UNUSUAL FOURTH ORDER DEFORMATION OF BEAM DUE TO SPACE CHARGE*

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Abstract

Unusual fourth order core deformation in y-phase space is observed in the simulation of SNS ring without lattice imperfections. The fourth order core deformation happens for a wide range of tunes, whereas an asymmetric deformation takes place when bare tune ν_y is 5.53 to 5.74. When ν_y is close to 6, 2:1 resonance appears in both planes. Extensive numerical simulation indicates that the difference resonance $2\nu_x$-$2\nu_y$=0 from space charge potential and dispersion effect are playing a crucial role. Similar effect is expected near the resonance $2\nu_x$-$2\nu_y$=P with a superperiod P. This is unusual because the SNS ring under model has a perfect four-fold symmetry and operating points are far from structural resonances. Also an interesting observation on **equipartitioning** is presented in section 4.

1 INTRODUCTION

All the simulation presented here is done using the ACCSIM code [1] and ORBIT code [2] for the full 2MW beam of the SNS ring. The tune depression is $\Delta\nu\approx0.08$. It was first shown in [3] that space charge induced resonance can generate an enormous emittance growth for an initially well-matched beam near the coupling resonance $2\nu_x$-$2\nu_y$=0. This clearly illustrates the importance of the choice of working points in high intensity rings. Also an unusual fourth order core deformation was first observed when bare tunes are ν_x=5.82, ν_y=5.77, as is shown in Fig. 1. This is highly unusual because the SNS ring lattice under model has a perfect four-fold symmetry. In the simulation, neither skew lattice elements nor nonlinear lattice elements are included. And space charge is the only source of nonlinearity.

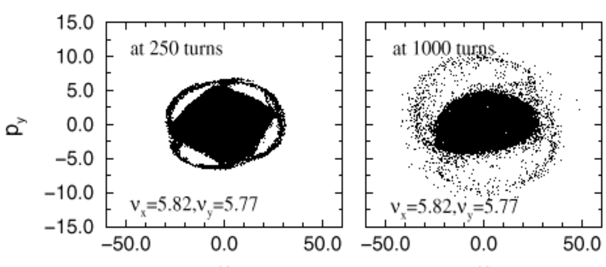

Figure 1: Plots of beam distribution in y phase space at two different turn numbers when bare tunes are ν_x=5.82, ν_y=5.77.

*Work supported by the DOE, under contract No. DE-AC05-00OR22725 with UT-Batelle, LLC for ORNL
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2 CONDITIONS FOR THE ONSET OF THE FOURTH ORDER DEFORMATION

There are a few conditions for the onset of this fourth order deformation in y phase space:

- Operating point should be close enough to the coupling resonance $2\nu_x$-$2\nu_y$=P with the lattice superperiod P.
- There should be dispersion effect: combination of dispersion and momentum spread of beam particles.
- Bare tunes should satisfy ν_x=ν_y+0.05 (ν_x=ν_y+0.03) for ACCSIM (ORBIT) code for the 2MW beam of SNS ring.

These unusual core deformations have been observed only for initially KV-like beams. The role of the coupling resonance $2\nu_x$-$2\nu_y$=0 (or P (superperiod)) is crucial. When the operating point is far away from the coupling resonance, the core deformation does not show up (see the right plots of Fig. 8 where ν_x=5.82, ν_y=5.67). However it is believed that the coupling resonance is not limited to $2\nu_x$-$2\nu_y$=P. Perhaps coupling resonances such as ν_x-$2\nu_y$=P could induce similar effect.

When there is no momentum spread in the beam or when there is no dispersion (such as uniform focusing lattice), the fourth-order core deformation does not show up, as shown in Fig. 2. Instead the usual 2:1 parametric resonance shows up in both planes.

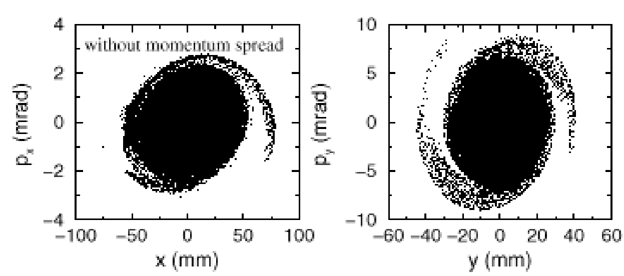

Figure 2: Plots of beam distribution in phase for a beam without momentum spread. Bare tunes are ν_x=5.82, ν_y=5.77.

The small difference in tune condition between ACCSIM and ORBIT in the third bullet comes from the small difference in depressed tunes between ACCSIM and ORBIT code for the same bare tunes (see Fig. 3). Under this condition, one corner of tune diagram touches the $2\nu_x$-$2\nu_y$=0 resonance line (see the blue dots in Fig. 3).

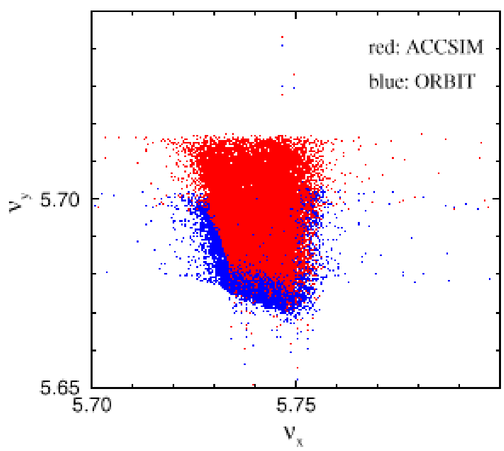

Figure 3: Depressed tune diagrams for the same bare tunes ν_x=5.82, ν_y=5.77 obtained from the two codes.

3 SIMULATION

Table 1: Summary of deformation types vs. tunes

ν_y	ν_x	y-plane	x-plane	Sym dist
5.53	5.58	Deformed	None	No
		4^{th}	None	Yes
5.70	5.75	Deformed	None	No
		4^{th}	None	Yes
5.73	5.78	Deformed	None	No
		4^{th}	None	Yes
5.74	5.79	Deformed	None	No
		4^{th}	None	Yes
5.77	5.82	4^{th}	None	No
5.80	5.85	4^{th}	None	No
5.85	5.90	4^{th}	None	No
5.87	5.92	4^{th}	None	No
5.87	5.92	4^{th}	None	No
6.05	6.10	2^{nd}	2^{nd}	No
6.27	6.32	4^{th}	None	No

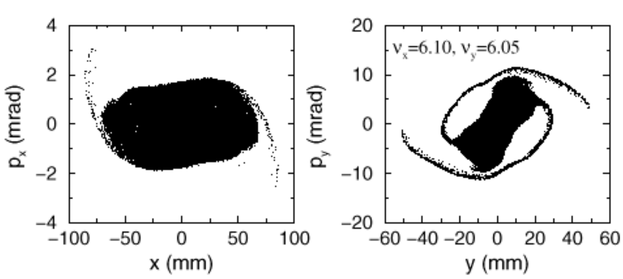

Figure 4: Beam distribution in phase space for bare tunes of ν_x=6.10, ν_y=6.05.

Extensive numerical simulation indicates that this deformation takes place over a wide range of y tunes. Table 1 lists the core deformation type depending on ν_y. Tunes in 1^{st} and 2^{nd} column are y and x bare tunes of operating points. Initial beam distributions are generated using random numbers. Due to the random generation, the generated initial distribution is not perfectly symmetric. At some tune values, this slight asymmetry manifests. The 5^{th} column indicates if the initial beam distribution is symmetrized as is described in Subsection 3.1.

For most of operating points listed in Table 1 (data obtained from the ACCSIM code and similar data obtained for the ORBIT code), the fourth order core deformation is observed. One distinct exception is for the operating point ν_x=6.10, ν_y=6.05, where the 2:1 parametric resonance is dominant in both planes as shown in Fig. 4. Due to the integer resonances ν_x=6 and ν_y=6, the beam deforms very fast. When bare tune ν_y is in the range from 5.77 to 6.27 (except for 6.05), the y space core deformation is symmetric fourth order (see Fig. 1 and 5), even though the initial beam distribution is not symmetrized. Figure 5 shows the beam distribution for ν_x=6.32, ν_y=6.27. However, asymmetric core deformation takes place, for ν_y ranging from 5.53 to 5.74.

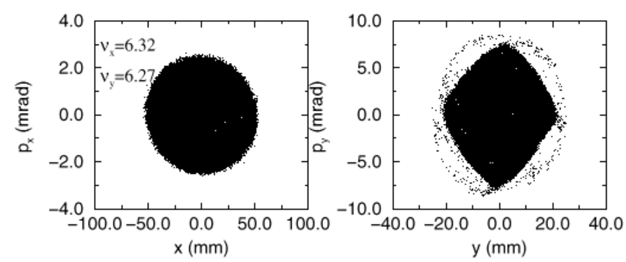

Figure 5: Beam distribution in phase space for bare tunes of ν_x=6.32, ν_y=6.27.

3.1 Symmetrized initial beam distribution

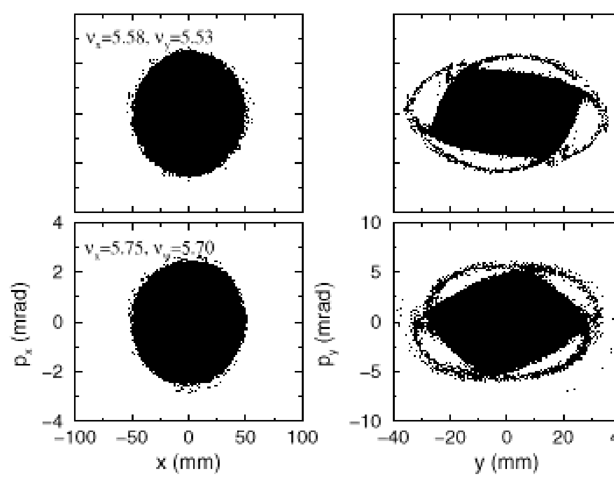

Figure 6: Beam distribution in phase space for bare tunes of ν_x=5.58, ν_y=5.53 (top plots) and ν_x=5.75, ν_y=5.70 (bottom plots). The initial distribution is symmetrized.

In order to track down the cause of the asymmetric deformation, symmetrized beam distributions in y phase space are used. Symmetrized beam distribution means that there is a beam particle with (x, px, -y, -py, E, ϕ) for every particle with (x, px, y, py, E, ϕ) in the beam. The symmetrization of beam distribution makes y space charge force perfectly symmetric. The lattice is also mid-plane symmetric. In this case, the y space charge force is perfectly symmetric. And all the deformation in y space is symmetric fourth order for all operating points listed in Table 1. The only exception is the ν_x=6.10, ν_y=6.05 case, where depressed tunes are close to integer resonance. Figure 6 shows beam distributions for ν_x=5.58, ν_y=5.53 and ν_x=5.75, ν_y=5.70, when symmetrized initial beam distributions are used.

3.2 Unsymmetrized initial beam distribution

When the initial beam is not symmetrized, asymmetric core deformation takes place for the cases such as ν_x=5.58, ν_y=5.53 and ν_x=5.79, ν_y=5.74 in Table 1. Seemingly a third order triangular deformation was observed using the ORBIT code [4] for ν_x=5.57, ν_y=5.54, etc. This is well illustrated in Fig. 7. Comparison of Fig. 7 and Fig. 6 is pretty interesting. These operating points are close to either the half integer resonance or third order resonance. It is not fully understood why the core deformation becomes sensitive to the slight asymmetry of the initial beam distribution, depending on tunes.

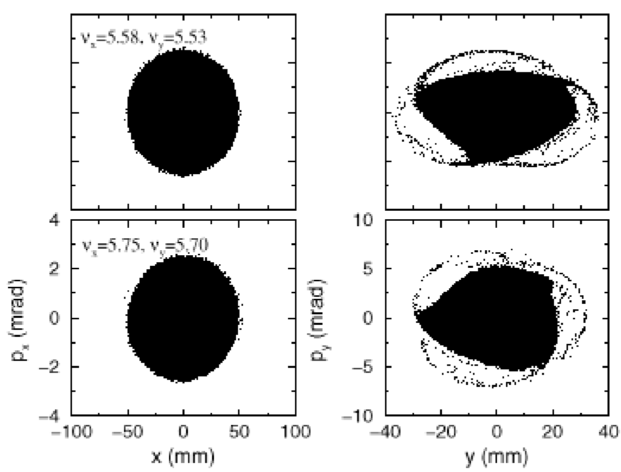

Figure 7: Beam distribution in phase space for bare tunes of ν_x=5.58, ν_y=5.53 (top plots) and ν_x=5.75, ν_y=5.70 (bottom plots). The initial distribution is not symmetrized.

4 EQUIPARTIONING

Table 2: Summary of rms emittance growth

Dist type	δ spread	ε_x	ε_y	$\Delta\varepsilon_x/\varepsilon_x$	$\Delta\varepsilon_y/\varepsilon_y$
KV-like	Yes	119.8	120.2	4.7%	-4.2%
KV-like	No	119.8	120.2	8.8%	-8.7%
Gaussian	No	89.1	88.3	2.5%	-1.8%

It is known that emittance exchange and growth does not happen when $\varepsilon_x=\varepsilon_y$ and $\nu_x=\nu_y$ due to equipartitioning. However, emittance exchange and growth seem to occur in the ring as shown in Fig. 8, even when $\varepsilon_x\nu_x/\varepsilon_y\nu_y=1$ (0.3% different). As input beam distributions, a few different types are used such as KV-like beams with and without momentum spread and Gaussian-like beam without momentum spread. A few examples were shown by Jeon *et al* [3]. Table 2 lists initial rms emittance and relative change ratio dominated by the coupling resonance for three different beam distributions. The operating point is ν_x=5.82, ν_y=5.82 (bare tunes) for all three cases. δ spread in the Table 2 means momentum spread of beam. This is not observed in the case of linac tracked 100 betatron periods [5]. The main difference between a ring and a linac is the existence of dipoles, introducing dispersion. However, this emittance exchange process is persistent even for beams without momentum spread (see the two cases in Table 2). One possibility is the very small initial anisotrophy in $\varepsilon_x\nu_x/\varepsilon_y\nu_y$.

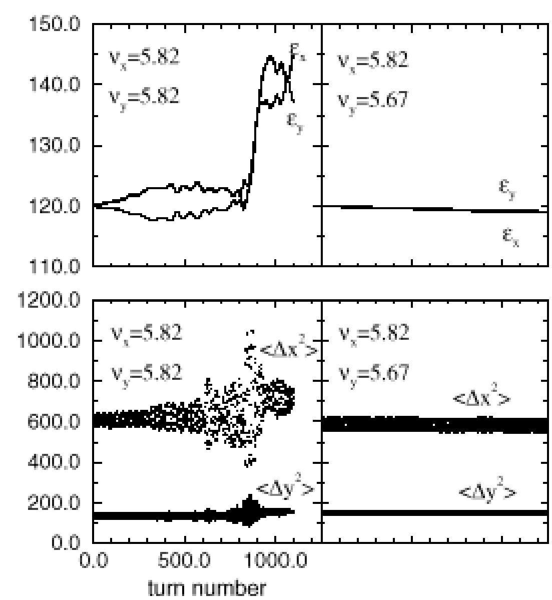

Figure 8: Plots of rms emittances (top plots) and second order moments (bottom plots) for two different operating points.

5 REFERENCES

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