SPATIAL-TEMPORAL MODES OBSERVED IN THE APS STORAGE RING USING MIA*

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Abstract

Singular-value decomposition of the data matrix containing beam position histories yields a spatial-temporal mode analysis of beam motion by effectively accomplishing the statistical Principal Component Analysis. Similar to the Fourier analysis, this mode analysis decomposes the spatial-temporal variation of the beam centroid into a superposition of orthogonal modes that are informative. We briefly review this mode analysis technique and show some interesting modes observed at the APS storage ring.

INTRODUCTION

Fourier analysis is commonly used to extract basic beam dynamics information in a storage ring. Generally it is a 1D harmonic mode analysis of beam temporal motion. In recent years, as a major part of Model-Independent Analysis (MIA), a spatial-temporal mode analysis technique has emerged for studying beam dynamics [1,2], where the spatial information comes from a large number of BPMs and the temporal information comes from turn-by-turn beam position histories at all BPMs. All the beam histories form a data matrix $B = (b_p^m)/\sqrt{P}$ where the column index m indicates the monitor, the row index p indicates the pulse or turn, and P is the number of turns. Usually B is normalized such that B^TB is the variance-covariance matrix of BPM readings. The spatial-temporal mode analysis uses the singular value decomposition (SVD) of B to decompose beam motion into a superposition of orthogonal spatial-temporal modes according to the Principal Component Analysis. We introduce the technique first, then show, with data from the Advanced Photon Source (APS) storage ring, that the spatial-temporal modes are interesting and informative.

SVD Mode Analysis

Mathematically, an SVD of the matrix B yields

$$B = USV^T = \sum_{i=1}^{d} \sigma_i u_i v_i^T, \qquad (1)$$

where $U_{P\times P} = [u_1, \cdots, u_P]$ and $V_{M\times M} = [v_1, \cdots, v_M]$ are orthogonal matrices, $S_{P\times M}$ is a diagonal matrix with nonnegative σ_i along the diagonal in decreasing order, $d = \mathrm{rank}(B)$ is the number of nonzero singular values, σ_i is the i-th largest singular value of B, and the vector u_i (v_i) is the i-th left (right) singular vector. The singular values reveal the number of independent variations and their magnitudes, while each set of singular vectors form an orthogonal basis of the various spaces of the matrix. These properties make SVD extremely useful. An SVD routine is commonly available in numerical packages. Thus it is as easy as Fourier analysis to obtain the SVD of B that yields a large set of $\{\sigma_i, u_i, v_i\}$. Each set of $\{u_i, v_i\}$ defines a spatial-temporal mode, where u_i gives the temporal variation, v_i gives the spatial variation, and σ_i gives the overall strength of this mode.

Principal Component Analysis

Principal Component Analysis is a major multivariate statistical data analysis technique. It is used to reduce a large set of observed variations to a minimum set of variables that account for the correlations observed in the sample variance-covariance matrix. The basic idea is to find the first "principal axis" in the data-point space such that the sample variance of the components of all data points along this axis is maximum, then find the next such axis that is orthogonal to the other principal axes, and so on. In our case, each BPM is one variable and the readings at the M BPMs for one turn become a data point in an M-tuple space. To find the first principal axis $v_1 = \{v_{11}, v_{12}, \cdots, v_{1M}\}^T$ with $v_1^T v_1 = 1$, we need to maximize the variation projected onto this axis, i.e., $\mathrm{var}(\sum_m v_{1m} b_p^m) = \mathrm{var}(Bv_1) = v_1^T B^T B v_1 = \max$. This maximization requires that the maximum variance is equal to the largest eigenvalue λ_1 of B^TB, and v_1 is the corresponding eigenvector. After finding the first principal axis, the associated variations can be subtracted out and the residual variation $\Delta B = B - (Bv_1)v_1^T$ is orthogonal to v_1. In the same way, we can find the principal axis v_2 for the residual variation ΔB. Since v_2 is orthogonal to v_1, v_2 must be an eigenvector of B^TB as well. By repeating this procedure we can find all the principal axes and all are eigenvectors of B^TB. Let the variations along the principal axes be $w_i = Bv_i$. It is easy to see that they are orthogonal to each other as well because $w_i^T w_j = v_i^T B^T B v_j = \lambda_i \delta_{ij}$. Furthermore, the variance of w_i is λ_i. Normalizing w_i by its standard deviation $\sigma_i = \sqrt{\lambda_i}$, we have orthonormal vector $u_i = w_i/\sigma_i$ with $u_i^T u_j = \delta_{ij}$. Putting the spatial vector v's into matrix $V = [v_1, v_2, \cdots]$, temporal vector u's into $U = [u_1, u_2, \cdots]$, and the standard deviations into diagonal matrix $S = \mathrm{diag}(\sigma_1, \sigma_2, \cdots)$, we get $B = USV^T$. Therefore, the SVD of B in fact accomplishes the statistical Principal Component Analysis of beam histories. In other words, statistical analysis is the foundation of the spatial-temporal mode analysis and SVD is the tool.

For more discussion on the technique and characteristics of the singular-value spectrum, see [2]. Usually a large number of modes are generated, but less then a dozen leading modes are due to beam motion; the rest are due to BPM noises. In the next section, we present a set of interesting spatial-temporal modes observed at the APS.

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MODES OBSERVED

The following modes are from horizontal BPMs with a horizontally kicked beam in the APS ring. There are nine modes above the noise floor. The first two are dominating betatron modes that are well understood and used for beam measurements. We skip these two modes here for lack of space. The next seven modes are shown in Figs. 1-7. In each figure, the spatial vector is on the top, the temporal vector is in the middle, and the Fourier spectrum of the temporal vector is at the bottom. The red dots are bad BPMs. Shown on the left-hand labels are the mode number and singular value in units of BPM count (7 μm). Brief comments are given in the figure captions. Note that the magnitudes of these modes are on the order of microns.

Remarks

The spectra of the temporal vectors indicate that each mode corresponds to certain excitations that have characteristic features in the frequency domain. This is remarkable since the statistical analysis knows nothing about the frequency domain. This also indicates that the associated spatial vectors should provide useful spatial information about the excitation, though more studies are required to fully understand and make use of such information. (In our case, bad BPMs make it even harder by breaking spatial periodicity.) What we have shown here are just more examples demonstrating that spatial-temporal mode analysis provides a useful way to investigate beam motion.

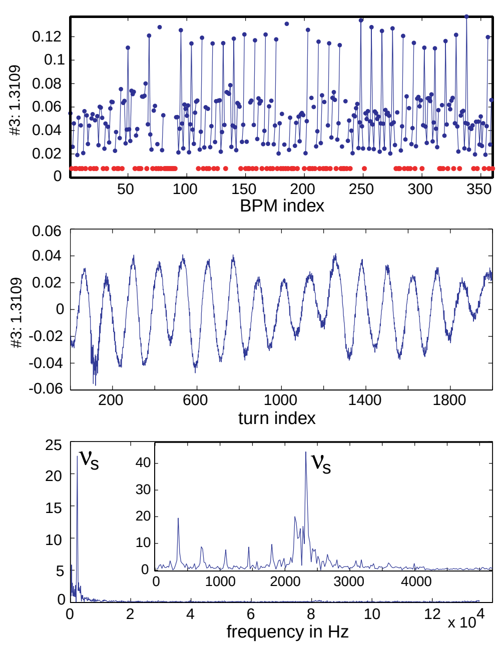

Figure 1: The third mode shows an oscillation unrelated to the kick. Its spectrum sharply peaked at the synchrotron tune and its spatial vector is consistent with dispersion. Thus this mode is due to residual synchrotron oscillation of magnitude 10 μm. The insert is the lower end of the spectrum that shows various power-line harmonics, which are the cause of the undamped synchrotron motion.

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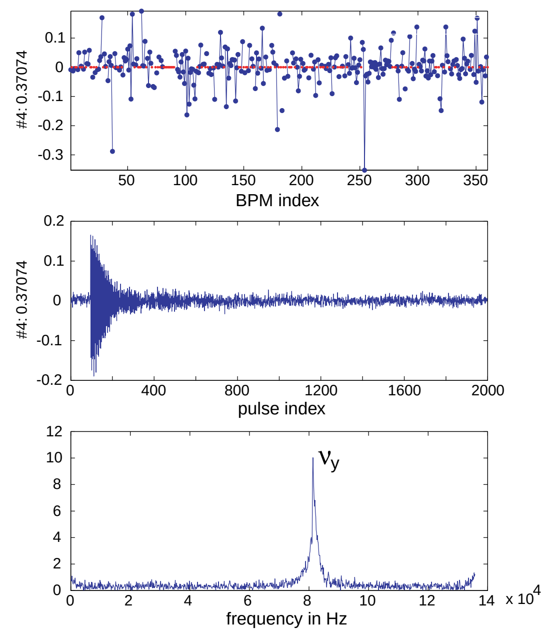

Figure 2: The fourth mode shows a kicked beam oscillating at the vertical tune. Since both the kick and observation are in the horizontal plane, this leads to transverse coupling. Misalignment of the kicker and BPMs might also contribute.

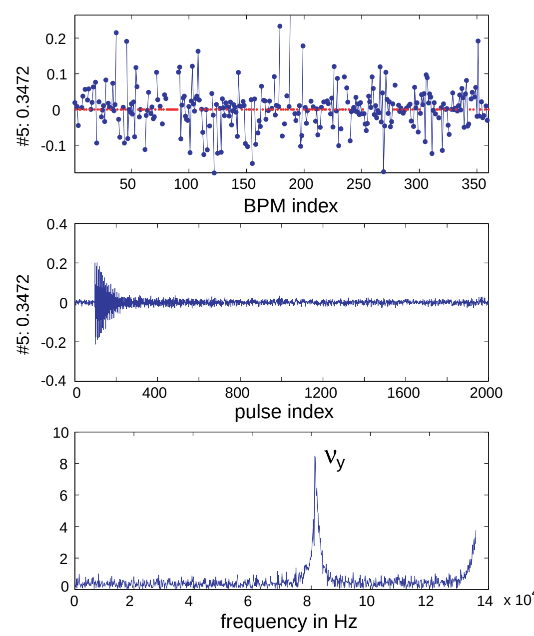

Figure 3: The fifth mode is the other vertical betatron mode paired with mode 4. See Fig. 5 for more comments.

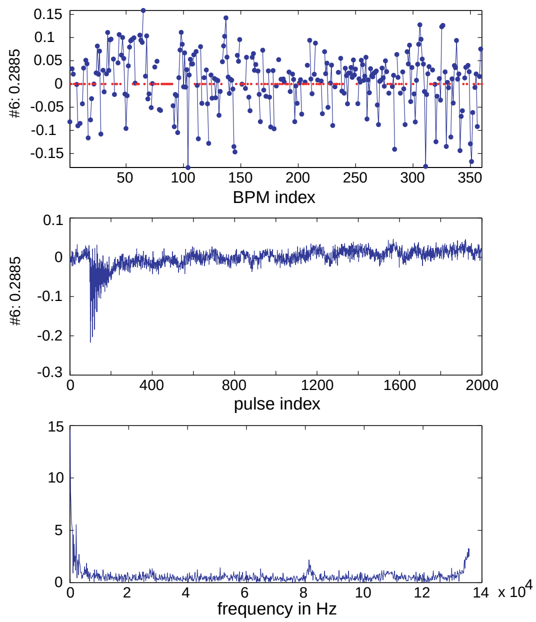

Figure 4: The sixth mode is rather different from others. The signal is excited by the kick and then smoothly damped instead of oscillating. Mixed with it are small oscillations at various frequencies.

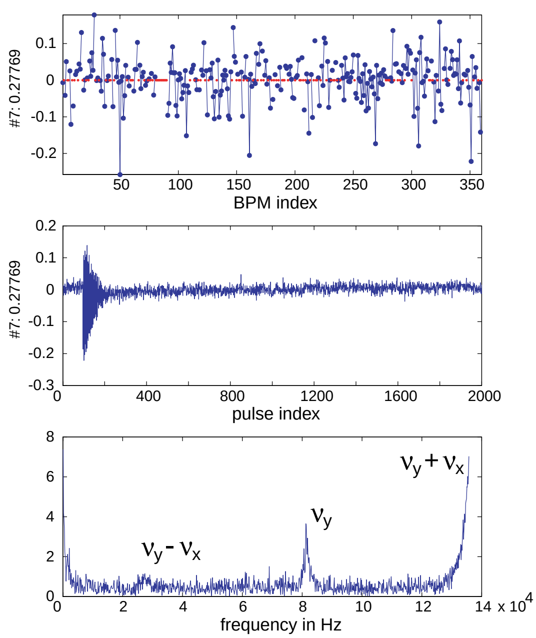

Figure 5: The seventh mode shows oscillation with frequencies ν_y, $\nu_y - \nu_x$, and $\nu_y + \nu_x$ ($\nu_x \simeq 35.20$, $\nu_y \simeq 19.26$, and $f_{\rm rev} = 271$ kHz). The right-most sum line is particularly strong, which suggests that sum resonance is much stronger than the difference resonance. Note that the sum signal also appeared in the vertical betatron modes, especially in Fig. 3. Thus the sum resonance might be the main driving force for transverse coupling.

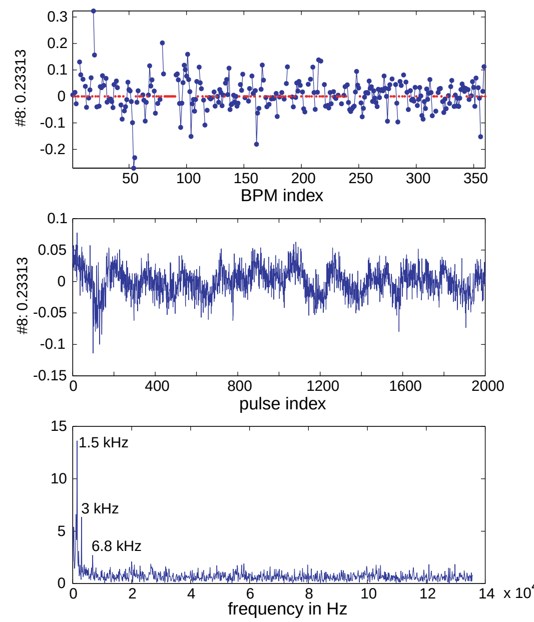

Figure 6: The eighth mode is unrelated to the horizontal kick. Its spectrum shows peaks around 0.36,1.5, 3, and 6.8 kHz. The zig-zag motion suggests that it may be due to feedback. In fact, some of these frequencies are connected with the real-time feedback system. Power line noise could be a factor as well.

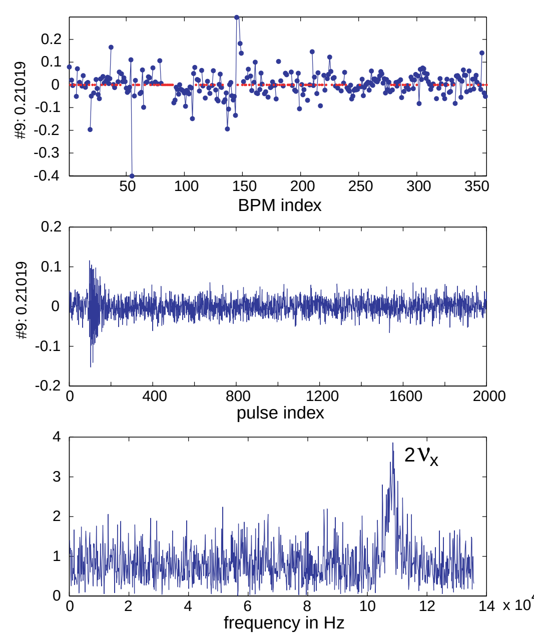

Figure 7: The ninth mode oscillates at $2\nu_x$ indicating nonlinear effects due to either lattice or BPM nonlinearity. The spatial vector suggests large effects around BPM 50 and 150. It is remarkable that the spatial-temporal mode analysis can clearly resolve this mode even at such a low signal level.