AN IMPROVED 1-D MODEL FOR ULTRA HIGH POWER RADIATION PULSE PROPAGATION IN THE HELICAL WIGGLER FREE ELECTRON LASER

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Abstract

A one dimensional model of a helical wiggler Free Electron Laser is presented that does not average the Maxwell-Lorentz equations describing the interaction between electrons, wiggler and radiation fields. Furthermore, no relativistic approximations in the equations governing electron motion are made and transverse motion of the electrons is self-consistently driven by both the wiggler and radiation fields. Numerical solutions of the resultant equations allow for the modelling of radiation pulses with peak powers orders of magnitude greater than the steady state saturation value and with pulse widths significantly less than one radiation period. Preliminary numerical modelling of such pulses was investigated by simulating injection of a suitable high power seed pulse into an FEL amplifier. These studies suggest a possible new regime of operation where sub-period radiation pulses may extract energy from the electrons in a periodic burst mode.

INTRODUCTION

In the non-linear regime of operation Free Electron Lasers can produce very high intensity spikes of radiation. One example of where such spikes arise is in FEL superradiance where a self-similar spike may grow and propagate through the electron pulse [1]. However, it is as yet unknown either analytically or through numerical simulation whether or how such spikes saturate and the self-similar solution breaks down. In this paper we present a model that should allow such investigation at least in the 1-D limit. We minimize any assumptions. In particular, none are made with respect to the relative strengths of the wiggler and radiation fields, and by not performing any average over a wiggler period, sub-period radiation evolution is included. Furthermore, no assumptions are made regarding the efficiency of energy exchange between electrons and radiation. While not answering the question regarding spike saturation our preliminary results nevertheless demonstrate some potentially interesting phenomena.

THE MODEL

The physics of the FEL in the 1-D limit may be described by the coupled Maxwell/Lorentz equations:

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}(z,t)=\mu_0\frac{\partial \mathbf{J}_\perp(z,t)}{\partial t} \qquad (1)$$

$$\frac{d\mathbf{p}_j}{dt}=-e\left(\mathbf{E}(z_j,t)+\frac{\mathbf{p}_j}{\gamma_j m}\times\mathbf{B}(z_j,t)\right) \qquad (2)$$

where $j = 1..N$, the total number of electrons, and the transverse current density may be written

$$\mathbf{J}_\perp(z,t)=-\frac{e}{m}\sum_{j=1}^{N}\frac{\mathbf{p}_{\perp j}}{\gamma_j}\,\delta\left(\mathbf{r}-\mathbf{r}_j(t)\right). \qquad (3)$$

The wiggler and radiation electric field are assumed to be

$$\mathbf{B}_w(z) \;=\; \frac{B_w}{\sqrt{2}}\left(\hat{\mathbf{e}}\, e^{-ik_w z}+c.c.\right) \qquad (4)$$

$$\mathbf{E}(z,t) \;=\; \frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}\,\mathcal{E}(z,t)e^{i(kz-\omega t)}+c.c.\right), \qquad (5)$$

where $\hat{\mathbf{e}} = (\hat{\mathbf{x}}+i\hat{\mathbf{y}})/\sqrt{2}$. The magnetic component of the radiation field $\mathbf{B}(z,t)$ required in the Lorentz equation (2) was calculated from the Maxwell equation $\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$. In performing this calculation for $\mathbf{B}$ the independent variables (z,t) are transformed to their scaled form $(\bar{z},\bar{z}_1)$ [2] which will be used in the final scaled equations. Then under the assumption that:

$$\left|\left(\frac{\partial}{\partial\bar{z}}+\frac{\partial}{\partial\bar{z}_1}\right)E_{x,y}\right| \ll \left|\frac{\bar{\beta}_z}{1-\bar{\beta}_z}\frac{\partial}{\partial\bar{z}_1}E_{x,y}\right|, \qquad (6)$$

where $c\bar{\beta}_z$ is the initial mean scaled z component of the electron velocity, the cartesian components of the radiation magnetic field $B_{x,y} \approx \mp E_{y,x}/c$ yielding

$$\mathbf{B}(z,t)=\frac{-i}{\sqrt{2}c}\left(\hat{\mathbf{e}}\,\mathcal{E}(z,t)e^{i(kz-\omega t)}-c.c.\right). \qquad (7)$$

By introducing the notation $p_\perp = p_x - ip_y$ (so that $\mathbf{p}_j\cdot\hat{\mathbf{e}}^* = p_\perp/\sqrt{2}$), taking the scalar product of the wave equation (1) with $\hat{\mathbf{e}}^*$, integrating over the common electron/radiation transverse area σ and making the assumption (6), the second order wave equation (1) reduces to:

$$\left(\frac{\partial}{\partial\bar{z}}+\frac{\partial}{\partial\bar{z}_1}\right)\mathcal{E}(z,t)=\frac{e}{2\sigma\epsilon_0 mc}\frac{\bar{\beta}_z}{1-\bar{\beta}_z} \quad \times$$

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$$\sum_{j=1}^{N} e^{-i\left(\frac{\bar{z}_1-\bar{z}}{2\rho}\right)} \frac{p_{\perp j}}{\gamma_j \beta_{zj}} \delta(\bar{z}_1 - \bar{z}_{1j}(\bar{z})) \tag{8}$$

Substituting the fields (4,5,7) into the Lorentz equation (2), taking the scalar product with $\hat{\mathbf{e}}^*$ and changing the independent variable from t to $\bar{z}$, the following equation for the perpendicular momentum is obtained

$$\frac{dp_{\perp j}}{d\bar{z}} = \frac{e}{2k_w\rho}\left(iB_w e^{-i\frac{\bar{z}}{2\rho}} - \frac{1-\beta_{zj}}{\beta_{zj}}\frac{\mathcal{E}}{c}e^{i\left(\frac{\bar{z}_{1j}-\bar{z}}{2\rho}\right)}\right). \tag{9}$$

Similarly, the z component of the Lorentz equation (2) yields

$$\begin{aligned} \frac{d\beta_{zj}}{d\bar{z}} &= -\frac{e}{4m^2c^3k_w\rho\gamma_j^2} \times \\ &\left[\left(\frac{1-\beta_{zj}}{\beta_{zj}}\right)\left(p^*_{\perp j}\mathcal{E}e^{i\left(\frac{\bar{z}_{1j}-\bar{z}}{2\rho}\right)} + c.c.\right) - \right. \\ &\left. icB_w\left(p_{\perp j}e^{i\frac{\bar{z}}{2\rho}} - c.c.\right)\right] \end{aligned} \tag{10}$$

We now introduce the following notation:

$$\begin{aligned} \epsilon Q_j &= \frac{1-\beta_{zj}}{\beta_{zj}}, \quad \epsilon = \frac{1-\bar{\beta}_z}{\bar{\beta}_z}, \quad \alpha = \left(\frac{2\rho\gamma_r}{a_w}\right)^2, \\ \bar{p}_\perp &= \frac{p_\perp}{mc}, \quad A = \frac{e\mathcal{E}}{mc\omega_p\sqrt{\gamma_r\rho}} \end{aligned} \tag{11}$$

which allow the previous equations describing the FEL interaction to be written as:

$$\frac{d\bar{z}_{1j}}{d\bar{z}} = 1 - Q_j \tag{12}$$

$$\begin{aligned} \frac{dQ_j}{d\bar{z}} &= \frac{a_w}{4\rho}\frac{Q_j(\epsilon Q_j+2)}{1+|\bar{p}_{\perp j}|^2}\left[-i\left(\bar{p}_{\perp j}e^{i\frac{\bar{z}}{2\rho}} - c.c.\right) + \right. \\ &\left. \alpha\epsilon Q_j\left(\bar{p}^*_{\perp j}Ae^{i\left(\frac{\bar{z}_{1j}-\bar{z}}{2\rho}\right)} + c.c.\right)\right] \end{aligned} \tag{13}$$

$$\frac{d\bar{p}_{\perp j}}{d\bar{z}} = \frac{a_w}{2\rho}\left[ie^{-i\frac{\bar{z}}{2\rho}} - \alpha\epsilon Q_j Ae^{i\left(\frac{\bar{z}_{1j}-\bar{z}}{2\rho}\right)}\right] \tag{14}$$

$$\begin{aligned} \left(\frac{\partial}{\partial\bar{z}} + \frac{\partial}{\partial\bar{z}_1}\right)A(\bar{z},\bar{z}_1) = \frac{\sqrt{\alpha}}{2\rho}\frac{1}{\bar{n}_{p\parallel}}e^{-i\left(\frac{\bar{z}_1-\bar{z}}{2\rho}\right)} \times \\ \sum_{j=1}^{N}\frac{\bar{p}_{\perp j}\sqrt{\epsilon Q_j(\epsilon Q_j+2)}}{\sqrt{1+|\bar{p}_{\perp j}|^2}}\ \delta(\bar{z}_1 - \bar{z}_{1j}(\bar{z})). \end{aligned} \tag{15}$$

Note that in the previous notation of [2] $Q_j = 1 - 2\rho p_j$. These are the working equations used for the following numerical simulations. In deriving these equations, and within the 1-D assumptions, the only approximations that have been made are the neglect of space charge effects and that of (6), equivalent to that made for the planar undulator case as described in more detail in [3]. Hence, very short high power pulse effects may be described where electrons may exchange a significant fraction of their energy in a short timescale. In the limit $(\epsilon, \rho) \ll 1$, and assuming a moderate, slowly varying radiation envelope $A(\bar{z}, \bar{z}_1) \not\gg 1$, the working equations may be expanded to first order in (ϵ, ρ) and a little analysis simplifies them to those of [2].

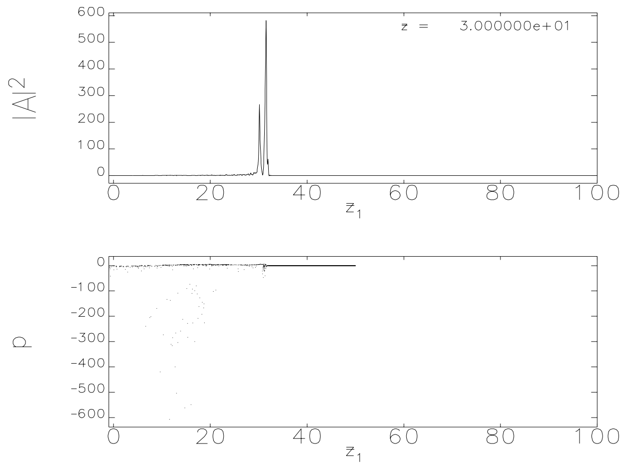

Figure 1: The scaled radiation power $|A|^2$ and electron phase space $(\bar{z}_{1j}, p_j)$ for a scaled distance through the interaction region of $\bar{z} = 30$.

AN NUMERICAL EXAMPLE

We now present a numerical solution of the equations (12..15) for the case of high power, short pulse injection into a FEL amplifier and compare this with the solution from an averaged model such as [1]. The electron pulse has a rectangular charge profile over the limits $0 < \bar{z}_1 < \bar{l}_e$ for $\bar{l}_e = 50$ and propagates a scaled length $\bar{z} = 50$ through the wiggler interaction region. The macroparticles are distributed uniformly over the electron pulse interval with initial conditions $p_{\perp j} = -a_w$ and $Q_j = 1\ \forall j$. Other parameters are $\rho = 1/4\pi$, $\gamma_r = 100$, $a_w = 2$. This value of ρ means that at fixed $\bar{z}$ one radiation period corresponds to an interval $\Delta\bar{z}_1 = 1$. A uniform power radiation pulse of two period duration is injected at $\bar{z} = 0$ with scaled field defined by $A_0(\bar{z}_1) = 10$ over interval $0 < \bar{z}_1 < 2$, and zero elsewhere. This corresponds to a power of the order of 10^2 greater than the steady-state scaled power of $|A|^2_{sat} \approx 1.4$ [1]. The equations (12..15) are solved using the streamline method of Finite Elements as in [3] with third order elements of scaled length in $\bar{z}_1$ of $\bar{l}_{el} = 0.1$, so that there are 10 elements per radiation period. The electron/radiation interaction was artificially switched off ahead of the injected radiation pulse i.e. for $\bar{z}_1 > \bar{z} + 2$. This suppressed any radiation evolution and allowed the radiation pulse to propagate into 'fresh' electrons. Although perhaps not strictly physically valid it does allow for a tentative preliminary investigation of the propagation characteristics of short high power pulses and is also similar to radiation pulse evolution over a large number of passes in a FEL oscillator.

In figure 1 we plot the scaled radiation power and the electron phase-space at an intermediate distance $\bar{z} = 30$ through the interaction region. It can be seen that the radiation pulse centre initially at $\bar{z}_1 = 1$ at $\bar{z} = 0$ has propagated through the electron pulse to $\bar{z}_1 \approx \bar{z} = 30$ and with a peak scaled power amplified to $\approx 6\times$ that at the begin-

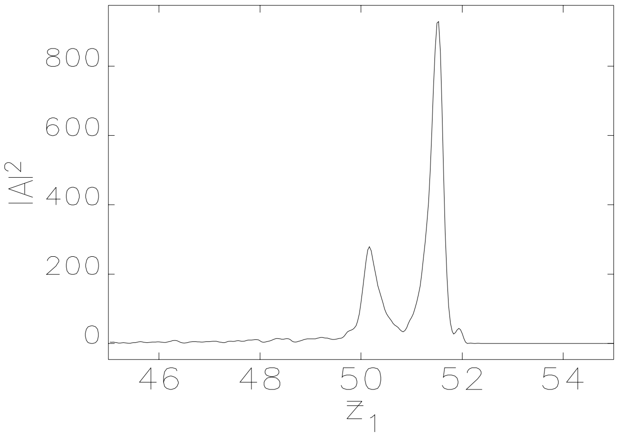

Figure 2: The scaled radiation power $|A|^2$ as a function of scaled pulse position $\bar{z}_1$ for a scaled distance through the interaction region of $\bar{z} = 50$.

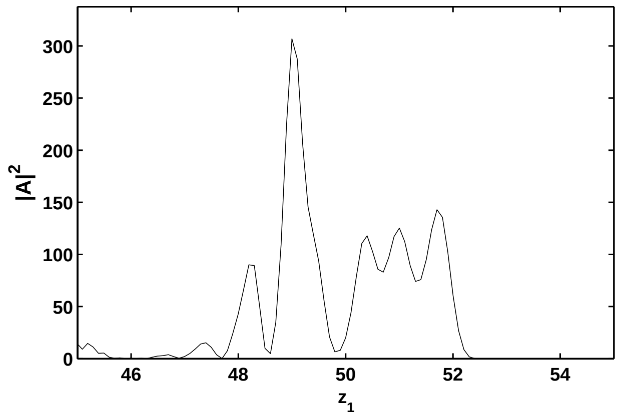

Figure 3: The scaled radiation power $|A|^2$ of the averaged model as a function of scaled pulse position $\bar{z}_1$ for a scaled distance through the interaction region of $\bar{z} = 50$.

ning of the interaction. The radiation pulse shape has been transformed from the rectangular shape to that suggesting a superradiant type spike [1]. The electrons react strongly in their interaction with the radiation spike and some may lose a large fraction of their initial energy to it. The electrons centred around $\bar{z}_1 \approx 15$ with large negative p have experienced a rapid deceleration to a mean of ≈ 0.2 of their original energy and rapidly propagate to negative values of $\bar{z}_1$ and out of the main body of the electron pulse. This deceleration and ejection of electrons from the pulse is a periodic feature of the radiation spike/electron interaction and occurs at the wiggler period. Figure 2 shows the scaled radiation power in a narrow window about the radiation spike at the end of the interaction $\bar{z} = 50$. The peak power is now a factor ≈ 10 greater than that at the beginning of the interaction and the pulse width (FWHM) is seen to be ≈ 0.25 of a radiation period. This very short pulse will clearly have different interaction dynamics with the electrons than that usually associated with the FEL interaction with a SVEA field where the transfer of energy from electron to radiation is considered a 'slow' process with one complete radiation wavelength passing over a resonant electron in one wiggler period. For very short radiation pulses the greatest rate of energy transfer between electrons and radiation will clearly be at the radiation spike. If this spike has duration less than a radiation period then it may only extract energy from the electrons during a fractional interval of the wiggler period when $\dot{\gamma} \propto \mathbf{v}_\perp \cdot \mathbf{E} < 0$. This may tentatively explain why, when radiation pulse lengths become shorter than the radiation period, the electrons are seen to loose energy in 'bursts' at the wiggler period as seen in figure 1.

We also include a simulation using an averaged model in the Compton limit [1]. We attempted to model the very short pulses generated with the above parameters by sampling the electron pulse at a sub-wavelength interval. We used 10 sample points per ponderomotive period and averaged about each sample point over a ponderomotive period. Each interval of $\Delta\bar{z}_1 = 4\pi\rho$ was therefore modelled by 10 overlapping but mutually non-interacting ponderomotive potentials. This averaged model result for the equivalent of the unaveraged model's figure 2 is shown in figure 3. It can be seen that, whereas for the unaveraged case of figure 2 where the original rectangular pulse has been amplified ($50 < \bar{z}_1 < 52$), for the averaged case of figure 3 the original rectangular pulse ($50 < \bar{z}_1 < 52$) appears to have been modulated with the growth of a less intense pulse appearing in its wake ($48 < \bar{z}_1 < 50$). This method of sampling at sub-period intervals appears to allow modelling of sub-period phenomena. However such a model is not physically valid as made clear in the comparison between figures 2 and 3.

CONCLUSIONS

A model has been derived that should allow the analysis and numerical modelling of high power radiation spikes in a FEL. It will be apparent that the numerical simulations presented here are of a preliminary nature and that much work remains to be carried out. Nevertheless, some interesting results have come to light, particularly the burst mode of interaction between the electrons and a sub-period radiation spike.

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