

# Feasibility Study of Storage Ring Based High Gain FEL\*

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## Abstract

The feasibility of a high gain FEL accelerator system based on storage-ring concept that employs high peak current, short bunch length, high repetition rate, and high efficiency, is revisited. The system is composed of a storage ring, bunch compression manipulation, external transport line with long undulator, and a transport line for re-injecting the beam into the storage ring. The performance of such an accelerator-FEL system is discussed.

## 1 INTRODUCTION

In recent years, great efforts has been devoted to push for the production of high brilliance high gain FELs. Since 1992, there have been several workshops devoted to the study of the fourth generation light sources [1, 2] The coherent production of FEL based on long undulators in storage ring has gradually lose its appeal because of the inherent difficulties in the beam peak current, bunch length, and the intrinsic beam instability associated with quasi-isochronous lattices [3].

Because of the unique quality of the high brilliance electron source developed in recent years, where the emittance and the bunch length of electron beam can be made small, e.g.  $\epsilon_{\perp, \text{normalized}} \sim 1\pi$  mm-mrad and  $\sigma_{\tau} \sim 1 - 10$  ps, modern design of coherent X-ray FELs is based on linacs, e.g. the linear coherent light source (LCLS) at SLAC and the TESLA-FEL at DESY. These light sources promise to produce peak brilliance of the order of  $10^{30}$  photons/(s mm<sup>2</sup> mrad<sup>2</sup> (0.1% bandwidth)) and beyond. Some of the system requires energy recovery in order to sustain a high power FELs with affordable wall-plug power.

On the other hand, the coherent X-ray source based on storage rings has been virtually stopped because the quality of electron beam is hard to compete with that of linac sources. Although there are concerted efforts to study the quasi-isochronous storage rings [3], the electron beam suffers from various instabilities and the difficulties of correcting high order momentum compaction factors.

This paper studies an alternative solution of coherent light source based on storage rings. In Section II, we will review beam requirements of the high gain FEL and the beam properties of electron storage rings. Section III introduce concepts for attaining high quality electron beam bunch for high gain FEL in the external beam line. The wasted beam will be re-injected back to the storage ring for synchrotron radiation damping. We will estimate the

efficiency of this process. The conclusion is given in Section IV.

## 2 QUALITY REQUIREMENTS FOR ELECTRON BEAMS

Modern high brilliance synchrotron light is normally produced through high quality beams in undulators. The undulator parameter for an electron passing through it is

$$K = \frac{eB_u}{k_u m_e c} = \frac{eB_u \lambda_u}{2\pi m_e c}, \quad (1)$$

where  $B_u$  is the dipole field of the magnets,  $\lambda_u$  is the length of the undulator period,  $k_u = 2\pi/\lambda_u$  is the undulator wave number, and  $m_e$  the electron mass. As an example, when the undulator field is  $B_u = 0.65$  T and the undulator period  $\lambda_u = 2.5$  cm,  $K = 1.517$ . The amplitude of the transverse velocity of the electron orbit is  $\beta_{\perp} = \frac{K}{\gamma}$ , where  $E = \gamma m_e c^2$  is the energy of the electron. The radiation field that is in resonance with the electron beam in the undulator has a wavelength

$$\lambda_1 = \lambda_u \frac{1 + K^2/2}{2\gamma^2}, \quad (2)$$

and a transverse emittance of  $\epsilon_{\gamma} = \frac{\lambda_1}{4\pi}$ .

The FEL characteristics are determined by a dimensionless FEL parameter  $\rho_{\text{fel}}$  given by

$$\rho_{\text{fel}} = \left[ \frac{r_e \lambda_u^2 K^2 [JJ]^2 n_e}{32\pi\gamma^3} \right]^{1/3}, \quad (3)$$

where  $r_e$  is the electron classical radius,  $[JJ] = J_0(\xi) - J_1(\xi)$  with  $J_0$  and  $J_1$  being the Bessel functions of order 0 and 1 of the first kind, and

$$\xi = \frac{K^2}{4 + 2K^2} = \frac{1}{2(1 + 2/K^2)}, \quad (4)$$

The peak electron density  $n_e$  is defined as

$$n_e = \frac{N_B}{\sqrt{2\pi}\sigma_{\tau} c \sqrt{2\pi}\sigma_x \sqrt{2\pi}\sigma_y} = \frac{I_p/e}{2\pi c \sigma_x \sigma_y} \quad (5)$$

where  $I_p$  is the peak current, and  $\sigma_x$  and  $\sigma_y$  are, respectively, the rms horizontal and vertical sizes of the electron beam.

In order to enhance interaction between the electrons and radiation along the undulator, the transverse electron beam size  $\sigma_x$  and  $\sigma_y$  must be approximately equal to the transverse size of the radiation  $\sigma_r$ , where

$$\sigma_r = \sqrt{\frac{\lambda_1}{4\pi} \frac{\lambda_u}{4\pi\rho_{\text{fel}}}}, \quad (6)$$

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Since the transverse emittance of the electron beam must be the same as emittance of the laser,  $\epsilon_\gamma = \frac{\lambda_1}{4\pi}$ , the average betatron amplitudes of electron-beam lattice in the long undulator section should be  $\langle\beta\rangle = \lambda_u/(4\pi\rho_{\text{fel}})$ . Substituting Eq. (6) into Eq. (3), we get

$$\rho_{\text{fel}}^2 = \frac{I_p \lambda_u K^2 [JJ]^2}{I_A 4\gamma^3 \lambda_1} = \frac{I_p 2\xi [JJ]^2}{I_A \gamma}, \quad (7)$$

where  $I_A = ec/r_e \approx 17$  kA is the Alfren current. Depending on the peak current and beam energy, the FEL parameter is typically  $10^{-3}$  in the undulator region.

### 3 NATURAL MOMENTUM SPREAD AND EMITTANCE

The fractional momentum spread is given by

$$\sigma_\delta \approx \sqrt{\frac{C_q \gamma^2}{J_E \rho}}, \quad (8)$$

where  $C_q = 55\hbar/(32\sqrt{3}mc) \approx 3.83 \times 10^{-13}$  m, and  $J_E \approx 2$  is the damping partition number. The bunch length is given by  $\sigma_\tau = \alpha_c \sigma_\delta / \omega_0 \nu_{\text{syn}}$ , where  $\alpha_c$  is the momentum compaction factor,  $\omega_0$  is the angular revolution frequency, and  $\nu_{\text{syn}}$  is the synchrotron tune. Typically, the bunch length ranges from 10 ps to 30 ps in storage rings. Efforts to attain short bunch length in storage rings have been unsuccessful.

The normalized emittance

$$\epsilon_n = \gamma \epsilon_x = \mathcal{F}_{\text{lattice}} C_q (\gamma\theta)^3. \quad (9)$$

depends essentially only on the lattice design factor  $\mathcal{F}_{\text{lattice}}$  for electron storage rings at constant  $\gamma\theta$ . Figure 1 compiles normalized emittances of some electron storage rings. Possible lattice design includes FODO cells, the double-bend achromat (DBA) or Chasman-Green lattice, three-bend (TBA), four-bend (QBA), and  $n$ -bend achromats ( $n$ BA), etc. Note that the emittances tend to be larger for machines with FODO cell lattices than for those with DBA or TBA lattices [5].

The top plot of Fig. 2 shows the emittances (in nm) of the electron and photon beams as a function of the electron beam energy with an undulator period of 2.5 cm. Note that the electron beam emittance can be less than the photon beam emittance at an electron beam energy less than 0.9 GeV. The bottom plot shows the natural momentum spread as a function of the electron beam energy for the magnetic bending radius  $\rho = 5.6$  m. Since the momentum spread must be less than the FEL parameter in the undulator, the ratio,  $\rho_{\text{fel}}/\sigma_\delta$ , corresponds to the maximum bunch compression-ratio that the electron beam can still meet the required coherent FEL generation criterion. At an electron energy less than 0.9 GeV, the bunch compression ratio is larger than 3.

Short bunch can be attained through careful beam manipulation techniques [6]. The ultimate bunch compression under the phase jump compression scheme is given by

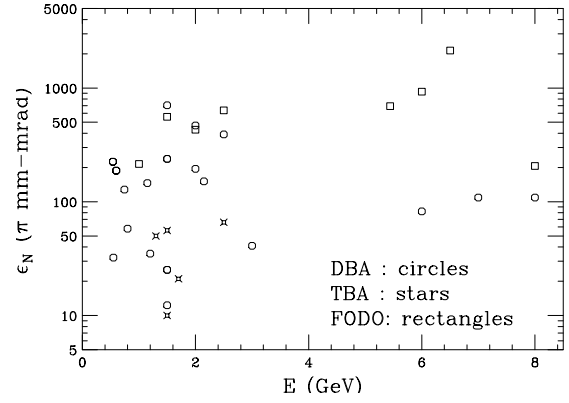


Figure 1: Normalized emittances of some electron storage rings plotted vs the designed beam energies.

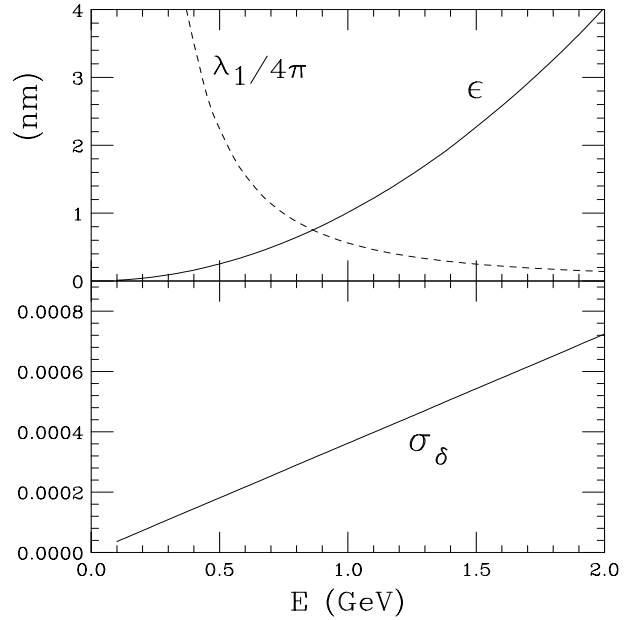


Figure 2: The top plot shows the natural emittance  $\epsilon$  of the electron beam with  $\theta = 0.1745$  rad and  $\mathcal{F}_{\text{lattice}} = 2/(4\sqrt{15})$ , and the emittance of the photon beam  $\lambda_1/4\pi$  as a function of the electron beam energy assuming the undulator period of 2.5 cm. The bottom plot shows the natural momentum spread of the electron beam in a storage ring as a function of the electron beam energy with a magnetic bending radius of  $\rho = 5.6$  m. The FEL parameter  $\rho_{\text{fel}}$  will be assumed constant in our discussion.

$r_{c,\text{max}}^D = \sqrt{2}/(\sqrt{3}\sigma_\phi)$ , where  $\sigma_\phi$  is the initial bunch length in rf phase coordinate. Once the desired bunch length is attained, the beam should be kicked out of the storage rings so that the  $R_{56}$  transport element can be used to rotate the beam. The short bunch can then pass through long undulators for the generation of coherent lights. After the undulator, electrons are re-injected into the storage ring for synchrotron radiation damping.

## 4 MICROWAVE INSTABILITY

Unfortunately, the high brightness beam may become microwave unstable [7]. The threshold longitudinal impedance  $[Z_{||}/n]_{\text{eff}}$  of the vacuum chamber averaged over the frequencies of the bunch is

$$\left[ \frac{Z_{||}}{n} \right]_{\text{eff}} \leq \frac{2\pi F |\alpha_c| E \sigma_\delta^2}{e I_p}, \quad (10)$$

where the form factor  $F \approx 1$  for linear Gaussian distribution,  $I_p = e N_B / \sqrt{2\pi} \sigma_\tau$  is the peak current of a bunch,  $N_B$  is the number of particles, and  $\sigma_\delta$  is the momentum spread.

Using the definition of FEL parameter, we find

$$\frac{\rho_{\text{fel}}^2}{\sigma_\delta^2} \leq \frac{\pi r_e F |\alpha_c| E K^2 [JJ]^2 \lambda_u}{2\gamma^3 c e^2 [Z_{||}/n]_{\text{eff}} \lambda_1} = \frac{Z_0 F |\alpha_c| \xi [JJ]^2}{[Z_{||}/n]_{\text{eff}}}. \quad (11)$$

This result bases only on the criterion of the microwave instability. The quality of the electron beam in a storage ring is determined by the microwave instability. This in turn is controlled by the momentum compaction factor. Figure 3 shows a compilation of the momentum compaction factor for all storage rings. The line is a simple model of  $\alpha_c \approx 0.01/(E(\text{GeV})^2)$ .

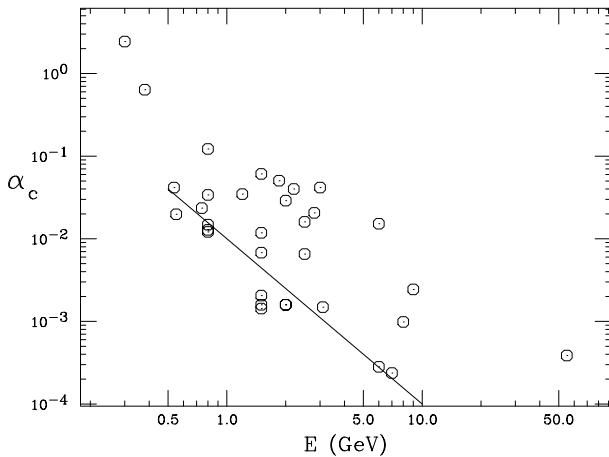


Figure 3: A compilation of the momentum compaction factors of electron storage ring lattices as a function of the electron beam energy in (GeV). The line corresponds to  $0.01/(E(\text{GeV})^2)$ .

Figure 4 shows  $I_{p,\text{peak}}$  for the threshold peak current of the microwave instability with an longitudinal broad-band impedance of  $0.5 \Omega$ . The actual peak current in the undulator region will be enhanced by the bunch compression ratio.

## 5 CONCLUSION

In conclusion, we have derived scaling properties for the generation of coherent synchrotron radiation based on the storage ring technology. We find that the peak current in the undulator region can be as high as 50 A limited by the

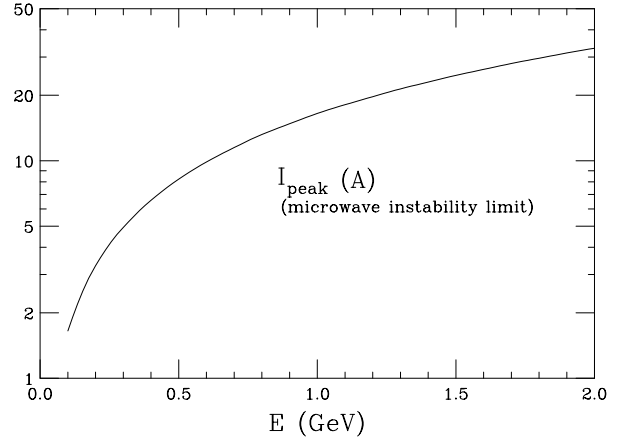


Figure 4: The threshold peak current for the microwave instability in the storage ring. The peak current in the undulator region is enhanced by the bunch rotation ratio. The longitudinal broadband impedance is assumed to be  $0.5 \Omega$ .

microwave instability. The actual bunch length depends on the rf system in the storage ring. The beam should be bunch compressed and extracted out of the storage ring for the generation of short bunch photon coherent light source.

The wasted beam should be bunch stretched and injected back to the storage ring for synchrotron radiation damping. The efficiency of such a storage ring based high gain FEL should be much higher than the LINAC based FEL sources.

In this study, we have neither calculated effect of the intrabeam scattering and nor examined the Touschek lifetime of the beam. A careful theoretical and experimental studies of low energy electron storage ring will be highly needed to address these questions.

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