# STRONG-STRONG SIMULATION OF BEAM-BEAM INTERACTION FOR ROUND BEAMS

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# Abstract

This paper presents the results of numerical simulations of beam-beam interaction in a  $e^+-e^-$  machine with round colliding beams. The calculations were done with the Particle-in-Cell computer code which was developed for the KEKB B-Factory [1]. The code was modified to include the effect of the machine nonlinearity and dynamical radiation emittance. A simulation study of the round beam collider VEPP-2000 justified the choice of the chromaticity correction scheme, working point and optics capable of achieving the design luminosity.

# **INTRODUCTION**

The single bunch luminosity at present circular  $e^{+} \cdot e^{-}$  colliders is limited by the beam-beam interaction. The nonlinear force of the counter-rotating bunch disturbs the betatron motion leading to the betatron coupling resonances. At some threshold beam intensity the resonances overlap, making the motion chaotic and causing the beam blow-up and life time degradation detrimental to the luminosity. The intensity of the beam-beam interaction is characterized by the dimensionless parameter  $\xi$ . In a collider with flat beams the beam-beam parameters for two transverse oscillation modes (x and y) differ, and the threshold value for  $\xi$  currently does not exceed 0.09.

Hence, various methods are proposed to overcome this limit. One of them makes use of the Round Colliding Beams. The first straightforward advantage of round beams is that  $\xi_x$  is equal to  $\xi_y$  and the cross-section of beams at the Interaction Point (IP) is round. This geometric property gives at least a factor of two increase in luminosity at constant  $\xi$  [2].

But the main idea of the method is aimed at enhancement of the maximum attainable beam-beam parameter. This can be achieved because of elimination of the betatron coupling resonances by introducing an additional integral of motion, namely the longitudinal component of the particle's angular momentum [3, 4]. Analytical models show the possibility to construct an integrable optics and predict the advantage of the round beams [4]. Some experimental tests of a Möbius accelerator [5] which is an option of round beam machine, were done at CESR (Cornell, USA) demonstrating accessibility of the beam-beam parameter as high as 0.1 [6]. However, the round beams in high luminosity regime were not obtained to this moment. The collider VEPP-2000 (BINP, Novosibirsk) with the design luminosity of  $1 \times 10^{32}$   $cm^{-2}s^{-1}$  at 1 GeV will use round beams as one of its operation options, thus presenting an opportunity to check the method [7].

In the past decade a number of computer codes appeared [1, 8, 9] using advanced models of the beam-beam interaction and allowing to perform numerical simulations of colliding bunches with sufficient accuracy. Although the computer simulations do not reproduce the experimental results precisely, they give a good answer about the main characteristics of the system: beam size and the beam-beam limit, coherent beam-beam tune shift, etc. Thus, a comprehensive numerical simulation of beam-beam effects for a new collider is highly advisable.

This report presents some results of simulation of beambeam interaction for the VEPP-2000 optics using a modified PIC code which has been originally developed for KEKB [1]. The calculations show availability of very high values of  $\xi$  in ideal optics and reveal some problems in a more realistic case. Based on this simulation, a corrected machine optics has been chosen.

# THE CODE

The code for calculating the beam-beam force is described in detail in [1]. This part did not need modification for our purposes. Here we describe only the main features of the algorithm.

Each colliding beam is represented with  $N_p$  macroparticles forming a 3-dimensional distribution in space, where  $N_p$  is taken big enough to avoid statistical effects and in our case was  $5 \times 10^4$ . The code uses 2D mesh to evaluate the transverse field which is calculated via the Poisson equation using FFT. Particles of the opposite bunch are tracked through the field. We used  $128 \times 128$  mesh with coordinate region covered approximately  $\pm 10$  of the transverse beam  $\sigma$ . Due to a comparatively long radiation damping time in VEPP-2000 ( $5 \times 10^4$  turns) we did not use longitudinal slicing of the beam, hence the beam-beam interaction was substantially 2-dimensional.

Modifications of the code concern particle tracking over the accelerator arc and radiation damping/quantum excitation. Nonlinearities of the machine optics are known to influence the beam-beam effect, therefore we included the chromaticity correction sextupoles as thin elements with linear transformations of betatron coordinates between them. Simulated dynamical aperture in the absence of beam-beam interaction reproduces very well the results of special codes.

Another important effect for electron machines consists

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in dynamical change of the beam emittance due to deformation of the machine lattice by linear part of the beambeam force. In the original code the equilibrium emittance was simulated by applying radiation damping and random quantum excitation once per turn:

$$X = \lambda X_0 + \sqrt{(1 - \lambda^2)\epsilon} \hat{F} \,.$$

Here X is 2-vector of dynamic variables of one normal mode,  $\lambda = e^{-\delta}$  with  $\delta$  being the damping decrement,  $\epsilon$  is the nominal equilibrium beam emittance, and  $\hat{F}$  is a vector of two Gaussian random numbers with the mean value equal to 0 and  $\sigma = 1$ . This mapping did not give the correct deformation of the beam emittance. To repair it, we introduced the modified mapping

$$X = \lambda X_0 + \sqrt{(1 - \lambda^2)\epsilon} M_d \hat{F} ,$$

where

$$M_d = \left(\begin{array}{cc} a+d & b \\ b & a-d \end{array}\right) \ ,$$

with

$$a = \sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - c^2 - s^2}}, \quad d = c/2a, \quad b = -s/2a.$$

The excitation coefficients c and s are calculated from the quantum excitation (or *diffusion*) matrix Q [10]. In the normal mode basis the matrix has the form

$$\tilde{Q} = 2\delta\epsilon \left( \begin{array}{cc} 1+c & -s \\ -s & 1-c \end{array} \right).$$

Figure 1 shows comparison of tracking using weakstrong linearized beam-beam interaction and the modified quantum excitation with calculation using conventional optics code, namely, SAD [11]. In SAD, the axisymmetric beam-beam lens was modelled using a pair of thin solenoids placed at the IP.



Figure 1: VEPP-2000, one IP. Beam emittance vs.  $\xi$ . Weak-strong linearized beam-beam interaction.

# **RESULTS FOR LINEAR OPTICS**

VEPP-2000 has the design with two IP's and symmetrical arcs between them [7]. Maximum operation energy is 1 GeV per beam, but all calculations were carried out at injection energy of 900 MeV. Final focus is formed by superconducting solenoids making beams round at IP and rotating the betatron oscillation plane by  $\pi/2$  per IP. The design parameters are  $\beta_x^* = \beta_y^* = 6.3 \text{ cm}, \epsilon_x = \epsilon_y = 1.7 \cdot 10^{-7} \text{ m}$  and fractional tunes  $\nu_x = \nu_y = 0.1$ . Results for the case of accelerator arc represented by linear transformation are presented in Figs. 2,3. For linear arc optics no luminosity degradation is observed until  $\xi = 0.15$ , where increase in the beam size is approx. 20%, mainly due to tails of the distribution. The spectrum of dipole oscillations with infinitesimally small amplitude is clean, showing no lines except the expected  $\sigma$  and  $\pi$  modes (the tune shift is twice the  $\xi$  due to two IPs).



Figure 2: Beam size at IP and square root of the luminosity per 1 IP vs. the nominal beam-beam parameter. Linear machine optics.



Figure 3: Fourier spectrum of the transverse dipole signal at  $\xi = 0.17$  (logarithmic scale).

## EFFECT OF SEXTUPOLES

Installing sextupoles (4 per half) in the accelerator arc leads to significant change in the beam-beam system behavior. Although the beam size growth (Fig. 4) is not very large as compared, for example, with the flat-beam collision, particle losses arise at rather moderate values of  $\xi$  (0.06-0.07, Fig. 5). Apparently, this happens because of 'shrinking' of the dynamical aperture in the optics distorted by the beam-beam force. This was justified by tracking simulation using SAD, where the dynamical aperture of the distorted lattice was as low as 6  $\sigma$ .



Figure 4: Beam size at IP and square root of the luminosity vs. the beam-beam parameter. Comparison of the sextupoles on and off options,  $\beta^*=6.3$  cm.



Figure 5: Simulated beam life time vs.  $\xi$ . Sextupoles on.

To overcome this problem, a new optics was adopted with improved phase relations between sextupoles and initial dynamical aperture increased to 25 beam  $\sigma$ . This optics has  $\beta^*=10$ cm (6.3 cm in the original one), but due to smaller beam emittance the beam size  $\sigma^*$  was conserved thus allowing to have the same luminosity at available beam intensity. The betatron working point was tuned closer to the integer resonance and now  $\nu=0.05$ . Simulation for this parameters does not show particle losses up to  $\xi=0.1$  (Fig. 6, 5). Further increase of the dynamical aperture may be achieved by lowering betatron tunes or tuning harmonic sextupole correctors.



Figure 6: Beam size at IP and square root of the luminosity vs. the beam-beam parameter,  $\beta^*=10$  cm.

## **SUMMARY**

Simulation of beam-beam effects for round colliding beams at the VEPP-2000  $e^+$ - $e^-$  collider using the strong-strong 2D PIC code predicts values of the maximum attainable  $\xi$  up to at least 0.15 in a lattice without nonlinearities. Inclusion of the machine sextupoles does not cause a serious beam blowup and reduction of luminosity. However, reduction of the dynamical aperture limits the beam life time in the optics with  $\beta^*$ =6.3 cm. Modification of optics with  $\beta^*$ =10 cm conserving the beam size at IP allows to enhance the dynamical aperture and reach the design luminosity.

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