SIMULATION OF PARTICLE DYNAMICS IN ACCELERATORS USING THE ENSEMBLE MODEL[*]

W. Ackermann, M. Krasilnikov, T. Weiland, TEMF, TU Darmstadt, Germany
W. Beinhauer, H.-D. Gräf, A. Richter, IKP, TU Darmstadt, Germany

Abstract

In beam dynamics simulation a particle beam is usually considered as a set of numerous macroparticles. As each macroparticle is described by six phase space coordinates an enormous amount of computational effort has to be performed for accurate studies. In order to reduce simulation time an ensemble model which also considers the motion of individual particles inside the regarded subspace is introduced. This so-called ensemble model is derived from the VLASOV equation and includes the average values of the coordinates and momenta as well as the second-order correlation parameter. Therefore the number of involved particles can be reduced significantly. Based on the ensemble model the V-Code was developed to perform fast and accurate calculations for complete accelerator simulations. Its database had been initially designed for the TESLA Test Facility accelerator component description but was modified to meet also the requirements of the S-DALINAC injector simulations. In a further step the object-oriented code has been successfully extended to enable the simulation of recirculation. In this paper the results for the beam simulation at the S-DALINAC are presented.

INTRODUCTION

A large amount of single particles is required for careful examination of the beam dynamics' behavior in accelerators. Since it is impossible to handle real particles effectively so-called macro particles are to be introduced to reduce the simulation time and to save computer resources. Usually, such a macro particle is composed of numerous individual particles interacting with the external fields and consequently less NEWTON equations for the coordinates and momenta have to be solved.

ENSEMBLE MODEL

In contrast to the common stiff macro particles there is a possibility to include also the internal movements inside such a macro particle using the ensemble model [1]. Instead of solving the VLASOV equation immediately to obtain the phase distribution function f of the particle density in the space of the coordinates and momenta it is convenient to consider additionally to the first order moments

$$M_u = <u> = \int u\, f\, \mathrm{d}\vec{r}\mathrm{d}\vec{p} \qquad (1)$$

also second and even higher order moments in the model. The first order moments represent the corresponding values of a single particle as a whole while the second order moments

$$M_{u,v} = <(u-<u>)(v-<v>)> \qquad (2)$$

allow to describe the behavior of the particle distribution due to its finite dimensions. The values u and v can be any coordinates from the phase space $\{x, y, z, p_x, p_y, p_z\}$ with the normalized momentum $\vec{p} = \vec{P}/(mc)$ in cartesian components.

General Approach

To calculate the beam parameter along the beam line for its different parts it is necessary to obtain appropriate time relations for the individual ensemble parameter. If the energy spread in the particle cloud is small enough compared to the mean energy

$$\gamma_m = \sqrt{1 + <p_x^2> + <p_y^2> + <p_z^2>} \qquad (3)$$

it is reasonable to implement a linear approximation of the encountered energy in a given operating point. Whenever the beam is exposed to forces which satisfy the condition

$$<\mu\, \mathrm{div}_{\vec{p}}(\frac{\vec{F}}{m\,c^2})> = 0 \qquad (4)$$

for each ensemble parameter $\mu \in \{M_u, M_{u,v}\}$ it is possible to obtain the differential equations

$$\frac{\partial <\mu>}{c\,\partial t} = <\mathrm{grad}_{\vec{r}}(\mu)\cdot\frac{\vec{p}}{\gamma}> + <\mathrm{grad}_{\vec{p}}(\mu)\cdot\frac{\vec{F}}{mc^2}> \qquad (5)$$

from VLASOV equation to calculate the time-dependent ensemble parameter [2]. Starting from a given set of parameters as an initial condition it is then possible to determine the beam characteristics along the whole beam line.

Time Relations

In order to implement an algorithm based on the ensemble model it is mandatory to derive a proper time equation for each involved ensemble parameter. The available degrees of freedom are apparently linked to the maximum order of moments. For an ensemble model which utilizes the statistical values up to the second order moments to describe the beam characteristics in an accelerator substructure it is proposed to formulate the individual time relations in the following way:

- coordinate relations

$$\frac{\partial M_u}{c \, \partial t} = \frac{p_u}{\gamma_m} - \frac{1}{\gamma_m^3} \sum_k < p_k > M_{p_u, p_k} \tag{6}$$

- momentum relations

$$\frac{\partial M_{p_u}}{c \, \partial t} = < \frac{F_u}{mc^2} > \tag{7}$$

- coordinate coordinate relations

$$\begin{aligned}\frac{\partial M_{u,v}}{c \, \partial t} &= \frac{1}{\gamma_m}(M_{u,p_v} + M_{v,p_u}) \\ &- \frac{< p_u >}{\gamma_m^3} \sum_k (< p_k > M_{v,p_k}) \\ &+ \frac{< p_v >}{\gamma_m^3} \sum_k (< p_k > M_{u,p_k}) \end{aligned} \tag{8}$$

- coordinate momentum relations

$$\begin{aligned}\frac{\partial M_{u,p_v}}{c \, \partial t} &= \frac{M_{p_u,p_v}}{\gamma_m} \\ &- \frac{< p_u >}{\gamma_m^3} \sum_k (< p_k > M_{p_v,p_k}) \\ &+ < (u - < u >)\frac{F_v}{mc^2} > \end{aligned} \tag{9}$$

- momentum momentum relations

$$\begin{aligned}\frac{\partial M_{p_u,p_v}}{c \, \partial t} &= < (p_u - < p_u >)\frac{F_v}{mc^2} > \\ &+ < (p_v - < p_v >)\frac{F_u}{mc^2} > \end{aligned} \tag{10}$$

These general time relations have to be applied to the different parts of the investigated accelerator structures. Special attention has to be paid to Eqs. (7,9,10) because of the ability to provide an interaction point with the essential external fields. All remaining Eqs. (6,8) are used to establish proper update relations.

V-CODE

Based on the ensemble model the simulation tool V-Code has been constructed [3]. It is organized to handle even a very long beam line with a large amount of individual beam line elements. Each element covers typical beam line properties without any interference to each other.

Beam Line Elements

So far a reliable database has been established to provide various beam line elements like cavities, drift spaces, steerers, quadrupoles, and bending magnets. Each part can be described by its geometry and by the predominant electric and magnetic field along the axes. It is possible to simulate the whole system in a step-by-step procedure. The software package had been initially designed for the TESLA Test Facility linear accelerator but was modified to meet also the requirements of a S-DALINAC injector simulation. The installed beam guiding system there allows to use the main linac several times and this emphasizes the necessity of the simulation code to handle even recirculation.

Recirculation

To illustrate the recirculating configuration of the S-DALINAC a principle layout is given in Fig. 1. It shows the injector part together with the main linac and the two established loops. The requirement to implement some modifications to the simulation code is obvious if the well proven linear approach of the program used so far is to be utilized further on.
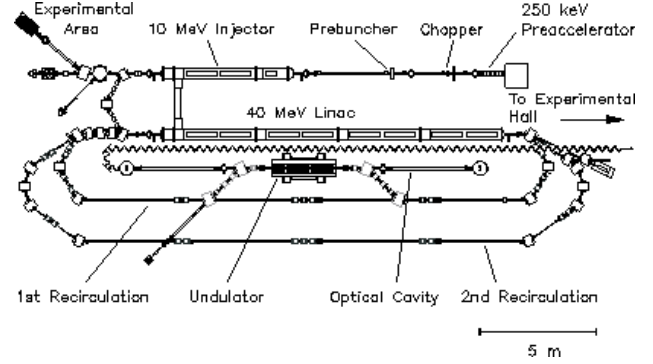

Figure 1: Layout of the S-DALINAC

Bending Magnets

The first approach to simulate a bending magnet is to assume a hard edge model with a constant magnetic flux density inside the magnet and a zero field outside. Some modifications have to be done if the effects caused by the finite extend of the fringe fields are to be also taken into account. All regarded external magnetic fields contribute to Eqs. (7,9,10) whereas the LORENTZ law $\vec{F} = q \, \vec{v} \times \vec{B}$ acts as the only force to calculate the particular contributions:

$$\frac{\partial M_{p_u}}{c \, \partial t} = \vec{e}_u \cdot \frac{q}{mc} < \frac{\vec{p}}{\gamma} > \times \vec{B} \tag{11}$$

$$\frac{\partial M_{u,p_v}}{c \, \partial t} = \vec{e}_v \cdot \frac{q}{mc} < (u - < u >)\frac{\vec{p}}{\gamma} > \times \vec{B} \tag{12}$$

$$\begin{aligned}\frac{\partial M_{p_u,p_v}}{c \, \partial t} &= \vec{e}_u \cdot \frac{q}{mc} < (p_v - < p_v >)\frac{\vec{p}}{\gamma} > \times \vec{B} \\ &+ \vec{e}_v \cdot \frac{q}{mc} < (p_u - < p_u >)\frac{\vec{p}}{\gamma} > \times \vec{B} \end{aligned} \tag{13}$$

Coordinate System

Originally the V-Code was created to simulate the beam dynamics in a linear accelerator. To meet the new requirements as close as possible a fixed cartesian coordinate system with its origin located in the gun of the accelerator and its z-axis oriented along the beam tube is used. Unfortunately, this approach does not fit the needs of a recirculating accelerator structure and a more convenient moving curvilinear system has to be used instead.

SIMULATION

All efforts to enable recirculating of the V-Code can concentrate on the study of the bending magnets. In order to obtain a comparison with a calculational approach entirely different from the ensemble model a usual tracking routine has been set up additionally. The obtained results refer to an ideal bending magnet with a reference path length of 0.25 m and a bending angle of $45°$. Into its homogeneous field a single bunch with the kinetic energy of 10 MeV is injected along the reference path. As an additional step a second bunch is tracked with an offset of 5 mm to simulate off-axes behavior. For detailed studies a subsequent drift space of 0.25 m is attached to the magnet.

In order to implement a moving coordinate system to the simulation code two different strategies were examined:

- fixed coordinate system for simulations inside the bending magnet and a one-time rotation after a particle reaches the end of the magnet;
- moving curvilinear coordinate system with successive rotations in every simulation step.

The results for the first approach are illustrated in Fig. 2. Due to the proposed fixed coordinate system even the particles on the reference path with no offset appear to possess a non-neglecting transverse component.
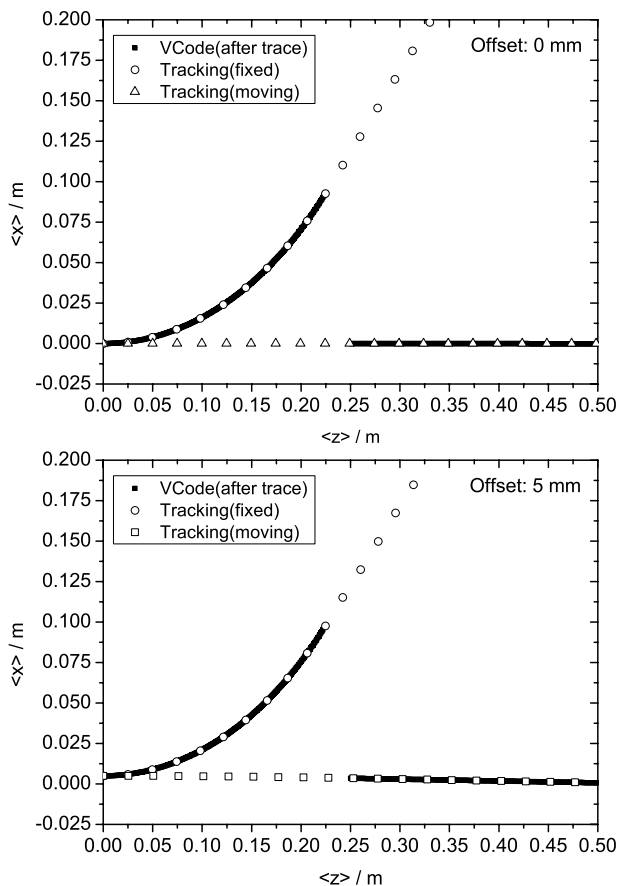
The simulation data for the unshifted and the intentionally shifted bunch in the case of a rotated coordinate system can been seen in Fig. 3. Ultimately, if one is interested in studies related to the longitudinal and transverse behavior of the beam within the magnet it is advisable to use a moving coordinate system. The simulation data also show that the output which has been acquired on a fixed coordinate system can be regarded as intermediate results only. The necessary corrections for further usage of the data are then carried out with a one-time rotation after the ensemble reaches the end of the magnet.
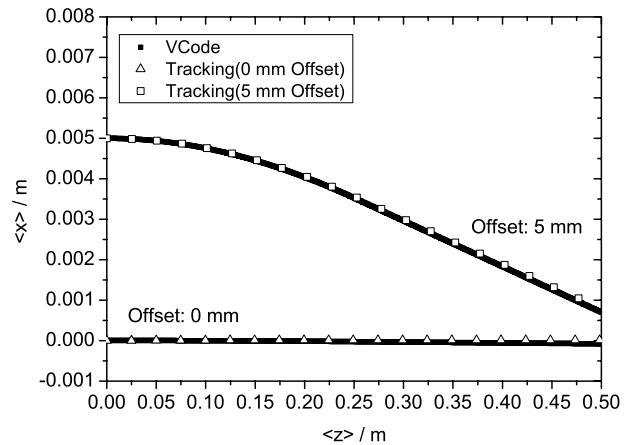

Figure 3: Beam location on a moving coordinate system

CONCLUSION

To enable the simulation of accelerator structures with V-Code with respect to recirculation the corresponding update equations for bending magnets are demonstrated. Two different possibilities to handle the rotation of the corresponding coordinate system are examined and the relevant numerical results are shown. In order to verify the results obtained by the V-Code, an additional tracking program was established. Both codes operate on the basis of a hard edge model for the bending magnet whereas no losses due to radiation are taken into account. The simulation data show that with a rotated coordinate system it is possible to unwrap the recirculating beam line configuration of the S-DALINAC and to apply the whole database from V-Code to further beam line elements beyond the injector part.

REFERENCES

[1] A. Novokhatski and T. Weiland, "Self-consistent Model for the Beams in Accelerators", *Proc. of ICAP*, Monterey, 1998

[2] A. Novokhatski and T. Weiland, "The Model of Ensembles for the Beam Dynamics Simulation", *Proc. of PAC1999*, New York, 1999

[3] M. Krassilnikov, A. Novokhatski, B. Schillinger, S. Setzer, T. Weiland, W. Koch, P. Castro, "V-Code Beam Dynamics Simulation", *Proc. of ICAP2000*, Darmstadt, 2000

Figure 2: Beam location on a fixed coordinate system