Coherent Synchrotron Radiation Calculations Using TraFiC^4: Multi-Processor Simulations and Optics Scans

Andreas Kabel, Stanford Linear Accelerator Center

Abstract

Coherent synchrotron radiation (CSR) is one of the most important effects in the design of high-quality beamlines with small bending radii. For a wide class of problems, numerical simulation is the only method to predict its effects. TraFiC^{4}is one of several simulation codes in existence. In this paper, we describe its extension to multi-processor machines, resulting in a substantial saving of CPU time. Also, we describe a semi-analytical approach to the problem of optics-dependence of emittance growth due to CSR.

1 INTRODUCTION

Coherent synchrotron radiation (CSR) is an important effect for short, low-emittance bunches of charged particles traveling on strongly bent trajectories. Its action on the bunch will in general lead to emittance degradation an an increase of energy spread.

If transient effects are important, so no analytical approximation can be used, numerical calculation is the only viable method. By know, a number of codes exist incorporating different approaches to the problem [1, 2], [3, 4]. TraFiC^{4}is a code using a weighted macroparticle approach, solving the Maxwell-Vlasov equations either perturbatively or self-consistently [5, 6, 7, 8, 9]. One of its biggest downside s is its CPU time consumption, particularly in the case of shielded CSR fields. These are calculated in the spatiotemporal domain using image charges, leading to slowly converging integrals and high CPU time consumption. As a remedy, a multi-processor version of TraFiC^{4}was developed, which we describe in this paper.

2 MULTI-PROCESSOR SUPPORT

TraFiC^4's kick algorithm (cf. [5] for a detailed description) is distributed to several CPU's the following way: A number N_P of TraFiC^{4}processes runs independently, reading the same input file. One process, the *root process*, is special in that it does all output processing. From the input file, all processes buid the internal cartesian representation of the beamline and the initial particle distribution.

After the input file is read, a process with process number n_p begins, in the *calculation step*, to calculate the CSR fields on all particles it *owns*, i. e. the ones with index $i = n_p \mod N_p + 1$. To do that, it must be able to access the trajectories of all the other particles. Thus, for self-consistent calculations, a *synchronization and update step* is required: after finishing its field calculation, a process broadcasts its results–the fields acting on the particles it owns–to all the other processes. It then waits for all other processes to broadcast their respective results. After this synchronization step, the process knows the total fields on all the particles in the process, as does every other process. Each process then uses these fields to advance all particles into the next timestep. After this *correction step*, all particles' corrected trajectories are known to each process, and the processes can continue with their next calculation step. After all process have gone through the last timestep, all processes but the root process are terminated. The root process then continues to postprocess the results and output the collective quantitities of the bunch.

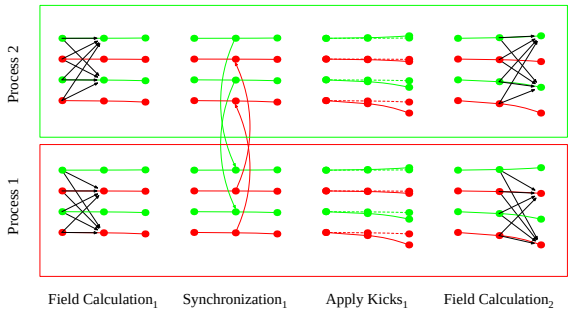

Figure 1: The MP algorithm for TraFiC^4

TraFiC^4/MP uses the MPI standard. It should compile and run on any platform supporting this standard and providing an ANSI C++ and Fortran77 compiler. It has been tested on Pentium class machines running FreeBSD 4.3 or Linux 2.4 and the freely available MPICH package. It has also been run on the IBM RS/6000 system at the National Energy Research Scientific Computing Center. It has been possible to assign a single node to each tracked macroparticle, thus reducing running times from 300 hours CPU time (for a long-beamline problem involving shielding, on a PC running Linux) to 5 hours of real time. TraFiC^4/MP was used to calculate some of the results in [1].

3 SIMPLIFYING OPTICS SCANS

One of the results of both experiments and simulations using TraFiC^{4}has been the optics-sensitivity of projective emittance growth, i. e. the fact that the emittance growth of a bunch depends on its initial Twiss parameters.

3.1 *Generalized emittance growth*

Consider a system of $D \geq d_\varepsilon$ degrees of freedom. We are interested in the correlation matrix C_ε in the first d_ε degrees

of freedom. The system evolves according to

$$\frac{\mathrm{d}}{\mathrm{d}s}x = T(s)x + F(s) \quad ,$$

where T describes a unimodular behavior. With

$$\frac{\mathrm{d}}{\mathrm{d}t}M(s) = T(s) \quad , \quad M(0) = 1 \tag{1}$$

and

$$I(s) = \int_0^s M^{-1}(s')F(s')\mathrm{d}s'$$

the system is determined by its initial conditions according to

$$x(s) = M(s)[x(0) + I(s)] \quad .$$

Now we are interested in the system's correlations matrix:

$$\begin{aligned} C(s) &= \left\langle x(s)x^\top(s)\right\rangle = M(s)\Big(C_0(0) \\ &+ \int_0^s\int_0^s M^{-1}(s_1)\left\langle F(s_1)F^\top(s_2)\right\rangle {M^{-1}}^\top(s_2)\mathrm{d}s_1\mathrm{d}s_2 \\ &+ \int_0^s \left\langle x(0)F^\top(s_1)\right\rangle \mathrm{d}s_1 + \text{transp.}\Big) M^\top(s) \\ &= M(s)(C_0 + C_{FF} + C_{Fx} + C_{Fx}^\top)M^\top(s) \quad , \end{aligned} \tag{2}$$

so the system behaves as if it had been transported linearly with an effective initial correlation given by the middle term in the last line. The emittance is given by $\sqrt{\det C(s)}$; (2) can be viewed as a generalization of the usual $\mathcal{H}$ quantity as it occurs in synchrotron radiation problems.

3.2 Emittance Growth by Dispersion Mismatch

We restrict all considerations to x, p, δ-subspace (which is approximately closed under the CSR-beam-interaction, if all dipoles bend in the same plane) of the full phasespace. We can introduce new coördinates by $(\bar{x}, \bar{p}) = (x, p) - \delta(\eta, \eta')$. The equations of motion for a single particle in a lattice of focal strength n and curvature ρ without external forces then read

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}s}\bar{x}(s) &= \bar{p}(s) - \eta(s)\frac{\mathrm{d}}{\mathrm{d}s}\delta(s) \\ \frac{\mathrm{d}}{\mathrm{d}s}\bar{p}(s) &= n(s)\bar{x}(s) - \left(\frac{\mathrm{d}}{\mathrm{d}s}\eta(s)\right)\left(\frac{\mathrm{d}}{\mathrm{d}s}\delta(s)\right) \end{aligned}\right\} \tag{3}$$

if we choose η to be the usual dispersion function. Then, (3) has the same form as (1) with an inhomogeneity of

$$F(s) = \delta'(s)\begin{pmatrix}\eta(s)\\ \eta'(s)\end{pmatrix} \quad .$$

Let's assume the energy kicks are uncorreleated across the bunch. Then, the total emittance is (because $\det(M) = 1$)

$$\varepsilon^2 = \det(C_0 + C_{FF}) \quad . \tag{4}$$

For a transport line, we are free to choose the shape of C_0, while its determinant is fixed by the initial emittance. It is easy to see that (4) becomes minimal if we choose $C_0 \propto C_{FF}$, in which case $\varepsilon_{total}^2 = \varepsilon_0^2 + \varepsilon_{FF}^2$. In terms of distibutions this means that the final distribution is the convolution of the initial distribution subject to the linear forces only, and a zero-emittance-distribution ("pencil bunch") subject to the perturbation and the linear forces. We can express the emittance growth analytically in terms of the energy spread $\sqrt{\langle\delta^2\rangle(s)}$ in two limiting cases:

When the energy noise is white, the correlation is just a Dirac function, $\langle\delta'(s_1)\delta'(s_2)\rangle = \delta(s_1 - s_2)\langle\delta^2(s)\rangle'$, where $\langle\delta^2(s)\rangle'$ is the change of the noise amplitude squared. In this case, we can write down the optimum values for the initial Twiss parameters in terms of the transport matrix and the noise amplitude:

$$\left.\begin{aligned} \alpha_0(0) &= \lambda\int_0^L \langle\delta^2\rangle'(s)\left(M_{12}(s)\eta'(s) - M_{22}(s)\eta(s)\right) \\ &\quad \left(M_{11}(s)\eta'(s) - M_{21}(s)\eta(s)\right)\mathrm{d}s \\ \beta_0(0) &= \lambda\int_0^L \langle\delta^2\rangle'(s)\left(M_{12}(s)\eta'(s) - M_{22}(s)\eta\right)^2\mathrm{d}s \\ \gamma_0(0) &= \lambda\int_0^L \langle\delta^2\rangle'(s)\left(M_{11}(s)\eta'(s) - M_{21}(s)\eta(s)\right)^2\mathrm{d}s \end{aligned}\right\} \tag{5}$$

where λ has to be chosen such that $\beta_0\gamma_0 = 1 + \alpha_0^2$ and $\beta_0 > 0$.

The other approximation we can make is that of an energy change strongly correlated with the longitudinal position in the bunch, but independent of the position along the beamline (such a situation might be CSR along a bend much longer than the overtaking length); in this case we get

$$\left.\begin{aligned} \alpha_0(0) &= \lambda\int_0^L \left(M_{12}(s)\eta(s)' - M_{22}(s)\eta(s)\right)\mathrm{d}s \\ &\quad \int_0^L \left(M_{11}\eta(s)' - M_{21}\eta(s)\right)\mathrm{d}s \\ \beta_0(0) &= \lambda\left(\int_0^L \left(M_{12}\eta' - M_{22}\eta\right)\mathrm{d}s\right)^2 \\ \gamma_0(0) &= \lambda\left(\int_0^L \left(M_{11}\eta' - M_{21}\eta\right)\mathrm{d}s\right)^2 \end{aligned}\right\} \tag{6}$$

For a bend of length L and radius R with a purely correlated, s-constant energy change, we can find the emittance growth in analytic form:

$$\begin{aligned} \frac{\Delta(\varepsilon^2)}{\varepsilon} &= \langle\delta^2\rangle(L)R^2/L^2[\beta_0(1-\cos(L/R))^2 \\ &\quad - 2\alpha_0(L - R\sin(L/R))(1-\cos(L/R)) \\ &\quad + \gamma_0(L - R\sin(L/R))^2] \end{aligned}$$

Note that the correlation matrix in this case has one eigenvalue of 0 because of the direct product structure of the integral for C_{FF}. This means the beam cannot be matched to C_{FF}, as it would require infinite extension in one direction in phasespace.

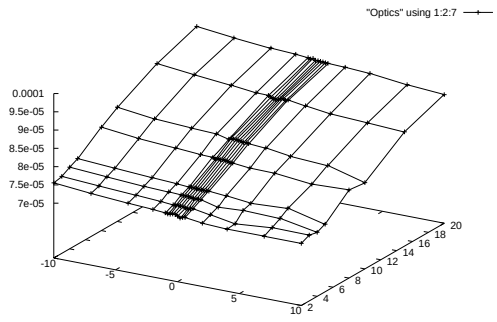

Figure 2: Emittance growth from numerical simulation for a single bend: $R = 2\text{m}, L = 1\text{m}, q = 1\text{nC}, E = 40.7\text{MeV}, \varepsilon_{\text{normalized}} = 70 \cdot 10^{-6}\text{m}$

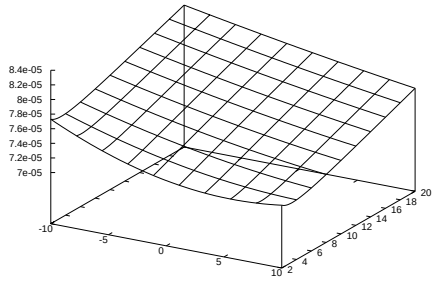

Figure 3: Emittance growth from C_{FF} extrapolation at $\alpha = 0, \beta = 10\text{m}$

3.3 Numerical results

The formulae given above have been evaluated and compared to numerical results for a toy problem (a single bend, fig 2). The analytical formulae for dispersion mismatch from above, however, fail to reproduce the numerical simulation behavior. Thus, the optics dependence of CSR induced emittance growth cannot be explained by dispersion mismatch alone.

Code has been added to `TraFiC`^{4}to output the matrices C_{Fx} and C_{FF} for the full six-dimensional forces. It turns out that the matrix C_{FF} is reasonably constant over a large area of parameter space, so $\sqrt{\det(C_0 + C_{FF})}$ can be used to extrapolate the $\varepsilon(\alpha_0, \beta_0)$ behavior of the simulation. Fig. 3 shows such an extrapolation from a C_{FF} from the center of the parameter range. While it reproduces the qualitiative behavior quite well, it fails to fully reproduce the magnitude of the emittance growth at the edges of the parameter range. (It should be noted, however, that the example chosen is somehat extreme as the bunch is wide compared to its length. Preliminary simulations with slimmer bunches indicate better agreement.) Inspection of the output for this problem shows that C_{Fx} gives a significant contribution to the determinant of the total correlation matrix (which means that the transverse gradients of the fields contribute significantly and/or the dynamics under CSR introduce significant cross-correlations). Thus, analytic models of the C_{Fx} contributions have to be developed to simplify optics calculations.

4 ACKNOWLEDGMENTS

I wish to thank the members of the Advanced Computational Department at SLAC for their support.

Work supported by Department of Energy contract DE–AC03–76SF00515.

5 REFERENCES

[1] Lars Groening, Hans Braun, Steffen Doebert, Michael Borland, and Andreas Kabel. Recent experiments on the effect of coherent synchrotron radiation on the electron beam of ctf ii. *This Conference*, 2001.

[2] H. H. Braun, R. Corsini, L. Groening, F. Zhou, A. Kabel, T. Raubenheimer, R. Li, and T. Limberg. Emittance growth and energy loss due to coherent synchrotron radiation in the bunch compressor of the CLIC Test Facility (CTF II). In *Proceedings of the 7th European Particle Accelerator Conferences (EPAC 2000), Vienna, Austria, 26-30 Jun 2000*, 2000.

[3] R. Li. Analysis and simulation on the enhancement of the csr effects. *eConf*, C000821:MOE01, 2000.

[4] R. Li. The impact of coherent synchrotron radiation on the beam transport of short bunches. Contributed to IEEE Particle Accelerator Conference (PAC 99), New York, 29 Mar - 2 Apr 1999.

[5] A. Kabel, M. Dohlus, and T. Limberg. Numerical calculation of coherent synchrotron radiation effects using TraFiC4. *Nucl. Instrum. Meth. A*, A455:185–189, 2000.

[6] M. Dohlus, A. Kabel, and T. Limberg. Efficient field calculation of 3D bunches on general trajectories. *Nucl. Instrum. Meth.*, A445:338–342, 2000.

[7] M. Dohlus, A. Kabel, and T. Limberg. Optimal beam optics in the TTF-FEL bunch compression sections: Minimizing the emittance growth. In *Proceedings of the IEEE Particle Accelerator Conference (PAC 99), New York, NY, 29 Mar - 2 Apr 1999*, 1999.

[8] M. Dohlus, A. Kabel, and T. Limberg. Uncorrelated emittance growth in the TTF-FEL bunch compression sections due to coherent synchrotron radiation and space charge effects. In *Proceedings of the 6th European Particle Accelerator Conference (EPAC 98), Stockholm, Sweden, 22-26 Jun 1998*, 1998. DESY-M-98-060l.

[9] M. Dohlus, A. Kabel, and T. Limberg. Design consequences of coherent synchrotron radiation beam dynamic effects on the TTF-FEL bunch compression system. In *Proceedings of the 19th International Conference on Free Electron Lasers, Beijing, China, 18-21 Aug 1997*, 1997. DESY-TESLA-FEL-97-06B, Oct. 1997.