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Dynamic Aperture Tracking for Fermilab Recycler Ring

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Abstract

The study of dynamic aperture in Fermilab Recycler Ring with the latest designed low beta straight section was done by MAD. The calculation shows that the optical design of the low beta section improves the optical properties of the ring. Without misalignments, tracking calculations for on-energy particles over 10^5 turns including all the field errors predict a dynamic aperture of 16.5σ, which is 1.6 times larger than the aperture in Recycler Ring with high beta straight section. Special attention was paid to treat gridiant magnets and their field errors in MAD and the detailed analysis was made in this paper.

1 INTRODUCTION

The high beta straight section ($HB30$) in Fermilab Recycler Ring [1] has been replaced by a low beta straight section ($LB30$) for stochastic cooling. The maximum beta functions in the lattice with $LB30$ now are 71.12m in x - plane and 56.18m in y - plane (Before, they were 243.50m and 241.53 m respectively). The fractional parts of the tunes in both plane are more closed to the desired values (0.425, 0.415). The integer part of the tune in x - plane in this lattice is 25, one integer larger than that in the lattice with $HB30$. We therefore expect that the low beta lattice will have a higher dynamic aperture than the high beta lattice. This report will quantitatively estimate the difference in the theoretical dynamic aperture.

2 DYNAMIC APERTURE TRACKING

The code used for the tracking is MAD ($Ver8.16$). The performance in the Recycler Ring is tested by launching an array of particles at different amplitudes in the presence of magnetic field errors. In the bare recycler lattice of *Version 20*, the designed angle and measured normal quadrupole and sextupole components of the gradient magnets are included in their definitions, and the measured normal quadrupole component of quadrupoles is reflected into quadrupole definitions. All the other high order multipole components of one magnet are treated in the lattice as two thin lenses, placed at the entrance and the exit of the magnet respectively. The measured dipole error is put in the lattice as an EFIELD element. The misalignments of the ring have not yet been included in the tracking simulations. The tune shifts resulted from field errors is $\triangle Q_x = 0.00076, \triangle Q_y = 0.000376$.

On the other hand, the chromaticity sextupoles are used to correct the linear chromaticities to the designed value Q_x = -2.0, Q_y = -2.0. Sextupoles are placed in the arc cells where the dispersion is non zero. In Recycler Ring, there are 8 and 16 sextupoles for the chromaticity correction in horizontal plane and in vertical plane, respectively. The corrected chromaticities $Q_x^{'}$ and $Q_y^{'}$ are -2.0 when the strengths of sextupoles $k2_F = 0.2627$ and $k2_D = -0.2794$.

The normalized $r.m.s.$ emittances (95%) are $18\pi mm.mrad$ in both planes. Particles are launched with a distribution of amplitudes with neighbouring particles differing in amplitude by either $1\sigma_{x0}$ or $1\sigma_{y0}$, which are the beam sizes at launched point. For each fixed x-amplitude, we search the largest y-amplitude for which the particle survives 100,000 turns. We also check that particles with smaller y amplitudes are stable over these numbers of turns. This is repeated for several x amplitudes until the largest y-amplitude falls to zero. The dynamic aperture is then defined as the average of all the largest stable radial amplitudes.

Include all the errors, the tracking over 10,000 turns in the lattices with both high beta and low beta section shows that the dynamic apertures are about 3σ in average, plotted in red line in Figure 1 and 2 . This value is much less than that expected. We attempt to split the gradient magnets into 16 pieces, each of them has the length of $L/16$, and the same magnetic field strengths. L is the magnet length, 3.099 meters for arc dipoles, 4.496 meters for dispersion magnets. Now by using MAD again, we find that the dynamic apertures are 9.46σ in average for high beta lattice and 16.07σ for low beta lattice over 100,000 turns. They are plotted in green line in Figure 1 and Figure 2 respectively.

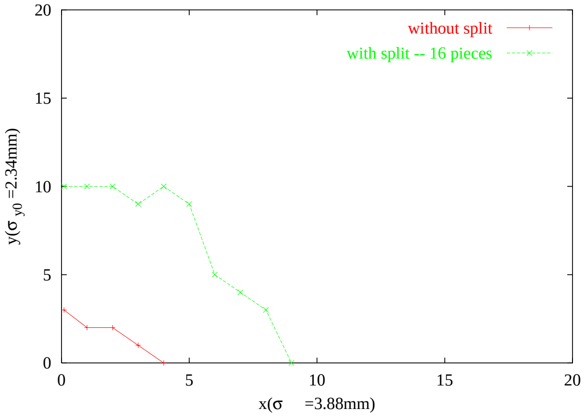

Figure 1: Dynamic aperture in Recycler Ring after 100,000 turns with the high beta insertion for two different models of the nonlinearites.

The tracking for the particles with momentum deviations with ±0.003 has been done, and the results are displayed

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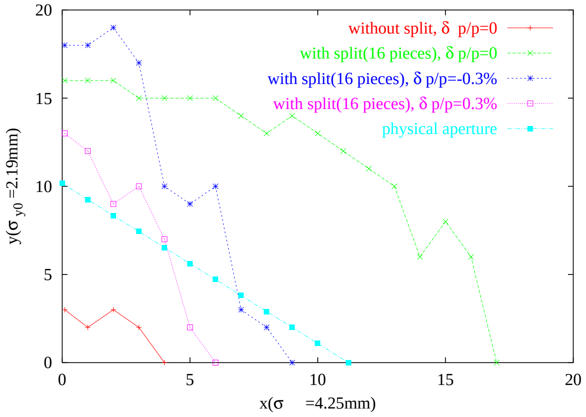

Figure 2: Dynamic Aperture in the Recycler Ring after 100,000 turns with the low beta insertion and for the same two models as in Fig. 1. Also shown are the dynamic apertures for particles with constant momentum deviations of ± 0.003. The beam pipe contour is drawn to illustrate the fact that in this tracking model, the dynamic aperture of particles with $\Delta p/p = 0$ exceeds the physical aperture.

Table 1: Average radial dynamic aperture calculated over 100,000 turns with the high and low beta lattices.

$\Delta p/p$ (%)	kick per magnet	$\langle DA \rangle$ (units of σ) High β lattice	Low β lattice
0.	1	3.0	3.0
0.	16	9.5	16.5
-0.3	16	-	13.0
+0.3	16	-	9.2

in Fig. 2. The results are summarized in Table 1. Particle with a constant negative momentum deviations have a large dynamic aperture than those with a positive momentum deviation. A possible exaplanation of this may be due to the tune dependence on amplitude and momentum deviation, and the resonances that are nearby. Fig. 3 shows that 7th and 12th order resonances are the closest resonances. Large amplitude particles e.g. at 7 σ with negative $\delta p/p$ are in the space between skew 7th order resonances which may not be strongly driven while those with positive $\delta p/p$ are closer to a normal 7th order resonance and several 12th order resonance.

3 ANALYSIS

The tracking results yielded a few surprises. The first surprise was that with only a single multipole kick in the center of each magnet, the dynamic aperture of both lattices was nearly the same. This result which is specific to this particular model of nonlinearities can be understood if the dynamic aperture is dominated by the nonlinear kicks from the arcs. Since the arcs have more magnets than the straight sections and the beta functions in the arcs are of the same order of magnitude as those in the straight sections for both lattice, this explanation is in the realm of being plausible but somewhat unexpected nonetheless. This explanation turns out to be wrong however when the nonlinear lattice model is changed to incorporate 16 multipole kicks, each with 1/16th of the integrated strength of the single kick in the previous model, along the length of each gradient magnet. We found that the dynamic aperture increases in this model for both lattices but the low beta lattice has the larger dynamic aperture - the result that we expected. The surprise in this model was the rather large increase in dynamic aperture when the number of kicks per magnet was increased to 16.

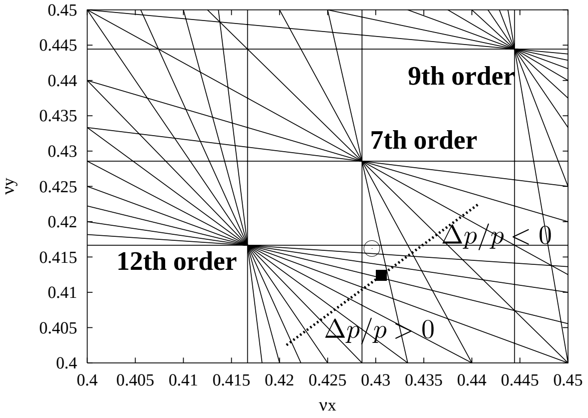

Figure 3: Resonances and the working point shown as a filled box. The dotted line shows the tunes for zero amplitude particles with momentum deviations in the range $-0.005 \leq \Delta p/p \leq 0.005$ due to a linear chromaticity of $Q' = -2$.

We observe that the beta functions are not varying rapidly along the length and furthermore the horizontal beta functions are in fact *larger* at several locations with the splits than the beta function at the center without the splits. The key to the source of the change lies in the observation that there is a phase advance of 7 degrees along the length of each gradient magnet. When there are several kicks along the length, each of these occurs at a slightly different phase. The resultant of these somewhat incoherent kicks will always be smaller than a single coherent kick which has the same length as the sum of all the individual kicks. This is a qualitative explanation of the effects of several incoherent kicks. A more exact calculation shows that the change in the linear Courant-Snyder invariant J decreases as the number of kicks is increased from 1 to 2 and based on these arguments we expect that the change in J will decrease with increasing number of kicks until we reach the asymptotic limit when the change in J remains nearly constant.

There are several ways of determining the minimum number of kicks necessary to reach the asymptotic limit. We compare the model where the gradient magnets are split into 16 pieces with another model where the gradient magnets are split into 32 pieces and the quadrupoles are split into 4 pieces. The tune shifts are determined by tracking two groups of particles for 1024 turns . Within each group the $x$amplitude is fixed, one at $0.1\sigma_{x0}$ and the other at $10\sigma_{x0}$ respectively, while the y amplitudes are varied from $1.0\sigma_{y0}$ to $11.0\sigma_{y0}$.

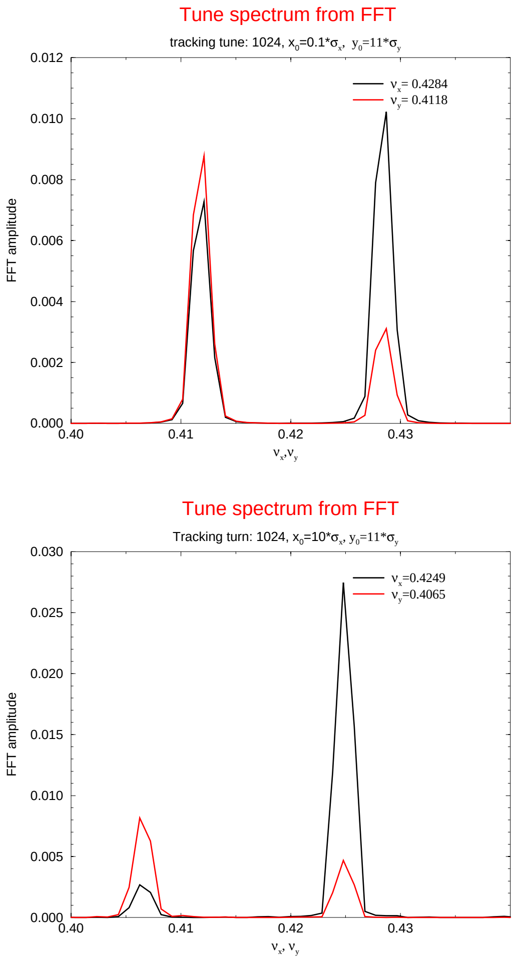

Figure 4: Tune spectrum of two particles at the same large y amplitude of $11\sigma_{y0}$ but at different x amplitudes of $0.1\sigma_{x0}$ (left figure) and $10\sigma_{x0}$ (right figure).

The FFT spectrum for each particle under the influence of all field errors is calculated from the tracking results. Figure 4 shows the FFT spectra for two selected particles with the same y amplitude but at two different x-amplitudes. A Hanning filter is applied to improve the resolution of the tune calculation. Even though the y amplitudes of these two particles are the same, there is a significant difference in the vertical tune $\Delta\nu_y = 0.0022$ due to the large cross-detuning term $\partial\nu_y/\partial J_x$ (=$\partial\nu_x/\partial J_y$) or in words due to the dependence of the vertical tune on the horizontal amplitude. We also observe a significant signal of the horizontal tune in the vertical spectrum (and vice versa) which indicates that the coupling is non-negligible.

The tune difference between the lattices with two split models vs the particle amplitudes is found within 1.4×10^{-5} which is also roughly the resolution of the tune calculation with the Hanning filter. We take this convergence in tune shifts to indicate that splitting the gradient magnets into 16 pieces suffices for the calculation of the dynamic aperture.

Figure 5 shows the variations of horizontal tune and vertical tune with the particle amplitudes.

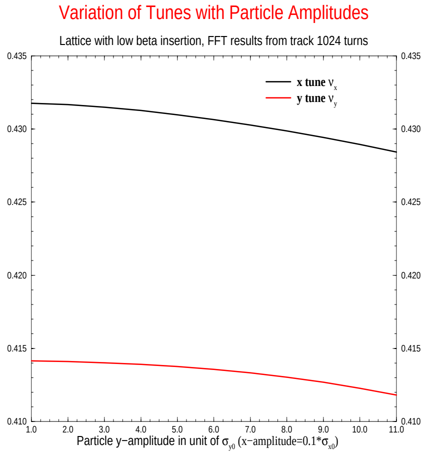

Figure 5: Variations of horizontal tune and vertical tune with the particle amplitudes

4 CONCLUSION

This study of dynamic aperture in Recycler Ring with low beta straight section shows that the optical design of the low beta section improves the optical properties of the ring. Without misalignments, tracking calculations for on-energy particles over 10^5 turns predict a dynamic aperture of 16σ which is larger than the physical aperture of $47.625mm \times 22.225mm$. Misalignments will reduce this dynamic aperture but, if small enough, they should not limit the performance of the Recycler.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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[3] Tanaji Sen, Personal Memo.