SIMULATIONS ON HARMONIC GENERATION WITH THE PARTICLE TRACKING CODE GPT

C.A. Thomas, J.I.M. Botman, Eindhoven University of Technology, The Netherlands.

Abstract

The feasibility of generating higher order harmonics from a Free Electron Laser (FEL) amplifier with a tabletop system is investigated. The system is based on a small period undulator able to wiggle femtosecond short electron bunches over several tens of periods. Harmonic generation is obtained using a FEL system as a pre-buncher for a 0.1 nC, 10 to 100 MeV electron bunch, which radiates in the second undulator tuned to a higher harmonic of the FEL. Simulations of the process, using the General Particle Tracer code with a new element that takes into account the radiation from particles, are presented.

1. INTRODUCTION

High harmonic generation using a FEL as a pre-buncher for the electron bunch and an undulator tuned to a higher harmonic of the FEL has been proposed since the start of FEL developments. The theoretical and experimental studies [1-4] pointed out the usefulness of such a system in the region of short radiation wavelengths where no mirror for an oscillator FEL cavity can be used.

Here we applied this system to a 100-fs electron bunch, at 10 and 100 MeV, generated by a tabletop accelerator [5]. The tabletop system is being built in our laboratory, and the first stage of this new accelerator will provide a 100 fs, 10 MeV electron bunch. Then this electron bunch can be then easily accelerated up to 100 MeV with a commercial accelerator. The layout of the harmonic generation device is sketched in figure 1. The FEL cavity is composed of 2 mirrors and an optical klystron. The optical klystron is composed of 2 undulators separated by a drift space. The first undulator, the modulator or buncher, induces electron beam bunching; the second undulator is the radiator or amplifier, tuned at the desired harmonic. As the electron bunches pass through the FEL cavity, the FEL intra-cavity intensity grows, associated with micro bunching in the modulator, which allows the growth of a coherent optical field until saturation occurs.

In this paper, we study the (high) harmonic generation of such a system by using a particle tracking simulation code, General Particle Tracer (GPT), with a new element that takes into account the radiative field from the particles. In the first part we present the new element to the code, in which a differential equation for the electro-magnetic field in the FEL optical cavity is coupled with the equations of motion of the charged particles. In the second part we present results from the code simulating the FEL harmonic generation device applied with the 100 fs e-bunch.

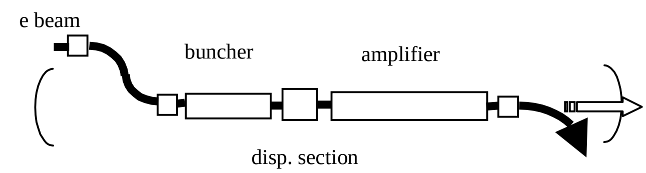

Figure 1: The FEL harmonic generation system.

2. DIFFERENTIAL EQUATIONS FOR THE RADIATIVE FIELD FROM CHARGED PARTICLES IMPLEMENTED IN THE GPT CODE

The GPT code is a 3D simulation platform for the study of charged particle dynamics in electromagnetic fields [6]. The code solves the equations of motion for each particle, taking into account the electromagnetic field felt by the particle. In order to include energy losses of the charged particles by radiation and to take into account this radiated field in the equations of motion, we derived differential equations for the electric field associated with longitudinal propagating eigen-modes of an optical cavity. It is based on the gaussian Transverse Electro-Magnetic field, TEM_{00}, which is a solution of the free space field equations in an optical cavity (Kogelnick [7]). The derivation consists of taking the field expression of the TEM_{00q} longitudinal mode, where q represents the q^{th} eigen mode of a given optical cavity, and evaluating the power at a certain point in the cavity together with the energy of that q^{th} mode. Then a variation in time of the q^{th} mode's energy is set equal to the energy loss from a particle crossing the cavity. We assume a cylindrical symmetry for the field, which propagates along axis z; $r_\perp$ is the orthogonal distance from the z-axis. The transverse shape of the field is assumed to be gaussian, and more over the field is assumed to be polarized in the x direction orthogonal to the z-axis. Finally, two coupled differential equations describing the evolution of the q^{th} mode field, are obtained:

$$\frac{\partial E_{0m}}{\partial t} = -\sum_i q_i v_{xi} \cos(\Phi) \frac{e^{-\frac{r_\perp^2}{w^2(z)}}}{\pi\varepsilon_0 w_0 w_q(z)},$$

$$\frac{\partial E_{0n}}{\partial t} = -\sum_i q_i v_{xi} \sin(\Phi) \frac{e^{-\frac{r_\perp^2}{w^2(z)}}}{\pi\varepsilon_0 w_0 w_q(z)}$$

where the electric field of the q^{th} mode is given by the sum of two fields 90 degrees out of phase:

$$E_q(t) = E_m + E_n = E_{0m}\cos(\Phi) + E_{0n}\sin(\Phi),$$

and where E_{0m} and E_{0n} are amplitudes of the two $E_{m,n}$ orthogonal fields; $w_q(z)$ is the waist of the field's profile of the q^{th} propagating mode, and w_0 is the waist at z = 0. The charge of each particle *i* is q_i and the velocity in the field orientation of the i^{th} particle is v_{xi} (x referring to the x-axis of polarisation). The phase information of the field $E_q(t)$ is given by arctan(E_{0n} / E_{0m}). Then the information from the field $E_q(t)$ can be restituted in the equations of motion of the charged particles.

Once these equations are coupled with the equations of motion, the growth of the TEM_{00} power spectrum generated by electrons crossing an undulator is obtained (fig. 2).

In the following section, simulations are presented of the harmonic generation from an electron bunch with 100 fs length, 0.1 nC charge, at 100 MeV, π mm mrad of emittance.

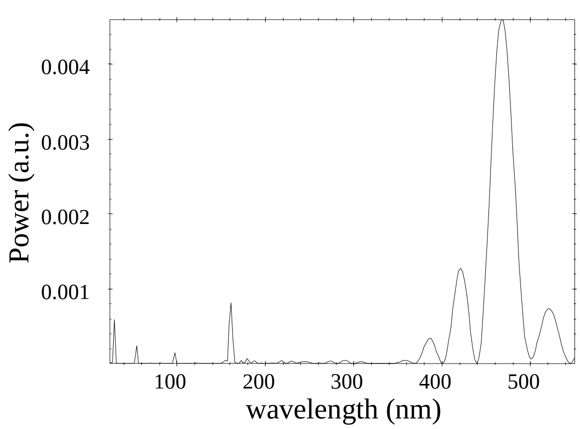

Figure 2: Spectral power from a 100 fs electron bunch at 100 MeV and 0.1 nC, crossing an undulator of 10 periods 25 mm length, *K*=1.

3. SIMULATION ON HARMONIC GENERATION WITH A 100-FS E-BUNCH USING A FEL SYSTEM

In this section we present results from the GPT code with the new element we described in the previous section, applied to the harmonic generation using a FEL system.

When relativistic electrons cross a planar undulator they radiate and the typical spectral power *P*, observed at infinity on the undulator axis is proportional to:

$$P(\lambda) \propto \sum_{n=1}^{\infty} \frac{JJ_{2n-1}^2}{\lambda^2} \left(\frac{\sin(\delta_{2n-1})}{\delta_{2n-1}} \right)^2, \qquad (1)$$

where:

$$\delta_p = \pi N \left(p - \frac{\lambda_r}{\lambda} \right)$$

$$JJ_p = J_{\frac{p+1}{2}}\left(\frac{nK^2}{4+2K^2} \right) - J_{\frac{p-1}{2}}\left(\frac{nK^2}{4+2K^2} \right),$$

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} \right)$$

$J_n(x)$ is the Bessel function of order n, λ is the wavelength of the radiation, *N* is the number of undulator periods, λ^u is the undulator period length, *K* is the undulator strength, $K = \frac{eB_u\lambda_u}{2\sqrt{2}\pi\, mc}$, B_u is the amplitude of the undulator magnetic field, *m* is the electron mass, γ is the electrons' Lorentz factor; λ_r is the fundamental radiation wavelength.

The intensity of the harmonics generated by the first undulator is lower than that of the fundamental, and depends on the undulator strength. The harmonic peak intensity increases for higher *K*, but at the same time the coherence of the generated light reduces. In order to enhance the harmonic peak intensity without degrading the coherence, the first undulator is used as an energy modulator [6].

The dispersive section of the optical klystron induces micro bunching, by transforming energy modulation to spatial modulation of the electron distribution.

An optimization of the energy modulation due to the laser field together with an optimization of the dispersive section parameters has to be done in order to provide an optimized micro bunched electron beam at the entrance of the second undulator, the amplifier. Then the bunch radiates coherently in the second undulator. Furthermore, the bunch has the potential to radiate at any harmonic of the fundamental λ_r, because of its coherently modulated longitudinal distribution, and the higher harmonics can be enhanced according to the degree of micro bunching of the electron bunch [8]. Finally, the growth of the harmonic radiation is exponential with the distance along the undulator.

In the GPT simulations using a 100 fs electron bunch crossing an optical klystron (tables 1 and 2) we obtained the expected spectral power as described by eq. 1. After passing the modulator and the dispersive section the electron energy has been modulated by the generated radiation, so the dispersive section already but insufficiently micro bunches the electrons. Then they radiate in the amplifier, tuned here at the 5^{th} harmonic (table 2).

In order to enhance micro bunching, a coherent light pulse is injected in the cavity. This pulse can represent either the FEL pulse or a seed laser. In this case, energy modulation of the electrons can be observed together with micro bunching at the end of the modulator or after the dispersive section [6] depending on the intensity of the pulse.

The efficiency of the micro bunching process can be seen either in the particle distribution or in the radiated spectral power, where a peak structure appears in the fundamental (fig. 4). More pronounced peaks indicate more coherence of the modulation of the electron energy. In figure 4 a simulation is shown in which a pre-injected light pulse is taken at the fundamental wavelength with an amplitude 20 times that of the spontaneous emission peak. The coherent modulation is seen from the peaks in the fundamental, but its amplitude is very small, explaining why the enhancement of the 5th harmonic is only 5 times more than without the modulation from the laser.

Table 1: Electron bunch parameters

E_0 (MeV)	10 – 100
σ_z (μm)	30
σ_x , σ_y (mm)	1
ε (mm mrad)	π

Table 2: Optical klystron parameters

	N	n_H	λ_u (mm)	*K*
Modulator	20	1	25	1
Radiator	100	5	5	1
Disp sect.	2		50	1

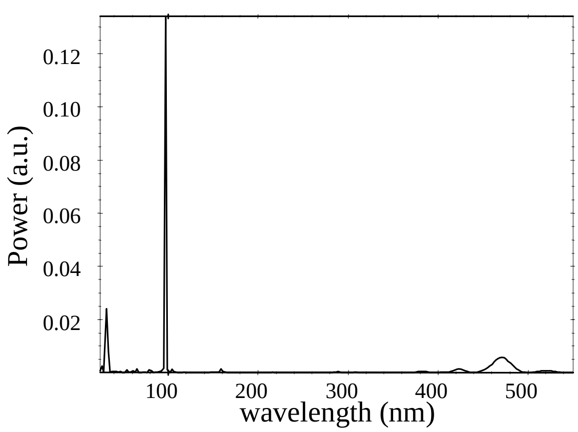

Figure 3: Same electron bunch as in fig. 2, at the end of the radiator. The 5th harmonic is enhanced.

4. CONCLUDING REMARKS

Simulations on FEL harmonic generation using a new element in the GPT code have been performed. With the new element the use of the simulation code is restricted to electromagnetic field systems with TEM_{00q} modes. This applies to most of the FEL devices.

Here 100 nm radiation was obtained from a 100 fs 100 MeV electron bunch passing through a radiator undulator with 5 mm period length.

Similar simulations in the FEL system described have been performed for a 10 MeV beam, using a shorter undulator period length and showing similar behavior at even higher harmonics. For a 20 periods 10 cm long modulator and a radiator at the 35th harmonic, harmonic generation was obtained at 340 nm.

In general the simulations scale with the electron energy, strength and period length of the undulator.

ACKNOWLEDGEMENTS

The authors thank B. van de Geer, M. de Loos, and P. K. Fung for fruitful discussions. This work was partly supported by the European Community under contract n° ERBFMRXC 980245.

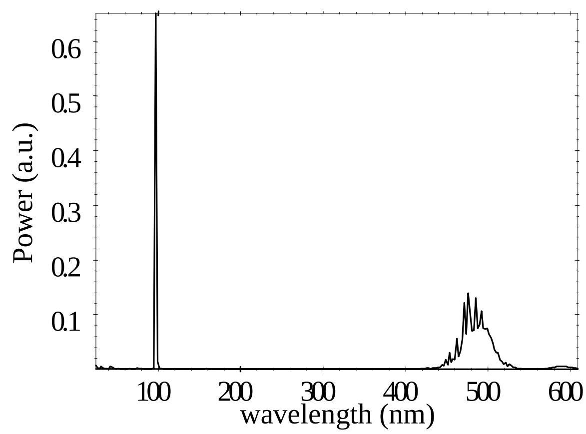

Figure 4: The same electron bunch as in fig.3 but with a seed laser, at λ_r. The enhancement of the 5th harmonic is 5 times bigger than without seed laser.

REFERENCES

[1] R. Bonofacio, L. De Salvo Souza, P. Pierini, E. T. Scharlemann, Nucl. Instrum. Meth. A 296, 787, (1990).

[2]L.H.Yu, Phys. Rev. A, Vol. 44, 5178-5192, (1991).

[3] R. Prazeres, P. Guyot-Sionnest, J. M Ortega, M. Billardon, D. Jaroszynski, IEEE Journal of Quantum Electronics, Vol. 27, 1061-1068, (1991).

[4] G. Dattoli, P. L. OttaViani, Journal of Applier Physics, Vol. 86, 10, 5331-5336, (1999).

[5] F. B. Kiewiet, O. J. Luiten, G. H. J. Brussaard, J.I.M. Botman, EPAC 2000, Austrian Academy of Science Press, Vienna, 1660-1663, (2000).

[6] C. Thomas, J.I.M. Botman, C.A.J. Van de Geer, M.E. Couprie, EPAC 2000, Austrian Academy of Science Press, Vienna, 776-779, (2000).

[7] J. A. Arnaud and H. Kogelnik, Appl. Opt., Vol. 8, 1687-1693, (1969).

[8] F. De Martini, Laser Handbook Vol. 6, W. B. Colson, C. Pelligrini and A. Renieri, (North Holland), (1990).