BROAD-BAND MODEL IMPEDANCE FOR THE ADVANCED PHOTON SOURCE STORAGE RING*

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Abstract

The vacuum chamber components of the Advanced Photon Source storage ring have been modified since the facility's initial construction. A notable modification includes narrow-gap chambers for insertion devices. We used the Bane-Heifets model to describe the broad-band longitudinal impedance of new and old vacuum components in the ring. We found that the model with a few expansion terms fitted the wake potential well. In this paper we present the impedances of individual components and the impedance of the ring.

1 INTRODUCTION

The broad-band impedance may be represented by the expansion over $\sqrt{\omega}$:

$$Z(\omega) = jL\omega + R + B(1+j)\sqrt{\omega} + Z_c(1-j)/\sqrt{\omega} + \ldots \tag{1}$$

This model was proposed by S. Heifets [1] as the further development of K. Bane's approach [2]. Each term used in the expansion has definite physical meaning as an inductor, a resistor, a resistive wall, or a cavity. Then, Equation (1) represents the impedance elements connected in series around ring.

We used this model to characterize the impedance-generating element in the Advanced Photon Source storage ring. The coefficient of each term was extracted by fitting the wake potentials given by ABCI [3] and MAFIA [4]. Similar work for DAΦNE main ring was reported in [5]. However, our methods are distinguished from DAΦNE work in two ways. First, we expanded model (1) to include high-order terms. Explicit expression is given as:

$$\begin{aligned} Z(\omega) &= j(L_1\omega + L_3\omega^3 + L_5\omega^5 + L_7\omega^7 + \ldots) \\ &+ (R_0 + R_2\omega^2 + R_4\omega^4 + R_6\omega^6 + \ldots) \\ &+ (B_1(1+jsign)|\omega|^{1/2} + B_3(1-jsign)|\omega|^{3/2} + \ldots) \\ &+ (Z_c(1-jsign)|\omega|^{-1/2} + \ldots) \end{aligned} \tag{2}$$

We found this extension necessary for the model to cover the spectrum of bunch length down to 10 mm. Second, we fit multiple wake potentials for different bunch lengths as a whole. If we fit for each bunch length individually, the coefficient varies from bunch to bunch. Our method constrains the coefficient to be unique, and makes the resultant fit the best fit for all the data in the least-square sense.

*Work supported by U.S. Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.
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In Section 2 we present the formulae for the model wake-function, which corresponds to each term in Eq. (2). Most of the formulae are new. In Section 3 we demonstrated the model in Eq. (2) applied to the vacuum components in the ring. The impedance of the ring is given as a final result.

2 BASIC FORMULAE

The wake potential of bunched beam is defined as:

$$W(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z(\omega)\lambda(\omega)e^{j\omega s/c}d\omega, \tag{3}$$

in which we used the $\exp(j\omega t)$ convention for time. We use this definition to derive model wake-functions by substituting $Z(\omega) = (j\omega)^n$ or $(j\omega)^{n/2}$ for Gaussian density, defined as:

$$\lambda(\omega) = \exp\left(-\frac{\omega^2}{2(c/\sigma)^2}\right) \Leftrightarrow \lambda(s) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{s^2}{2\sigma^2}\right). \tag{4}$$

We substitute $Z(\omega) = jL_{2n+1}\omega^{2n+1}$ in Eq. (3) to find the wake-function for the generalized inductive impedance:

$$W(s) = \frac{L_{2n+1}}{\sqrt{2\pi}}\frac{(-1)^n}{2^n\sqrt{2}}\left(\frac{c}{\sigma}\right)^{2n+2}\exp\left(-\frac{s^2}{2\sigma^2}\right)\mathrm{H}_{2n+1}\left(\frac{s}{\sqrt{2}\sigma}\right), \tag{5}$$

where $H_n(x)$ is the Hermite polynomial.

The wake-function corresponding to the generalized resistance, $Z(\omega) = R_{2n}\omega^{2n}$, is in the form:

$$W(s) = \frac{R_{2n}}{\sqrt{2\pi}}\frac{(-1)^n}{2^n}\left(\frac{c}{\sigma}\right)^{2n+1}\exp\left(-\frac{s^2}{2\sigma^2}\right)\mathrm{H}_{2n}\left(\frac{s}{\sqrt{2}\sigma}\right). \tag{6}$$

We now consider a generalized resistive-wall impedance $Z(\omega) = b_n(j\omega)^{n/2}$. For n = 1, the impedance can be written as $Z(\omega) = B_1(1 + j\,\mathrm{sign}(\omega))|\omega|^{1/2}$, and the corresponding wake-function given by:

$$\begin{aligned} W(s>0) &= \frac{-B_1}{4}\left(\frac{c}{\sigma}\right)^{3/2}\left(\frac{|s|}{\sigma}\right)^{3/2}e^{-b}\left[I_{1/4}^{+}(b) - I_{3/4}^{+}(b)\right] \\ W(s<0) &= \frac{+B_1}{4}\left(\frac{c}{\sigma}\right)^{3/2}\left(\frac{|s|}{\sigma}\right)^{3/2}e^{-b}\left[I_{1/4}^{-}(b) + I_{3/4}^{-}(b)\right], \end{aligned} \tag{7}$$

where $I_n^{\pm}(b) = I_{-n}(b) \pm I_n(b)$, $I_n(b)$ is a modified Bessel function of first kind, and $b = s^2/4\sigma^2$.

For n = 3, the impedance can be written as $Z(\omega) = B_3\,(1 - j\,\mathrm{sign}(\omega))\,|\omega|^{3/2}$, and the corresponding wake-function is given by:

$$W(s>0) = \frac{-B_3}{8}\left(\frac{c}{\sigma}\right)^{5/2}\left(\frac{|s|}{\sigma}\right)^{5/2} e^{-b}\left[2I^{+}_{1/4}(b) - 3I^{+}_{3/4}(b) + I^{+}_{5/4}(b)\right]$$
$$W(s<0) = \frac{+B_3}{8}\left(\frac{c}{\sigma}\right)^{5/2}\left(\frac{|s|}{\sigma}\right)^{5/2} e^{-b}\left[2I^{-}_{1/4}(b) + 3I^{-}_{3/4}(b) + I^{-}_{5/4}(b)\right]. \tag{8}$$

For n = 5, the impedance can be written as $Z(\omega) = B_5\,(1 + j\,\mathrm{sign}(\omega))\,|\omega|^{5/2}$, and the corresponding wake-function is given by

$$W(s>0) = \frac{+B_5}{16}\left(\frac{c}{\sigma}\right)^{7/2}\left(\frac{|s|}{\sigma}\right)^{7/2} e^{-b}\left[5I^{+}_{1/4} - 9I^{+}_{3/4} + 5I^{+}_{5/4} - I^{+}_{7/4}\right]$$
$$W(s<0) = \frac{-B_5}{16}\left(\frac{c}{\sigma}\right)^{7/2}\left(\frac{|s|}{\sigma}\right)^{7/2} e^{-b}\left[5I^{-}_{1/4} + 9I^{-}_{3/4} + 5I^{-}_{5/4} + I^{-}_{7/4}\right]. \tag{9}$$

For the cavity impedance in the form of $Z(\omega) = Z_c\,(1 - j\,\mathrm{sign}(\omega))\,|\omega|^{-1/2}$, the formula was reported in Ref. [3].

3 BROAD-BAND IMPEDANCE OF RING

3.1 RF Stations

There are four rf stations in the APS storage ring. Each station, accommodating four 352-MHz rf cavities, starts and ends with tapered transitions. We used ABCI to calculate the wake potential of an rf cavity. The wake potentials of the different bunch lengths, which are 10, 15, 20, and 25 mm, are used in determining the impedance. By fitting the wake potentials to the model in Eq. (2), we found the impedance of an rf cavity in the form:

$$\frac{Z}{n} = 3.200\,\frac{1-j}{n\sqrt{n}}\ \mathrm{k}\Omega\,, \tag{10}$$

where *n* is the harmonic number: $n = \omega/\omega_0$.

Transition was approximated as the sum of taper-in and taper-out. Interference effects between the cavities and the tapered section were in general neglected because it tends to reduce the total wakefield. However, for the bunch length of 10 mm, we found that the interference effects almost doubled the simple sum. By taking this into account, the resultant impedance for the rf-transition is

$$\frac{Z}{n} = j\,(8.972\,\mathrm{x}10^{-3} - 3.927\mathrm{x}10^{-12}\ n^2) + (\frac{-4.680}{n} + 2.761\mathrm{x}10^{-7}\ n - 5.436\mathrm{x}10^{-17}\ n^3)\,. \tag{11}$$

3.2 ID Chambers

The vacuum chamber for the insertion device (ID) has an elliptical cross-section. Currently there are nineteen 8-mm gap (height of minor axis), two 5-mm gap, and one 19.8-mm gap chambers in the ring. The dimensions of the cross-sections, as well as the transition, are depicted in Fig. 1.

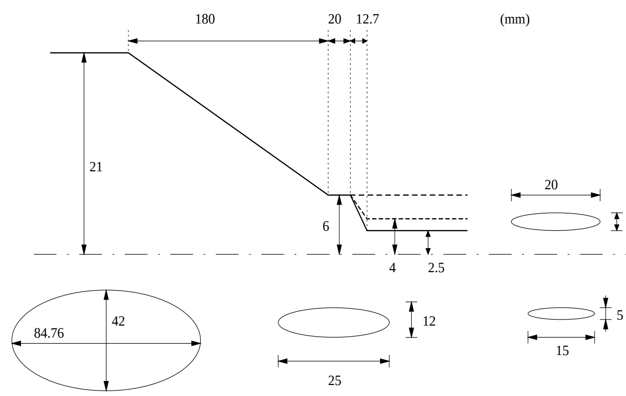

Figure 1: Schematic drawing showing transition from regular to ID chamber.

Because of 3-D geometry, we used MAFIA to calculate the wake-potential for gaps of 5 mm, 8 mm, and 12 mm. We used a long attached tube after the ID transition to calculate the wake potential accurately (DIRECT-method problem). This restricted us to simulate shortest bunch lengths up to 10 mm. This is generally true for all 3-D MAFIA simulations considered in this paper. We fitted the wake potentials of the bunch lengths from 10 mm to 30 mm. We found the impedance model for 5 mm to be:

$$\frac{Z}{n} = j\,(2.124\mathrm{x}10^{-3} - 3.562\mathrm{x}10^{-13}\ n^2) + (\frac{-0.1832}{n} + 4.341\mathrm{x}10^{-9}\ n - 1.743\mathrm{x}10^{-18}\ n^3)\,, \tag{12}$$

for 8 mm:

$$\frac{Z}{n} = j\,(1.758\mathrm{x}10^{-3} - 3.274\mathrm{x}10^{-13}\ n^2) + (\frac{-0.2098}{n} + 4.571\mathrm{x}10^{-9}\ n - 1.956\mathrm{x}10^{-18}\ n^3)\,. \tag{13}$$

3.3 X-Ray Absorber

Each sector among 40 in the ring has five x-ray absorbers: two crutch absorbers and three end absorbers. Crutch absorbers intrude from ante-chamber into the beam chamber by 1 cm. Impedance is mostly inductive. The fit of wake potential from MAFIA calculation shows the impedance as:

$$\frac{Z}{n} = j\,(2.129\mathrm{x}10^{-5} - 4.681\mathrm{x}10^{-15}\ n^2) + (\frac{-0.0134}{n} + 3.230\mathrm{x}10^{-10}\ n + 2.120\mathrm{x}10^{-19}\ n^3)\,. \tag{14}$$

The wake potentials generated by the model in Eq. (14) and given by MAFIA calculations are shown in Fig. 2 for a comparison; wake potentials are normalized by the maximum and the positions in the bunch are normalized by sigma; symbols represent MAFIA results and the lines are fit. We can see that the model in Eq. (14) fits the wake potentials of different bunch lengths pretty well.

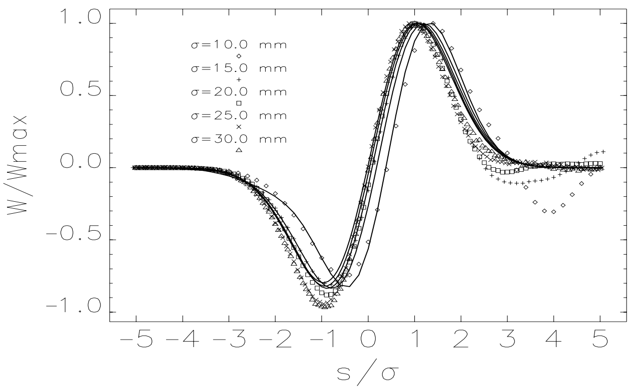

Figure 2: Wake potentials from MAFIA simulation.

End absorbers intrude into the beam chamber less than crutch absorbers. However, we used Eq. (14) for the end absorber too.

3.4 Beam Scraper and Stripline Monitor

We have two horizontal scrapers, two vertical scrapers, and one window-type scraper. The component consists of the scraper itself and vacuum housing with elliptic-to-circular transitions. The horizontal scraper shows inductive behavior; however, we found that the impedance of the vertical scraper is the resistive-wall type.

Impedance of the horizontal scraper takes the form:

$$\frac{Z}{n} = j\,(1.000\text{x}10^{-2} - 2.557\text{x}10^{-11}\ n^2) + (\frac{-10.55}{n} + 8.507\text{x}10^{-7}\ n - 1.050\text{x}10^{-15}\ n^3), \tag{15}$$

whereas the vertical scraper impedance has the form:

$$\frac{Z}{n} = (\frac{-127.6}{n} - 2.883\text{x}10^{-6}\ n) + (\frac{2.420\,(1+j)\sqrt{n}}{n} + \frac{2.629\text{x}10^{-4}\,(1-j)\sqrt{n^3}}{n} + \frac{5.292\text{x}10^{-9}\,(1-j)\sqrt{n^5}}{n}). \tag{16}$$

The stripline monitor has a cavity-like structure housing the electrode. The voltage induced between the electrode and adjacent chamber wall gives rise to wake potential. By fitting the wake potentials from MAFIA, we found the model impedance is

$$\frac{Z}{n} = j\,(7.927\text{x}10^{-4} - 1.334\text{x}10^{-12}\ n^2) + (\frac{-0.1825}{n} + 4.286\text{x}10^{-9}\ n - 1.649\text{x}10^{-18}\ n^3). \tag{17}$$

3.5 Injection Point

The injection point is the complex region where the septum and inner chamber meet. The simulated region covers about 75 cm of slanted structure and about 125 cm worth of empty tubes necessary for accurate simulation. The five-term model results in a good fit, which is:

$$\frac{Z}{n} = j\,(1.008\text{x}10^{-3} - 5.338\text{x}10^{-13}\ n^2) + (\frac{-0.8216}{n} + 3.543\text{x}10^{-8}\ n - 1.432\text{x}10^{-17}\ n^3). \tag{18}$$

3.6 Inductive Elements and Others

Table 1 shows the several known inductive elements in the ring. These elements contribute 0.134 Ω to the budget.

Table1: Inductance of Components

Component	Quantity	Im (Z/n)
Shielded Bellow	160	0.040
Shielded Transition	80	0.020
Welded Flange	480	0.010
Gate Valve	80	0.010
BPM	360	0.054

The resistive-wall impedance of elliptic vacuum chamber can be estimated by using the formula:

$$\frac{Z}{n} = \frac{Z_0\delta}{2<b>}\frac{1+j}{\sqrt{n}}F\ , \tag{19}$$

where Z_0 = 377 Ω, $<b>$ = 18.5 mm is the average chamber height, δ = 168 μm is the skip depth at the revolution frequency, and form factor F is close to unity. This results in $Z/n = 1.712\ (1+j)/n^{1/2}$.

Synchrotron radiation contributes about 0.15 Ω.

4 CONCLUSION

Total impedance of the Advanced Photon Source storage ring is

$$\frac{Z}{n} = j\,(0.237 - 8.090\text{x}10^{-11}\ n^2) + \frac{1.712(1+j)}{\sqrt{n}} + \frac{3.2x10^3(1-j)}{n\sqrt{n}} + (\frac{-48.6}{n} + 3.023\text{x}10^{-6}\ n - 2.338\text{x}10^{-15}\ n^3).$$

This does not include impedance due to synchrotron radiation, vertical scrapers, or miscellaneous components.

5 ACKNOWLEDGMENT

We thanks Accelerator System Division's Mechanical Group for providing all the drawings used in this work.

6 REFERENCES

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