THREE-DIMENSIONAL THEORY OF COMPTON SCATTERING AND ADVANCED BIOMEDICAL APPLICATIONS

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Abstract

A complete, three-dimensional theory of Compton scattering is described, which fully takes into account the effects of the electron beam emittance and energy spread upon the scattered x-ray spectral brightness. This formalism is then applied to Compton scattering in a three-dimensional laser focus, and yields a complete description of the influence of the electron beam phase space topology on the x-ray spectral brightness; analytical expressions including the effects of emittance and energy spread are also obtained in the one-dimensional limit.

1 INTRODUCTION

In the linear regime, where the 4-potential amplitude satisfies the condition $eA/m_0c \ll 1,$ and in the absence of radiative corrections, in the so-called Thomson scattering regime, where the frequency cutoff is $\omega \ll m_0c^2/\hbar,$ as measured in the electron frame, the spectral photon number density scattered by an electron interacting with an *arbitrary* electromagnetic field distribution *in vacuum* is given by the momentum space distribution of the incident vector potential at the Doppler-shifted frequency [1]:

$$\frac{d^2N\left(k_\mu^s\right)}{d\omega_s d\Omega} = \frac{\alpha}{(2\pi)^4}\frac{1}{\gamma_0^2\omega_s}\left|\boldsymbol{k_s}\times\int_{\mathbb{R}^3}\left\{\left[1+\left(\frac{\boldsymbol{k}}{\kappa_s}\right)\boldsymbol{u}_0\cdot\right]\times \tilde{\boldsymbol{A}}\left[\omega_s-\frac{\boldsymbol{u}_0}{\gamma_0}\cdot\left(\boldsymbol{k}_s-\boldsymbol{k}\right),\boldsymbol{k}\right]\exp\left(i\boldsymbol{k}\cdot\boldsymbol{x}_0\right)\right\}d^3\boldsymbol{k}\right|^2 \tag{1}$$

Here, $k_\mu^s = \left(\omega_s,\boldsymbol{k}_s\right) = \omega_s\left(1,\hat{\mathbf{n}}\right)$ is the 4-wavenumber of the wave scattered in the observation direction $\hat{\mathbf{n}}$, at the frequency ω_s; $\alpha = e^2/2\varepsilon_0 hc \simeq 1/137.036$ is the fine structure constant; $u_\mu^0 = \left(\gamma_0,\boldsymbol{u}_0\right)$ is the electron initial 4-velocity; $x_\mu^0 = \left(0,\boldsymbol{x}_0\right)$ is its initial 4-position, and we have introduced the scattered light-cone variable, $\kappa_s = -u_0^\mu k_\mu^s = \gamma_0\omega_s - \boldsymbol{u}_0\cdot\boldsymbol{k}_s.$ The term $\left[1+\left(\boldsymbol{k}/\kappa_s\right)\boldsymbol{u}_0\cdot\right]$ is to be considered as an operator acting on the Fourier transform of the spatial components of the 4-potential, $A_\mu = \left(\mathcal{V},\boldsymbol{A}\right),$

$$\tilde{A}_\mu\left(k_\nu\right) = \frac{1}{\sqrt{2\pi}^4}\int_{\mathbb{R}^4} A_\mu\left(x^\nu\right)\exp\left(ik_\nu x^\nu\right)d^4k_\nu, \tag{2}$$

while the term $\exp\left(i\boldsymbol{k}\cdot\boldsymbol{x}_0\right)$ gives rise to the coherence factor.

2 THREE DIMENSIONAL LASER FOCUS

The transverse laser profile is specified at the focal plane, and propagated using the method discussed in [2], where the vector potential derives from a generating function: $\boldsymbol{A} = \nabla\times\boldsymbol{G}$; in this manner, the Coulomb gauge condition, $\nabla\cdot\boldsymbol{A} = 0,$ is automatically satisfied. For a linearly polarized Gaussian-elliptical focus, with focal waists w_{0x} and w_{0y}, and a monochromatic wave at the central frequency $\omega_0 = 1,$ with a Gaussian envelope of duration $\Delta t,$ the 4-potential is represented in momentum-space by

$$\tilde{A}_\mu\left(k_\nu\right) = \frac{\sqrt{\pi}}{2}A_0 w_{0x}w_{0y}\Delta t\times \exp\left\{-\left[\frac{w_{0x}k_x}{2}\right]^2-\left[\frac{w_{0y}k_y}{2}\right]^2-\left[\frac{\Delta t\left(\omega-1\right)}{2}\right]^2\right\}\times \delta\left(k_z-\sqrt{\omega^2-k_x^2-k_y^2}\right)\left[i\left(-\hat{\mathbf{x}}k_z+\hat{\mathbf{z}}k_x\right)\right]. \tag{3}$$

Here, we recognize the $\boldsymbol{k}_\perp$-spectrum, the frequency spectrum, the propagator, $\delta\left(k_\mu k^\mu\right)$, and the curl operator, as expressed in momentum space.

2.1 Spectral Brightness

The scattered radiation can now be determined by using Eq. (1); to obtain an analytical answer than can be further exploited to include the phase space topology of the electron beam interacting with the laser pulse, the paraxial propagator formalism is used: $\delta\left(k_z-\sqrt{\omega^2-k_x^2-k_y^2}\right),$ is replaced by $\delta\left[k_z-\omega+\left(k_x^2/2k_0\right)+\left(k_y^2/2k_0\right)\right].$

Finally, the normalized vector potential is given by $A_0 = \dfrac{e}{\omega_0 m_0 c}\sqrt{\dfrac{2}{\varepsilon_0 c}\dfrac{\mathcal{W}_0}{\pi w_0^2\Delta t}},$ as expressed in terms of the laser pulse energy $\mathcal{W}_0$, duration $\Delta t,$ frequency ω_0, and focal spot size w_0.

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It proves convenient to introduce the "cold" spectral brightness, defined as

$$S_0(\omega,\gamma,\theta,\varphi)=\omega\exp\left\{\frac{-\Delta\phi^2}{2}\left[\chi(\omega,\gamma,\theta,\varphi)-1\right]^2\right\}\times \frac{\left[\gamma\cos(\theta+\varphi)-u(\gamma)\cos\theta\right]^2}{\left[\gamma-u(\gamma)\cos\varphi\right]^4}. \quad (4)$$

Note that S_0 is a function of the electron initial energy, γ, scattering angle, θ, and incident angle, φ; we can then perform incoherent summations over the electron initial energy and momentum distributions to study the effects of energy spread and emittance. For conciseness, the scattered frequency is now labeled ω.

We start with the beam energy spread; the "warm" beam brightness is given by

$$S_\gamma(\omega,\gamma_0,\Delta\gamma,\theta,\varphi)=\frac{1}{\sqrt{\pi}\Delta\gamma}\times \int_1^\infty S_0(\omega,\gamma,\theta,\varphi)\exp\left[-\left(\frac{\gamma-\gamma_0}{\Delta\gamma}\right)^2\right]d\gamma, \quad (5)$$

where we have used a Gaussian distribution to model the beam longitudinal phase space, and where the analytical value of the normalization constant is very nearly equal to the exact value.

The integral over energy given in Eq. (5) can be expressed analytically by Taylor expanding the spectral brightness around the mean value of the beam energy, γ_0. Since the angular function $\mathscr{f}=\left|\frac{\hat{\mathbf{n}}\times(\kappa_0\hat{\mathbf{x}}_0+u_{0x}\hat{\mathbf{z}})}{\kappa_0^2}\right|^2$ is a slow-varying function of the energy, we can first write

$$S_\gamma(\omega,\gamma_0,\Delta\gamma,\theta,\varphi)\simeq\frac{\omega\,\mathscr{f}(\gamma_0,\theta,\varphi)}{\sqrt{\pi}\Delta\gamma}\times \int_{-\infty}^{+\infty}\exp\left\{-\frac{\Delta\phi^2}{2}\left[\omega\left(\frac{\gamma-u\cos\theta}{\gamma-u\cos\varphi}\right)-1\right]^2-\left(\frac{\gamma-\gamma_0}{\Delta\gamma}\right)^2\right\}d\gamma; \quad (6)$$

we then use the following approximation: $\left[\omega\left(\frac{\gamma-u\cos\theta}{\gamma-u\cos\varphi}\right)-1\right]^2\simeq\left[\mathscr{a}(\gamma-\gamma_0)+\mathscr{b}\right]^2$, where $\mathscr{a}=\frac{\omega}{\gamma_0^3}\frac{(\cos\varphi-\cos\theta)}{(1-\cos\varphi)^2}$ and $\mathscr{b}=\omega\left(\frac{\gamma_0-u_0\cos\theta}{\gamma_0-u_0\cos\varphi}\right)-1=\chi(\omega,\gamma_0,\theta,\varphi)-1.$

The integral in Eq. (6) can now be performed analytically to yield the spectral brightness degradation due to energy spread:

$$S_\gamma\simeq\frac{\omega\,\mathscr{f}(\gamma_0,\theta,\varphi)\exp\left(\frac{\mathscr{u}^2}{\mathscr{v}}-\mathscr{w}\right)}{\sqrt{\left[1+\frac{1}{2}\left(\Delta\phi\frac{\Delta\gamma}{\gamma_0}\right)^2\left[\frac{\omega}{\gamma_0^2}\frac{\cos\varphi-\cos\theta}{(1-\cos\varphi)^2}\right]^2\right]}}, \quad (7)$$

where $\mathscr{u}=\frac{1}{\Delta\gamma^2}\left[1+\frac{\mathscr{a}}{2}(\Delta\phi\Delta\gamma)^2\right]$, $\mathscr{v}=\frac{\Delta\phi^2}{2}\mathscr{a}\mathscr{b}$, and $\mathscr{w}=\frac{\Delta\phi^2}{2}\mathscr{b}^2$. Since $\mathscr{v}$ and $\mathscr{w}$ are both linear functions of $\mathscr{b}$, which is equal to zero at the peak of the x-ray spectrum, the exponential is equal to one for $\omega=\omega_x$. In addition, the factor $\left[\Delta\phi(\Delta\gamma/\gamma_0)\right]^2$ in the square root shows that the relative energy spread must be compared to the normalized laser pulse duration, which is equivalent to the number of electromagnetic wiggler periods; this indicates that to increase the x-ray spectral brightness by lengthening the drive laser pulse, the requirement on the electron beam energy spread becomes increasingly stringent. Finally, we note that as the normalized Gaussian energy distribution tends to a Dirac delta-function for zero energy spread, we have $S_\gamma(\omega,\gamma_0,\Delta\gamma=0,\theta,\varphi)=S_0(\omega,\gamma_0,\theta,\varphi)$.

2.2 Emittance Effects

We now turn our attention to the influence of the electron beam emittance:

$$S_\varepsilon(\omega,\gamma_0,\Delta\gamma,\theta,\varphi_0,\Delta\varphi_\varepsilon)=\frac{1}{\sqrt{\pi}\Delta\varphi_\varepsilon}\times \int_0^{2\pi}S_\gamma(\omega,\gamma_0,\Delta\gamma,\theta-\delta\varphi,\varphi+\delta\varphi)\exp\left[-\left(\frac{\delta\varphi}{\Delta\varphi_\varepsilon}\right)^2\right]d\delta\varphi, \quad (8)$$

where the spread of incidence angle is given in terms of the beam emittance by $\Delta\varphi_\varepsilon=\frac{\varepsilon}{\gamma_0 r_b}$, and where φ_0 is the mean incidence angle, defined by the laser and electron beams. Note the important geometrical correction term, $\theta-\delta\varphi$, which corresponds to the fact that the scattering angle is measured with respect to the initial electron velocity. The effects of emittance are found to be independent of φ_0. Considering the on-axis x-ray spectral line, it is clear that emittance both broadens the spectrum and decreases the peak spectral brightness; in addition, a low energy tail develops, with a structure related to the interference of the different x-ray cones radiated by the focusing electron beam.

At this point, the combined effects of energy spread and emittance can be studied by varying the bunch charge and modeling the behavior of the electron beam phase space as follows:

$$\frac{\Delta\gamma}{\gamma_0}(q) \simeq \sqrt{\left[\frac{\gamma_0}{2}\left(\omega_{rf}\Delta\tau\right)^2\right]^2 + \left(\frac{e}{m_0c^2}\frac{q}{2\pi\varepsilon_0 c\Delta\tau}\right)^2}, \quad (9)$$

where the first term is the spread due to the finite duration of the bunch in the rf accelerating bucket of frequency $\omega_{rf}/2\pi$, while the second term corresponds to space-charge; for the emittance, an empirical linear scaling with charge is chosen, with $\varepsilon(q) \simeq \sigma q$, and $\sigma = 1\,\pi$-mm mrad/nC. This results in brightness first scaling linearly with the charge, reaching a maximum near 0.5 nC, and starting to degrad thereafter under the combined influences of energy spread and emittance. This optimum value of the charge is quite interesting as it very nearly corresponds to the state-of-art for high-brightness photoinjectors.

3 CONCLUSIONS

In conclusion, we have presented a detailed theoretical description of the influence of the electron beam phase space on the brightness of Compton x-ray sources that are being developed for a number of new research areas, including the advanced biomedical applications presented in the Introduction. The main results obtained are the following: first, a fully covariant and nonlinear solution to the motion of an electron in plane wave of arbitrary intensity has been presented; second, the longitudinal phase space of the electron beam has been modeled in terms of energy spread, and an analytical expression of the corresponding x-ray brightness degradation has been derived, which clearly emphasizes the relation between the laser pulse bandwidth and the energy spread in determining the x-ray spectral brightness; third, the effects of the transverse electron bunch phase space have been included and shown to be independent from the interaction geometry; in addition, emittance can cause a low energy tail to develop in the x-ray spectrum, with a structure related to interference effects between x-ray cones pointing in different directions; fourth, the optimum bunch charge as been determined by taking into account the combined influence of the energy spread (both space-charge and bunch duration effects) and beam emittance; fifth, the laser focusing also degrades the x-ray brightness, and this effect is found to be strongly dependent upon the incidence angle between the laser and electron beams; the optimum geometry corresponds to head-on collisions, for which the laser focusing is a second-order correction; finally, 3D effects, including the shaping of the spectrum due to the convective terms induced by the electron motion through the focus, and timing jitter have been modeled precisely.

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5 REFERENCES

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