of actual coupler cavity.
6. Fine-tune the coupler cavity according to calculated result.

4 ONE EXPERIMENTAL TUNING PROCESS

This section describes one experimental tuning process to verify this quantitative method.

Figure 5 shows the procedure.

(a) Original cavity is undercoupling (β=0.74), so the coupling aperture is expanded.
(b) After expanding, the coupling coefficient is increased from 0.74 to 0.85, but the frequency is still high.

(c) Expanding the radius of coupler cavity, the frequency deviation is decreased from 21MHz to 6.8MHz, and the coupling coefficient is increased from 0.85 to 0.95.
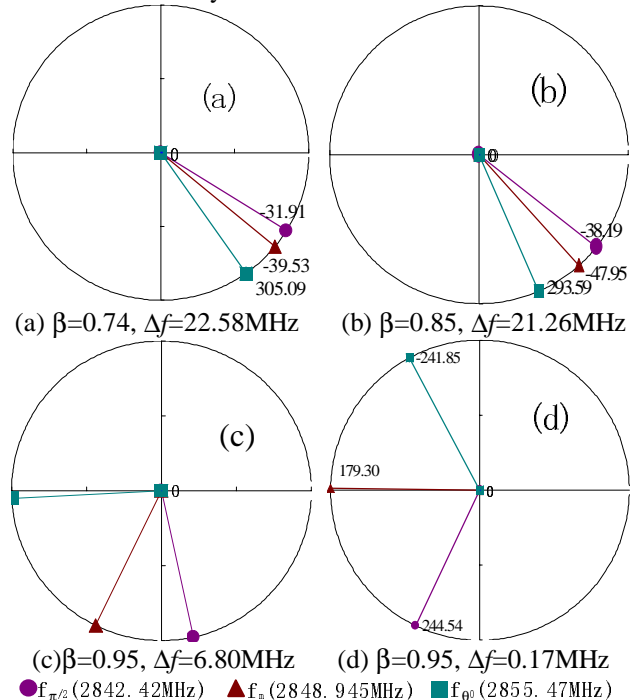(d) Result of slightly expanding the radius eccentrically.

(a) β=0.74, Δf=22.58MHz   (b) β=0.85, Δf=21.26MHz

(c)β=0.95, Δf=6.80MHz   (d) β=0.95, Δf=0.17MHz

● $f_{\pi/2}$ (2842.42MHz) ▲ f_{π} (2848.945MHz) ■ f_{θ^0} (2855.47MHz)

Figure 5. Example of one travelling wave accelerator coupler cavity tuning process

5 CONCLUSION

The method described here can be used in coupler cavity tuning and simulation. We only need to concern one coupler cavity in tuning and simulation process because matching frequency ω_{c0} and external quality factor Q_{ec0} of coupler cavity is calculated at first. So the tuning and simulation process is simplified.

6 REFERENCES

[1] E. Westbrook, Microwave Impedance Matching of Feed Waveguides to the Disk-Loaded Accelerator Structure Operating in the $2\pi/3$ Mode, SLAC-TN-63-103, 1963.
[2] Marie Chanudet, Matching of the Coupler Cavity to Travelling Wave Structures at any Operating Mode, LAL/RT 93-06, 1993