# PROPOSAL OF PARTIAL SIBERIAN SNAKE BASED ON HELICAL MAGNETS FOR AGS 

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#### Abstract

A scheme of partial Siberian snake is considered for polarized proton acceleration at AGS. The snake consists of four identical helical magnets with the field amplitude about 4.0 T on axis and provides proton spin rotation by about 32 degrees per one pass at the injection energy 2.5 GeV . Orbit distortions do not exceed 2 cm inside the insertion. The influences of the snake fields' nonlinearities on the beam dynamics are discussed.


## INTRODUCTION

Adiabatical crossing of imperfection spin resonance with a partial Siberian snake has been suggested, tested and successfully used already 30 years ago [1]. The application of this method at AGS has provided an acceleration of polarized protons up to 21 GeV with the $50 \%$ beam polarization [2]. Introduction of an RF dipole field inducing a coherent vertical betatron oscillation results in adiabatical spin flip on four strong intrinsic spin resonances with vertical tune at AGS. However, a coupling of the betatron oscillations introduced by the strong snake solenoid enhances depolarizing resonances with the horizontal tune and dramatically reduces the polarization level during acceleration up to the top energy of 25 GeV . To avoid these problems, a design of the partial snake based on the room temperature helical magnet was suggested in 1998 [3]. It was shown that the helical snake approach creates much smaller coupling with the same snake strength $5 \%$.
The helical approach for the partial snake has an evident advantage. In contrast to the solenoid, a DC current in the helix coil will rotate spin approximately on the same angle during entire AGS acceleration cycle.
This paper considers a scheme of partial Siberian snake which consists of four 0.5 -meter identical helical magnets with the field value up to 4.0 T . An analysis of particle and spin motions yielded a configuration of the helical magnets that can provide proton spin rotation by 32 degrees per one pass. At that, orbit distortions are zero outside the snake and do not exceed 2.2 cm inside the insertion on the injection energy 2.5 GeV .

## MAIN PARAMETERS OF THE SNAKE AND MAGNETIC FIELD DISTRIBUTION

It's clear that the 4.0 T magnetic field level can be realized only in a superconducting helix. Main parameters of the helix are determined by a number of simple requirements. The total length of the snake insertion is 10 feet; the aperture of the magnet does not restrict the AGS acceptance; a number of the helixes results from symmetry. Taking into account fringe fields, we need to
insert four identical helixes to exclude outside orbit excursions in both transverse directions. So we come to the parameters: period of the helix is 50 cm ; internal diameter is 10 cm , end conductors are each 10 cm in length.


Fig. 1 A cross section view of the MERMAID input file model.

A practical design of the helical magnet has to be based on the realistic fields including edges of coils. A calculation of the field configuration was done by computer code MERMAID [4]. The helix was presented as a number of consecutive slices in the transverse cross section. The coil size was optimized for coils wound by 1.26 mm NbTi wire with well known critical parameters. Fig. 1 shows a cross section view of the helix. 2-D distributions of transverse ( $\mathrm{Bx}, \mathrm{By}$ ) and longitudinal (Bz) components of the magnetic field in the middle cross section are presented in Figs. 2-4.


Fig. 2 Transverse distribution of Bx component.


Fig. 3 Transverse distribution of By component.


Fig. 4 Transverse distribution of Bz component.

An interpolated behaviour of the on-axis transverse components along the helix is shown in Fig. 5.


Fig. 5. Longitudinal distribution of the Bx and By components on axis.

Our analysis has shown that the spin rotation is not very sensitive to the initial field direction. For simplicity we always consider the magnetic field on the helix entrance along " $y$ "-axis.

## EQUATIONS OF THE ORBITAL MOTION

Consideration of the particle and spin motions in the calculated fields was done in the rectangular Cartesian frame with z-coordinate along the helix axis. Exact equations of particle trajectory excursions $x(z)$ and $y(z)$ are:

$$
\begin{align*}
& x^{\prime \prime}(z)=\frac{1}{B \rho} q\left(x^{\prime}(z) y^{\prime}(z) B_{x}-\left(1+x^{\prime}(z)^{2}\right) B_{y}+y^{\prime}(z) B_{z}\right) \\
& y^{\prime \prime}(z)=-\frac{1}{B \rho} q\left(x^{\prime}(z) y^{\prime}(z) B_{y}-\left(1+y^{\prime}(z)^{2}\right) B_{x}+x^{\prime}(z) B_{z}\right) \\
& q=\sqrt{1+x^{\prime}(z)^{2}+y^{\prime}(z)^{2}} \tag{1}
\end{align*}
$$

We calculated two options of the snake with four helixes with the right helicity but different polarities. The first set of helixes had the polarities: "+ --+ ". More serious orbit excursions appear at the AGS injection energy of 2.5 GeV . Some residual mismatching of the trajectory can be adjusted be existing steering coils in the ring.

The other snake option can be constructed by the helix sequence: " +-+- ". This option has no symmetry. It results in a big enough x -angle at the snake exit. To compensate this angle we calculated dipole correctors incorporated in the helix module above the edge commutations. Fig. 6 presents a picture of the field on the axis with correctors "on".


Fig. 6 Longitudinal distribution of the Bx and By components with correctors.

The particle trajectory in the 2 nd snake option, when the first and last helixes are equipped by the correctors is given in Fig. 7.


Fig. 7 Particle's trajectory at 2.5 GeV in case of "+ ++ " scheme. $1^{\text {st }}$ and $4^{\text {th }}$ snakes have correctors.

## EQUATIONS OF SPIN ROTATION

Similarly to the particle motion, the BMT-equation for the spin vector $\vec{S}$ can be presented in the same frame in the form:
$S_{x}^{\prime}=S_{y} \frac{q}{B \rho}\left[(1+\gamma a) B_{z}-\frac{a}{q}\left(1-\frac{1}{\gamma}\right)\left(x^{\prime} B_{x}+y^{\prime} B_{y}+B_{z}\right)\right]-$ $-S_{z} \frac{q}{B \rho}\left[(1+\gamma a) B_{y}-\frac{a}{q}\left(1-\frac{1}{\gamma}\right) y^{\prime}\left(x^{\prime} B_{x}+y^{\prime} B_{y}+B_{z}\right)\right]$ $S_{y}^{\prime}=S_{z} \frac{q}{B \rho}\left[(1+\gamma a) B_{x}-\frac{a}{q}\left(1-\frac{1}{\gamma}\right) x^{\prime}\left(x^{\prime} B_{x}+y^{\prime} B_{y}+B_{z}\right)\right]$ $-S_{x} \frac{q}{B \rho}\left[(1+\gamma a) B_{z}-\frac{a}{q}\left(1-\frac{1}{\gamma}\right)\left(x^{\prime} B_{x}+y^{\prime} B_{y}+B_{z}\right)\right]$ $S_{z}{ }^{\prime}=S_{x} \frac{q}{B \rho}\left[(1+\gamma a) B_{y}-\frac{a}{q}\left(1-\frac{1}{\gamma}\right) y^{\prime}\left(x^{\prime} B_{x}+y^{\prime} B_{y}+B_{z}\right)\right]-$ $-S_{y} \frac{q}{B \rho}\left[(1+\gamma a) B_{x}-\frac{a}{q}\left(1-\frac{1}{\gamma}\right) x^{\prime}\left(x^{\prime} B_{x}+y^{\prime} B_{y}+B_{z}\right)\right]$ $q=\sqrt{1+x^{\prime 2}+y^{\prime 2}}$

Here $\gamma$ is the relativistic factor, $a=1.7928$ is the proton magnetic anomaly; all derivatives are taken with respect to longitudinal coordinate and field components are taken along the particle trajectory given by solution of Eq. (1).
As one can see from the Fig. 8, the final spin rotation corresponds to precession around the longitudinal axis by
the angle: $\quad \arccos \left[S_{y}(z=L)\right] \approx 32^{\circ}$. This angle practically does not depend on the beam energy and gives snake strength $\approx 0.18$.


Fig. 8 Spin rotation along the snake.

## CONCLUSION

Presented snake design requires a more detailed analysis before the realization phase. Numerical estimations shown that the tune shifts are quite moderate, $\square 0.02$. Since the snake occupies about $1 \%$ of the machine circumference we hope that relatively high local field nonlinearities in the helixes can be easily compensated by existing correctors. In any case further investigations of particle and spin dynamics should be done.

## REFERENCES

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