# SIMULATION OF ACCELERATED ELECTRON SPECTRA IN LASER WAKEFIELD ACCELERATORS\*

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### Abstract

The generation of high quality electron beams in a laser wakefield accelerator (LWFA) is generally thought to require phased injection of an ultrashort electron bunch. However, simulations have shown that longer bunch, unphased injection may also produce high quality, short electron bunches if the injection energy is properly chosen. The process involves pruning of electrons that move into defocusing portions of the wakefield, along with strong phase bunching and rapid acceleration. Simulation results are consistent with a simple Hamiltonian model that numerically integrates particle orbits in an idealized wake potential. Simulations are also presented for a channel-guided LWFA system using optical injection

## INTRODUCTION

A laser wakefield accelerator (LWFA) uses a high power (>TW), short laser pulse to produce a large amplitude plasma wave that can accelerate electrons to high energies over extraordinarily short distances [1-3]. The plasma wave has a phase velocity that is approximately equal to the group velocity of the laser pulse in the plasma.

A general strategy for producing high quality electron beams is being pursued at a number of institutions. This strategy involves three basic components. First, laser and plasma parameters are chosen to be in the "standard" LWFA regime, where the laser pulse length  $c \tau_p$  is less than the plasma wavelength  $\lambda_p = 2\pi c/\omega_p$ . In this regime, the laser produces a strong, well-defined wakefield while limiting the growth of dangerous instabilities. The second part of the strategy is to use a plasma channel to provide optical guiding and thus increase the acceleration length. The acceleration length is usually limited by dephasing that arises from the difference in speed between the beam electrons and the group velocity of the laser pulse. The third part of strategy is to use an external electron beam source that provides precisely-timed ultrashort bunches injected into the optimal phase of the wake. The short period of the wakefield places difficult limitations on the bunch length and timing jitter of the injected electrons.

This precisely timed or phased electron injection is expected to result in high quality electron beams with small energy spread. It is inherently difficult for conventional RF injectors to achieve femtosecond precision, so most conceptual LWFA designs have employed some form of all-optical injection [4-6]. Integrated experiments demonstrating all-optical injection and LWFA acceleration are planned at several laboratories, but this milestone has not yet been achieved.

However, simulations and theoretical models presented in this paper will show that phased injection of preciselytimed ultrashort bunches may not be necessary. If the electron injection energy is properly chosen, a high quality accelerated electron beam is possible even if electrons are injected over a broad range of phases. This strategy involves matching the injection energy to the strength of the wakefield so that most injected electrons experience a defocusing radial electric field at some point and are lost or "pruned". Those electrons that survive experience strong phase bunching and can be accelerated to high energies with small energy spread.

### **UNPHASED INJECTION SIMULATONS**

TurboWAVE is highly a parallelized particle-in-cell simulation code that has been used extensively to model intense laser propagation in plasmas [7]. For long range propagation in regimes where the plasma frequency is much less than the optical frequency, it has an option to use a ponderomotive guiding center (PGC) model that averages over the fast (optical) time scale. The simulations are fully relativistic and fully electromagnetic and include the transverse structure of the plasma wave, nonzero emittance and energy spread of injected electrons, and the nonlinear evolution of the driving laser pulse in the plasma channel. The channel-guided LWFA simulations described here have been run in a 2-D Cartesian geometry.

An example of a channel-guided LWFA TurboWAVE simulation with monoenergetic, unphased injection is shown below. The laser has  $\lambda = 0.8 \ \mu m$ ,  $\tau_p = 80$  fsec, peak power  $P_0 = 8$  TW, and initial Gaussian radius  $r_0 = 30 \ \mu m$ . The channel on-axis density  $n_0 = 5 \times 10^{17} \ \text{cm}^{-3}$ , and the density doubling radius of the parabolic channel profile [3] is chosen to have a matched guiding spot size  $r_M = r_0$ . The injected electrons had an initial energy  $W_0 = 1.6 \ \text{MeV}$  and were loaded uniformly in phase behind the primary laser pulse. The normalized emittance  $\varepsilon_n$  for this bunch was 1  $\pi$  mm-mrad.

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Figure 1 shows the location of the laser pulse in the simulation box at injection (z = 0) and after 3.62 cm of propagation. The axial coordinate z - ct defines position within the laser frame. Since the channel parameters were chosen to provide guiding at the initial spot size, the laser

intensity contours at z = 3.46 cm show little change except for the usual group-velocity-driven phase slippage. However, there is a dramatic change in the injected electrons, which form very short, highly focused bunches.



Figure 1: Contours of laser intensity and injected particle positions at injection (z = 0, left frame) and after 3.46 cm of propagation (right frame).

Figure 2 shows the energy distribution of the injected electrons at two different locations from this simulation. At z = 3,46 cm, the average energy  $\langle W \rangle$  of the electrons is 230 MeV, with a small energy spread. At z = 6.62 cm, the average energy has increased to 400 MeV, and the energy spread has increased. Most of the original injected electrons have been lost due to expulsion by the strong radial electric fields in the defocusing portions of the wake. However, those that survive form short, well-defined beam bunches with modest energy spread.



Figure 2: LWFA particle energy spectra at two locations

# HAMILTONIAN ANALYSIS OF UNPHASED INJECTION

Trapping and acceleration of injected electrons are often analyzed by plotting particle phase space trajectories in a frame moving at the group velocity of an ideal guided laser pulse [2]. The Hamiltonian in this frame is given by  $H(\gamma, \psi) = \gamma(1 - \beta_g \beta)mc^2 + e\phi_0 \sin \psi$ , where  $\beta$  and  $\beta_g$  are the

particle velocity and wave group velocity, normalized to c,  $\phi_0$  is the peak wake potential, and  $\psi = \alpha_p(z/\beta_s c - t)$  is the particle phase. Although the analysis is one-dimensional, it is assumed that any particle that enters the defocusing portion of the wake ( $\pi < \psi < 2\pi$  in the first wake bucket) is immediately lost.

The largest phase space closed orbit in the first bucket that satisfies  $0 < \psi < \pi$  contains all orbits of particles that never cross into the defocusing region. This orbit is defined by  $H(\gamma, \psi) = H(\gamma_{e}, 0)$ , where  $\gamma_{e} = (1 - \beta_{e}^{2})^{-1/2}$ . The lowest point on this retaining orbit defines the minimum injection energy  $\gamma_{min}$  for trapping. For  $\gamma_{min} < \gamma_0 < \gamma_g$ , where  $\gamma_0$  defines the injection energy, one can define a range of phases  $\psi_{min} < \psi < \psi_{max}$  for particles that remain focused. The collection efficiency for particles loaded uniformly in phase is given by  $\eta = (\psi_{max} - \psi_{min})/2\pi$ . With an appropriate change of variables, the equations of particle momentum p(z) and phase  $\psi(\zeta)$  can be solved numerically for an idealized wake to give final output energy and phase as a function of input energy and phase. The final average energy  $\langle \gamma \rangle$ , relative energy spread  $\delta \gamma \langle \gamma \rangle$ , mean output phase  $\langle \psi \rangle$ , and rms pulse width  $\delta \psi^2$  can be obtained as functions of  $\gamma_0$  and  $\phi_0$ .

# **COMPARISON WITH SIMULATONS**

For the parameters used in the previous simulation the normalized potential  $e\phi_0/mc^2 = 0.1$ . Figure 3 plots the collection frequency and normalized energy spread as functions of the unphased injection energy. The plot compares the Hamiltonian model with the TurboWAVE results at z = 6.62 cm for the parameters used in that simulation.

The Hamiltonian model shows that the collection efficiency rises much faster than the relative energy spread for injection energies slightly above the minimum trapping energy, which is 1.72 MeV in the theoretical model. The simulation collection efficiency follows the basic trend of the Hamiltonian model but is consistently higher. For  $W_0 = 1.6$  MeV, which is slightly below the theoretical trapping threshold energy,  $\eta$  actually exceeds 0.2. The energy spread in the simulations also agrees well with the Hamiltonian model for injection energies near the trapping threshold.



Figure 3: Collection efficiency (red) and energy spread (blue) as functions of the injection energy for the laser and plasma channel parameters used in Figs. 1 and 2. The curves are from the Hamiltonian analysis, and the symbols are from the TurboWAVE simulation.

### **MULTI-STAGE LWFA SIMULATIONS**

This Section describes simulation of a channel-guided LWFA that would employ phased injection from an alloptical injector. This "end-to-end" simulation uses the 3-D version of TurboWAVE to generate an injection spectrum for a LIPA (Laser Ionization and Ponderomotive Acceleration) injector [5]. The LIPA simulation employs a 2 TW, 50 fsec Ti-sapphire pulse tightly focused onto a nitrogen gas jet and produces a 2 MeV, 20 fsec long, phased electron bunch with a modest energy spread. The output particles from this simulation are used to initialize a 2-D TurboWAVE simulation of the main LWFA acceleration stage. The 8 TW, 50 fsec, 30 µm radius LWFA pulse is guided in a three stage, ablative-wall capillary discharge plasma channel [8]. The on-axis channel density is 8x10<sup>17</sup> cm<sup>-3</sup> in the first stage and is slightly higher in the other stages in order to extend the dephasing length. Laser ionization of inner shell carbon electrons from the capillary discharge is included.

Figure 4 shows the simulation output spectrum at the end of each stage. The simulations predict average energies of almost 300 MeV after the first stage and 800 MeV after the third stage. The energy spread remains small throughout the acceleration process.



Figure 4: Energy spectra after each stage for a three-stage, channel-guided LWFA with LIPA injection.

## **SUMMARY**

The TurboWAVE simulation code has been used to model acceleration of injected electrons in a channelguided laser wakefield accelerator. It is often assumed that phased injection of precisely-timed, ultrashort electron bunches will be required to produce high quality accelerated electron beams with low emittance and energy spread. However, simulations show that high quality, ultrashort bunches can be produced during LWFA acceleration if the injection energy is slightly above the minimum energy for trapping. The process involves removal or pruning of electrons that move into defocusing portions of the wakefield, combined with strong phase bunching. The results suggest that extremely precise timing may not be necessary, making conventional RF injectors a more viable alternative to all-optical injection. "End-to-end" simulations of a three-stage LWFA with alloptical LIPA injection predicts almost 300 MeV gain after the first stage and 800 MeV after the third stage, with modest energy spread.

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