The Chaotic Behavior of the Bunched Beam

Z. Parsa*, Brookhaven National Laboratory, Physics Dept., 510 A, Upton, NY 11973, USA.
V. Zadorozhny†, Institute of Cybernetic, National Academy of Sciences of Ukrain.

Abstract

Using the self selfconsistent Vlasov equation we discuss a wave dynamical system to describe the chaotic behavior of the bunched beam, present some results of the existence of the global solutions as the generalized functions. Disappearance of the first integral, and appearance of the wave packet chaos due to birth of the continuous spectrum in Vlasov system is studied. We propose a new concept of wave packet chaos to describe the chaotic behavior of the wave dynamical system.

1 INTRODUCTION

The Vlasov system for the beam distribution function f has the form

$$\frac{\partial f(r,v,t)}{\partial t} + div_r v f + div_v <\dot{v}>= 0 \tag{1}$$

$$<\dot{v}>= \frac{\int\limits_{(\infty)} \dot{v}\, f(r,v,\dot{v},t) d\,\dot{v}}{f(r,v,t)} = \frac{e}{m}(E + \frac{1}{c}[vH]), \quad rotH - \frac{1}{c}\frac{\partial E}{\partial t} = \frac{4\pi}{c} e \int\limits_{(\infty)} v f dv,$$

$$divE = 4\pi e \int\limits_{(\infty)} f dv \tag{2}$$

$(rotE + \frac{1}{c}\frac{\partial H}{\partial t} = 0, \quad divH = 0)$,
where r is the three - dimensional vector, $r = (x_1, x_2, x_3)$ are stochastic coordinates of the particles, $v = (v_1, v_2, v_3)$ is their velocity, E and H are the electric and magnetic fields correspondently. Notice, the fields are sum of both external and excited fields, namely in this there is a selfconsistent sense. Let us find the solution of (1) in the form

$$f(r,v,t) = f_0(r,v)e^{-iwt},$$

here w is real frequency, the function f_0 is sought in the form of the Fourier series

$$f_0 = \sum C_k(v)e^{ikr} \tag{3}$$

where

$$kr = k_1x_1 + k_2x_2 + k_3x_3,$$

i.e. the vector $k = (k_1, k_2, k_3)$.

Under these conditions (1) may be rewritten as

$$i(k \cdot r - w)C_k + \frac{\partial C_k}{\partial v}(E + \frac{1}{C}[vH]) = 0 \tag{4}$$

By using (4) we introduce a linear differential operator L_0 :

$$L_0 g = (E + \frac{1}{c}[vH])\frac{\partial g}{\partial v} + i(kr + w)g,$$

where L_0 is an operator in a given Hilbert space $L^2(\Omega_r)$ and $g = g(\cdot)$ is element of the same space. Here Ω_v is a domain of admissible velocities.

The motion of particles of bunched beam are fulfilled in the space $\Omega_r \times \Omega_v$, $\Omega_r = \{r : |r| \leq \infty\}$, $\Omega_v = \{v : |v| \leq \infty\}$.

The variables r, v are governed by the following equation

$$\begin{aligned} \dot{r} &= v \\ \dot{v} &= \frac{e}{m}(E + \frac{1}{c}[vH]) \end{aligned} \tag{5}$$

as well known, here $\Omega_r \times \Omega_v \subset R^6$.
Thus

$$\sum_1^3 \frac{\partial v_s}{\partial x_s} = \sum_1^3 \frac{\partial [vH]_k}{\partial v_k} \equiv 0$$

for this reason (well known Liouville theorem) the measure $\partial\mu = dr \times dv$ is the invariant measure for a group T_t (5), i.e. $T_t\mu_0 = \mu_t$ for all $t \in [-\infty, \infty]$, that is easy to see. Consider an invariant measure on $\Omega_r \cdot \Omega_v$, simplify to solve linear partial differential (4) by eigenvalue method, because we now have the eigenvalue problem with electromagnetic dependent coefficients and the zero eigenvalue. We claim that the eigenvalues will be points of the continuous spectrum and eigenvector of (4) will be chaotic in the phase space in the present case. It is interesting to know if it is the case and how should one solve this kind of eigenvalue problem when the system (4) is chaotic.

2 MAIN RESULT

Let us consider the following operator L that is selfadjoint extensions of the operator L_0 in Hilbert space $L^2(\Omega_v)$. In accordance with the Stone theorem, the operator $L = L^*$ generates the group of transformation $U_t = e^{itL}$, such that

$$iL = \lim_{t\to 0} \frac{U_t\varphi - \varphi}{t}.$$

Let $e_k(\lambda)$ be the eigenfuction of the group U_t then

* Supported by US Department of Energy contract Number DE-AC02-98CH10886. E-mail: parsa@bnl.gov
† zvf@umex.istrada.net.ua

$$U_t e_k(\lambda) = e^{i\lambda t} e_k(\lambda),$$

$k = 1, ..., \dim L_\lambda$, here L_λ is a multiple of the point $\lambda \in \sigma(L)$ and σ is the spectrum of the operator L.

The element $e_k(\lambda)$ belongs to the space $H_{-1}(\Omega_v) = H_1(\Omega_v)^*$ ([2] p.387).

It is a direct consequence of the existence of the invariant measure in dynamical system (5).

It is well known that $e_k(\lambda) \in H_{-1}(\Omega_v)$ and $e_k(\lambda) \notin C(\Omega_v)$ iff it is the point of the continuous spectrum.

In this case the first integral will be absent for dynamical system (5) and it has become the transitive system. In particular this reasoning yields the first integral destruction. A. Einstein, [1] has given conditions under which the first integral disappears.

Summary 1 *The electro - magnetic field in (5) can generate the ergodic or chaotic motion. Assuming ergodic is equivalent to the chaos. It is possible to develop the chaos to form vortex, solution like vortex and so on. The mathematical aspects of this theory and implications in chaos theory will be given in the future.*

3 REFERENCES

[1] A. Einstein Eine bleitung des Theorems non Jacobi. preuss. Akad. Wiss.,1917 pt.2, 606-608

[2] I. Gelfand, A. Kostjuchenko *On decomposition of differential and other operators onto eigenfunetions*, DAN SSSR, 103, N3, 1955,349-352.

[3] M. Stone On one-parameter unitary groups in Hilbert space. Ann. of Math., 33,1932,643-648.

[4] Z. Parsa, V. Zadorozhny, Focusing and Acceleration of Bunched Beams, in AIP conf. Proc. 530, pp. 249-259 (1999).