ON THE COHERENT X-RAY RADIATION BY RELATIVISTIC ELECTRONS IN A CRYSTAL

N.F. Shul'ga* [a,b], V.I. Truten' [a]

[a] NSC "Kharkov Institute of Physics and Technology", Kharkov 61108, Ukraine

[b] Belgorod State University, Belgorod 308007, Russia

Abstract

The results of the coherent x-ray radiation simulation of relativistic electrons in a crystal are presented. We show, that the multiple electron scattering may cause a coherent radiation suppression in the low frequency region for electron energies of about several hundreds MeV.

1 INTRODUCTION

It is known that for ultra-high electron energy in amorphous medium the Landau-Pomeranchuk-Migdal effect of bremsstrahlung suppression in the low foton energy region may take place [1,2]. The effect is caused by the influence of the multiple particle scattering on bremsstrahlung. It arises when the condition for the dipole radiation of electrons in the matter is violated [3]. In [4] it was shown, that the similar effect is possible in a crystal even in the case, when the condition for the dipole radiation of relativistic electrons is valid. In this case, however, the effect is caused by the influence of the multiple particle scattering by the crystal atomic strings on the coherent radiation process. The problem considered in [4] is of the case, when electron collisions with different crystal atomic strings may be viewed as random. In the present paper this effect is considered on the basis of simulation in the actual dynamics of relativistic electrons in the periodic field of crystal atomic strings. It is shown, that the multiple particle scattering by the crystal atomic strings may significantly influence the coherent radiation even for particles with energy of about several hundreds MeV. In this case the coherent radiation suppression takes place for X-rays. The comparative analysis of the effects that are connected with the influence of the multiple scattering and medium polarization (Ter-Mikaelian effect) on coherent radiation in crystal is conducted also.

2. METHOD OF COHERENT RADIATION PROCESS SIMULATION

The process of relativistic electron radiation develops in a large length along particle motion direction. That region was titled as the coherence radiation process length [3,5]. This length is determined with an account of the effect of medium polarization on the radiation according to a formula

$$l_c = \frac{2\gamma^2}{\omega}\frac{1}{1+\gamma^2\omega_p^2/\omega^2}, \qquad (1)$$

where γ is the Lorenz factor of the electron, ω and ω_p are the radiation and plasma frequencies. If within the limits of the coherence length the electron collides with a large number of medium atoms, the radiation in low frequency range may be considered on the basis of the classical radiation theory. In the dipole approximation, i.e. when the particle scattering angle ϑ_e in the media at the distances of order l_c is small compared to the characteristic value of the relativistic electron radiation angle $\sim\gamma^{-1}$, the radiation spectral density may be presented as

$$\frac{dE}{d\omega} = 2\pi e^2\omega\int_{\delta}^{\infty}\frac{dq}{q^2}\left[1-2\frac{\delta}{q}\left(1-\frac{\delta}{q}\right)\right]|\mathbf{W}(q)|^2, \qquad (2)$$

where $\delta = \frac{\omega}{2\gamma^2}\left(1+\frac{\gamma^2\omega_p^2}{\omega^2}\right)$ and $\mathbf{W}(q)$ is the Fourier transform of the transversal component of electron acceleration

$$\mathbf{W}(q) = \int_{-\infty}^{\infty} dt\ddot{\boldsymbol{\rho}}(t)\exp(iqt) \qquad (3)$$

When fast electron is moving in a crystal under a small ψ angle to one of the crystallographic axes (z axis) the electron trajectory is determined, basically, by the continuous potential of a crystal atomic string, i.e. by the lattice potential, averaged along the z-axis [6,7]. In the continuous potential field the component of particle momentum parallel to crystal axis is conserved. In this case the electron trajectory in the transversal plane is determined by a two-dimensional equation [7]

$$\ddot{\boldsymbol{\rho}} = -\frac{e}{\varepsilon}\frac{\partial}{\partial\boldsymbol{\rho}}U(\boldsymbol{\rho}), \qquad (4)$$

where e and ε are the charge and energy of an electron, $U(\boldsymbol{\rho})$ is the continuous potential of the crystal atomic

* shulga@kipt.kharkov.ua

.

string and $\boldsymbol{\rho} = (x, y)$ is the coordinate, orthogonal to *z*-axis.

The continuous potential $U(\boldsymbol{\rho})$ represents the sum of continuous potentials of a separate crystal atomic strings $U_R(\boldsymbol{\rho} - \boldsymbol{\rho}_n)$, whose axes $\boldsymbol{\rho}_n$ are periodically positioned in the transversal plane. Moving in such field, the electron sequentially collides with various crystal atomic strings. Essentially for this case is that the electron motion in the periodic field of the crystal atomic strings may be regular as well as chaotic with respect to crystal atomic strings [8]. Especially noticeably the various motion modes exhibit themselves at particle incidence angle, comparable to the critical angle of axial channeling $\psi_c = \sqrt{4Ze^2/\varepsilon d}$, where *Z* is the atomic number of lattice atoms and *d* is the distance between atoms in a crystal atomic string.

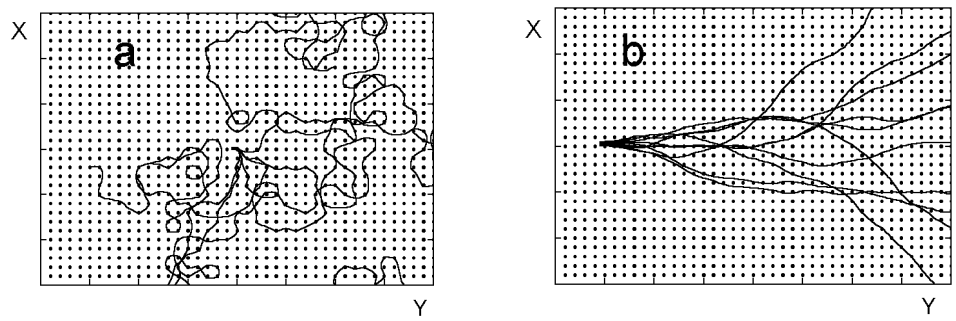

Figure 1: Trajectories of electrons in the plane (100) of silicon crystal, for $\psi = 0.5\psi_c$ (a) and $\psi = 1.5\psi_c$ (b).

Fig. 1 represents the results of calculation of electron trajectories in the plane, orthogonal to crystallographic axis <100> of silicon crystal. The points in these figures correspond to positions of axes of crystal atomic strings in the transversal plane. The various trajectories correspond to various initial values of $x = x_0$ coordinate within a unit cell. It is supposed here, that the angle ψ of the misalignment of the crystalline axis <100> relative to beam direction lies in (*z, y*) plane.

The obtained results show that for $\psi \sim \psi_c$ the electron motion in a periodic field of the crystal atomic strings is of a rather complicated behaviour. Therefore, for investigation of the relativistic electron radiation process in a crystal the simulation of the radiation process especially in conditions of actual particles dynamics in crystal, is very instrumental. Such method was proposed in paper [9]. It is based on the presentation of a particle trajectory in the transversal plane as of the broken line and consequent determination of Fourier component of acceleration from such trajectory using the formula

$$\mathbf{W}(q) = \sum_n \Delta t_n \dot{\mathbf{v}}_{\perp n} e^{iqt_n} \ , \qquad (5)$$

where t_n is the time instant of trajectory breaking, $\dot{\mathbf{v}}_{\perp n}$ is the acceleration at this moment and Δt_n is the time intervals breaking the trajectory. Such approach is valid, if within the limits of the coherence length of radiation process the large number of trajectory partitions is placed. Below we present the results of calculation of radiation in the case, when within the unit cell in the transversal plane up to several tens divisions of trajectory are present. It allows us to take into account the distinctions of radiation process in the field of each crystal atomic string, as well as the effect of interference of radiation at various crystal atomic strings.

Some results of simulation of coherent radiation process of electrons in a crystal that were obtained on the basis of above model were presented in paper [9]. In particular, it was shown, that for $\psi >> \psi_c$, when the particle trajectory in periodic field of a crystal atomic string is close to rectilinear, results of simulation are in good agreement with corresponding results of the Ter-Mikaelian [10] and Überall [11] theory of coherent radiation of relativistic electrons in crystal. Radiation spectrum in this case contains sharp maxima in the low frequency region with intensity considerably exceeding radiation intensity in amorphous medium.

With decreasing angle ψ the radiation intensity in the region of coherent maxima sharply increase. This is the frequency region, for which the length l_c is comparable with the particle run between collisions with crystal atomic strings. In [9] it was shown that for ψ of order of several ψ_c the simulation results rather noticeably differ from the Born theory of coherent radiation The simulation results in this case are close to corresponding results of the Überall [12] theory of coherent radiation, which is based on the random strings model.

3 EFFECT OF MULTIPLE SCATTERING ON THE COHERENT X-RAY RADIATION

Now we consider the dipole radiation in the frequency region, for which within the length l_c the electron collides with a large number of atomic strings. For electrons with energy about several hundreds MeV this frequency region corresponds to X-ray range. The results

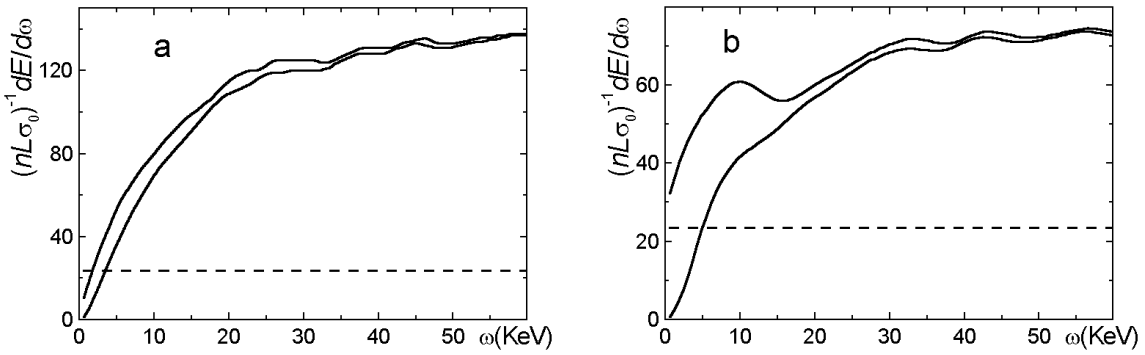

Figure 2: Spectrum of coherent X-ray radiation by electrons with energy ε=100 MeV, moving in silicon crystal at angles $\psi = 0.5\psi_c$ (a) and $\psi = 1.5\,\psi_c$ (b) to the <100> axis. Dashed line is Bethe-Heitler radiation spectrum; *n* is the atomic density, *L* is the target thickness, $\sigma_0 = Z^2 e^6 m^{-2}$.

of simulation of radiation process for electrons with 100 MeV energy in silicon crystal at angles $\psi = 0.5\psi_c$ (a) and $\psi = 1.5\,\psi_c$ (b) to <100> axis are presented in Fig. 2. The upper and lower curves correspond to simulations without and with effect of medium polarization on radiation.

The simulation results show that with ω decreasing in the considered frequency region the radiation intensity rapidly decreases. For $\psi = 0.5\psi_c$ the effect is due mainly to the influence of the multiple scattering of electron by crystal atomic strings on the coherent radiation process. For $\psi = 1.5\,\psi_c$ the decrease in radiation is caused mainly by the effect of medium polarization on the electron radiation in crystal. The above simulation results are in the good agreement with the corresponding predictions of the theory [4] which describes the influence of multiple scattering and medium polarization on coherent radiation of relativistic electrons in crystal and is based on the model of random collisions of electrons with crystal atomic strings. Notice, that according to prediction of random string model of Überall [11], the spectrum of radiation in the considered frequency region does not depend on frequency. It is so because this model does not account for the influence of multiple scattering on coherent radiation.

The presented results are valid if the dipole condition for the relativistic electron radiation in crystal $\gamma\vartheta_e >> 1$ is fulfilled. For $\psi < \psi_c$ the characteristic angle of electron scattering in crystal ϑ_e is of order of critical angle of axial channeling ψ_c and the dipole radiation condition takes the form

$$\gamma\psi_c << 1 . \qquad (6)$$

With the increase in particle energy this condition becomes invalid. In this case the effect of multiple scattering on coherent radiation stipulated by a non-dipole nature of electron radiation in crystal (see, e.g., [13]) becomes essential. This effect is similar to the Landau-Pomeranchuk-Migdal effect [1,2] of the influence of multiple scattering on bremsstrahlung of relativistic electrons in amorphous medium.

Therefore, even in conditions (6) of dipole radiation the multiple scattering can significantly influence the coherent electron radiation in crystal. With the increase in the electron energy the frequency region, in which the effect takes place, is widened.

Work is supported in parts by RFFR (project # 00-02-16337).

4 REFERENCES

[1] L.D. Landau, I.Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR, 92 (1953) 735.
[2] A.B. Migdal, Dokl. Akad. Nauk SSSR, 96 (1954) 49.
[3] A.I. Akhiezer and N.F. Shul'ga, High-Energy Electrodynamics in Matter (Amsterdam: Gordon and Breach, 1996).
[4] N.V. Laskin at al., JETP, 62 (1985) 438.
[5] M.L. Ter-Mikaelian, High Energy Electromagnetic Processes in Condensed Media (Wiley, New York, 1972).
[6] J. Lindhard, K.Dansk.Vid.Selsk.Math. Phys.Medd. 34, no. 14 (1965).
[7] D.S. Gemmell, Rev. Mod. Phys. 46 (1974) 129.
[8] A.I. Akhiezer, V.I. Truten', N.F. Shul'ga, Phys. Rep. 203 (1991) 289.
[9] N.F. Shul'ga at al., Nucl. Instr. and Meth. B 145 (1998) 128; B 115 (1996) 405.
[10] M.L. Ter-Mikaelian, Zh. Eks.Teor. Fiz. 25 (1953) 296.
[11] H. Überall, Phys. Rev., 103 (1956) 1055.
[12] H. Überall, Z.Naturforsch., A 17 (1960)332.
[13] N.F. Shul'ga, S.P. Fomin, JETP, 86 (1998) 32.