SPECTRAL-ANGULAR DISTRIBUTIONS OF RELATIVISTIC ELECTRONS' RADIATION IN A THIN LAYER OF MATTER

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Abstract

The formulas for spectral-angular density of bremsstrahlung of relativistic electron in a thin layer of matter are obtained. The influence of electron multiple scattering on medium atoms on spectral-angular performances of radiation in thin amorphous target is investigated. It is shown, that at large values of root-mean-square angle of multiple scattering in comparison with a characteristic angle of relativistic electron radiation the suppression of radiation effect similar to the Landau-Pomeranchuk-Migdal effect takes place. .

1 INTRODUCTION

The multiple scattering of high energy electrons in a thin layer of matter can significantly influence the bremsstrahlung process. A case of large coherence length as compared with a target thickness is the one of particular interest. It was shown in [1-3] that in this case an effect analogous to the Landau-Pomeranchuk-Migdal effect is possible. An experimental investigation of this effect has been recently realized on the SLAC accelerator [4,5]. The spectral characteristics of bremsstrahlung in the low frequencies range have been investigated there. In the present work the spectral-angular characteristics of bremsstrahlung in a thin layer of matter are examined. Particular attention is paid to the condition, at which the bremsstrahlung suppression effect reveals itself stronger in the angular distribution, than in the spectral distribution. It is shown that at the large values of root-mean-square angle of multiple scattering as compared with the characteristic radiation angle $\theta \sim \gamma^{-1}$ (where γ is the Lorenz factor), the angular distribution of radiation in the range of $\theta < \gamma^{-1}$ from the incident beam direction does not practically depend on the target thickness. Besides, the maximum of the angular distribution is situated at the angle $\theta \sim \gamma^{-1}$ to the initial direction of the electron beam.

2 GENERAL FORMULAS

The spectral-angular density of radiation of electron driven on the trajectory $\vec{r}(t)$ is determined in classical electrodynamics by the expression [6,7]

$$\frac{d^2E}{d\omega do} = \frac{e^2}{4\pi^2}\left[\vec{k}\times\vec{I}\right]^2, \qquad (1)$$

where $\vec{k}$ and ω are wave vector and frequency of the radiated wave and

$$\vec{I} = \int_{-\infty}^{\infty} \vec{v}(t)\, e^{i(\omega t - \vec{k}\vec{r})} dt\,. \qquad (2)$$

In a thin layer of matter the characteristic values of scattering angles of relativistic electron ϑ_e are small in comparison with a unit. If the coherence length of radiation process is big in comparison with thickness of the target

$$l_c \approx \frac{2\gamma^2}{\omega}\frac{1}{1+\gamma^2\theta^2+\gamma^2\vartheta_e^2} \gg T\,, \qquad (3)$$

then $\vec{I}$ can be represented as [7]

$$\vec{I} \approx \frac{i}{\omega}\left(\frac{\vec{v}'}{1-\vec{n}\vec{v}'} - \frac{\vec{v}}{1-\vec{n}\vec{v}}\right), \qquad (4)$$

where $\vec{v}$ and $\vec{v}'$ are velocities of an electron before and after scattering.

The spectral-angular density of electron radiation in this case is determined only by the scattering angle of a particle in matter. Putting (4) into (1), we shall obtain

$$\frac{d^2E}{d\omega\, do} = \frac{e^2\gamma^2}{\pi^2 p}\left[\frac{1+\alpha^2+\alpha^2\beta^2+2\alpha\beta\cos\varphi}{\left(1+\alpha^2\right)^2} - \frac{1}{p}\right], \qquad (5)$$

where $p = 1+\alpha^2+\beta^2-2\alpha\beta\cos\varphi$, $\alpha = \gamma\theta$, $\beta = \gamma\vartheta_e$, θ and φ are polar and azimuth angles of radiation, ϑ_e is a scattering angle of an electron. The angles θ and ϑ_e are counted from the direction of the initial velocity of an electron $\vec{v}$. The angle φ is an angle between vectors $\vec{k}_\perp$ and $\vec{v}'_\perp$ in a plane, orthogonal to $\vec{v}$.

The formula (5) is necessary for averaging on scattering angles of a particle in matter. If the distribution function of scattered particles $f(\vec{\vartheta}_e)$ is known, then the average value of spectral-angular density of radiation will be determined by the expression

$$\left\langle\frac{d^2E}{d\omega do}\right\rangle = \int d\vec{\vartheta}_e\, f(\vec{\vartheta}_e)\frac{d^2E}{d\omega do}\,. \qquad (6)$$

We shall note, that the formula (6) is fair for any targets. It is required only, that the target thickness was small in comparison with the coherence length of radiation. The different character of the scatterer will be exhibited only in a concrete kind of the distribution function $f(\vec{\vartheta}_e)$.

For an amorphous target particle distribution on scattering angles is determined by the function of Bethe-Molière [8,9]

$$f_{B-M}\left(\vec{\vartheta}_e\right) = \frac{1}{2\pi}\int_0^\infty \eta\, d\eta\, J_0(\eta\vartheta_e)\exp\left\{-nT\int d\sigma(\chi)\left[1-J_0(\eta\chi)\right]\right\}, \quad (7)$$

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where $d\sigma(\chi)$ is the differential cross-section of electron scattering by a separate atom at a small angle χ.

The distribution function (7) does not depend on φ, therefore in (6) the integration over φ can be executed

$$\left\langle \frac{d^2E}{d\omega\, do} \right\rangle = \int_0^\infty \vartheta_e d\vartheta_e\, f_{B-M}(\vartheta_e)\, \Phi(\theta,\vartheta_e),$$

$$\Phi(\theta,\vartheta_e) = \frac{e^2\gamma^2}{\pi^2}\left\{ \frac{2+\beta^2}{\sqrt{q}\left(1+\alpha^2\right)} - \frac{\left(1+\alpha^2+\beta^2\right)}{q\sqrt{q}} - \frac{1}{\left(1+\alpha^2\right)^2} \right\},$$

$$q = (1+\alpha^2+\beta^2)^2 - 4\alpha^2\beta^2. \tag{8}$$

At the experimental research of the spectral-angular density of radiation, one is usually interested in radiation into a definite solid angle, which is cut by a specific collimator. If we are interested in radiation into a small solid angle along the direction of an incident particle beam, the formula (8) should be integrated over the angles of radiation, which are cut by the collimator. For a round collimator with the collimation angle of radiation θ_c we have

$$\frac{dE_c}{d\omega} = 2\pi \int_0^{\theta_c} \theta\, d\theta \left\langle \frac{d^2E}{d\omega\, do} \right\rangle. \tag{9}$$

For large values of θ_c in comparison with the characteristic values of scattering angles $\vartheta_e \approx \sqrt{\overline{\vartheta^2}}$ and the radiation $\theta \approx 1/\gamma$ the integration in (9) can be executed in general form. In the outcome we shall obtain a spectral density of radiation in a thin layer of matter (see formulas (3.3) and (3.5) in paper [3]).

3 DISCUSSION OF OBTAINED RESULTS

Paying attention to some features of spectral-angular distributions of relativistic electrons radiation in a thin layer of amorphous matter, let's first consider angular distribution of electron radiation of an in the plane $(\vec{v},\vec{v}')$. Turning in (5) to the Cartesian coordinates $(\alpha_x = \alpha\cos\varphi, \alpha_y = \alpha\sin\varphi)$, we find, that for $\alpha_y = 0$

$$\frac{d^2E}{d\omega\, do} = \frac{e^2\gamma^2}{\pi^2}\left[\frac{\alpha_x}{1+\alpha_x^2} - \frac{\alpha_x - \beta}{1+(\alpha_x-\beta)^2} \right]^2 \tag{10}$$

At small values of scattering angle ($\beta << 1$)

$$\frac{d^2E}{d\omega\, do} \approx \frac{e^2\gamma^2}{\pi^2}\beta^2 \frac{\left(1-\alpha_x{}^2\right)^2}{\left(1+\alpha_x{}^2\right)^4} \tag{11}$$

This formula shows, that at $\beta << 1$ the maxima of angular distribution of radiation are located at $\alpha_x = 0$, and that at $\alpha_x = \pm 1$ the spectral-angular density of electron radiation equals zero. The main body of the spectral density of radiation in this case is concentrated in the range of angles $\alpha_x \approx 1$.

At large values of scattering angles ($\beta >> 1$) the angular distribution of radiation (10) has maxima at the angles $\alpha_x \approx 1$ and $\alpha_x \approx \beta - 1$, and equals zero at $\alpha_x \approx -1/\beta$ and $\alpha_x \approx \beta + 1/\beta$. The formula (10) also shows, that the angular density of radiation decreases rapidly at the angles $\alpha_x \le -1$ and $\alpha_x \ge \beta + 1$, and in the range of angles $1 \le \alpha_x \le \beta$ the angular density of radiation has comparable values in a rather broad interval of scattering angles β. In particular, for $\beta = 10$ the angular density of radiation has a minimum at $\alpha_x = \beta/2$ with emission intensity in a minimum only 50 % less than emission intensity in maxima. It means that at $\beta >> 1$ the main body of spectral density of electron radiation is concentrated in the range of angles $0 \le \alpha_x \le \beta$.

At $\beta >> 1$ (5) angular density of radiation in the directions close to the initial velocity of a particle is determined by the expression

$$\frac{d^2E}{d\omega do} \approx \frac{e^2\gamma^2}{\pi^2}\frac{\alpha^2}{\left(1+\alpha^2\right)^2},\ \alpha << \beta. \tag{12}$$

In this case angular density of radiation does not depend on a scattering angle of a particle.

Let's consider now the influence of multiple scattering on angular distribution of bremsstrahlung. At small angles of radiation in (8) the expansion of function $\Phi(\vartheta,\vartheta_e)$ in terms of β can be executed. As the first approximation of such an expansion the following expression for spectral-angular density of radiation is discovered

$$\left\langle \frac{d^2E}{d\omega do} \right\rangle = \frac{2e^2\gamma^2}{\pi^2}\overline{\beta^2}\frac{1+\alpha^4}{\left(1+\alpha^2\right)^4}, \quad \overline{\beta^2} = \gamma^2\overline{\vartheta_e^2}. \tag{13}$$

This formula coincides with corresponding result of the Bethe and Heitler theory for spectral-angular density of radiation, according to which the emission intensity grows linearly with the target thickness increasing (see, for example, formula (5.9) in paper [10]).

The target thickness increasing leads to breaking the condition $\overline{\beta^2} << 1$, and then the account of non-dipole effect at radiation is required.

At arbitrary $\overline{\beta^2}$ realization of the procedure of averaging in (8) is possible only on the basis of numerical methods.

In Fig. 1 the dependence of spectral-angular density of radiation from a polar angle of radiation θ at various values of the parameter $\sqrt{\overline{\beta^2}}$ is represented. The indicated curves show, that at $\overline{\beta^2} < 1$ the angular distribution of radiation has a maximum in the direction of the initial velocity of a particle and decreases rapidly with the growth of the radiation angle θ (see (13)). For $\overline{\beta^2} \sim 1$ the maximum of angular distribution of radiation shifts to the area of the radiation angles $\vartheta \sim \gamma^{-1}$. Then the linear growth of the emission intensity with the target thickness increasing, characteristic of the Bethe and Heitler theory, weakens.

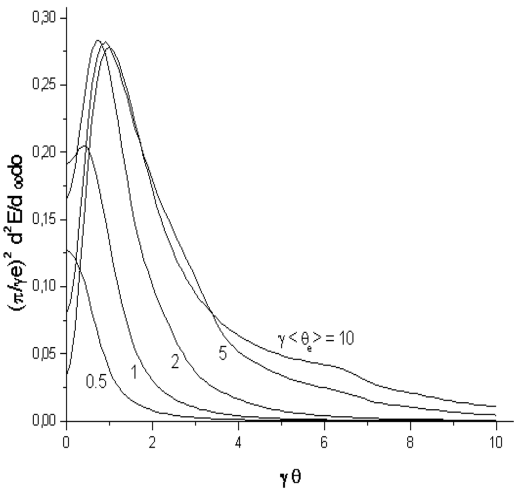

Figure 1: Angular distribution of radiation.

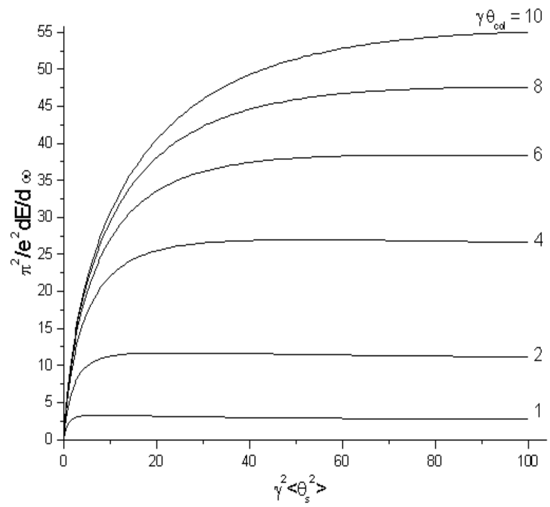

Figure 2: Spectral density of collimated radiation.

For $\overline{\beta^2} > 1$ the maximum of angular distribution of radiation is located in the range of angles $\vartheta \sim \gamma^{-1}$. The magnitude of the emission intensity in these maxima practically does not depend on the target thickness. The width of angular distribution of radiation, however, increases with the growth of the target thickness. In the range of angles $\vartheta << \gamma^{-1}$ the emission intensity decreases rapidly with the growth of $\overline{\beta^2}$.

At $\overline{\beta^2} >> 1$ the angular distribution of radiation in the range of angles $\vartheta << \sqrt{\overline{\beta^2}}$ is determined by the expression (18). In this range of radiation angles the angular distribution of radiation is universal, it does not depend on the kind of the distribution function of particles in scattering angles.

In Fig. 2 the dependence of spectral density of radiation in the definite collimator (13) on the value $\overline{\beta^2}$ is represented. The indicated curves show, that at $\overline{\beta^2} >> 1$ for small collimation angles $\vartheta_c << \sqrt{\overline{\vartheta_e^2}}$ the spectral density of radiation reaches maximum value and, unlike the corresponding result of Bethe and Heitler (13), does not depend on the value $\overline{\beta^2}$ (which is proportional to the target thickness).

With collimation angle increasing the value of the target thickness, starting with which the spectral-angular density of radiation practically does not depend on $\overline{\beta^2}$, grows. If the collimation angle of radiation is big enough ($\vartheta_c > \sqrt{\overline{\vartheta_e^2}} >> \gamma^{-1}$), then the radiation density grows logarithmically with the target thickness increasing . In this case practically all the radiation of a particle hits in the collimator and, actually, we deal with a spectral density of radiation (13).

Thus, at $\overline{\beta^2} >> 1$ in the range of radiation angles $\vartheta_c << \sqrt{\overline{\vartheta_e^2}}$ the effect of bremsstrahlung suppression (comparing with the Bethe and Heitler results) takes place. This effect is similar to the suppression effect of bremsstrahlung spectral density in a thin layer of matter, which was pointed at in works [2,3]. For spectral density of radiation, however, it is characteristic that the linear growth of emission intensity with the target thickness increasing is replaced by logarithmic growth. In the case under consideration the linear dependence of emission intensity on the target thickness is replaced by the constant. It means, that in the spectral-angular distribution of radiation the effect of bremsstrahlung suppression appears brighter, than in a spectral density of radiation.

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