# COHERENT EMISSION OF SYNCHROTRON RADIATION AND LONGITUDINAL INSTABILITIES\*

M. Abo-Bakr, J. Feikes, K. Holldack, P. Kuske, G. Wüstefeld, BESSY, Berlin, Germany

## Abstract

At BESSY bursts of coherent synchrotron radiation around 20cm<sup>-1</sup> have been observed above a certain threshold current. The repetition rate of these bursts depends on the beam current and the thresholds vary strongly with the bunch length. Observed thresholds are in agreement with the theory of beam instability and microbunching due to coherent synchrotron radiation (CSR) [1].

### **INTRODUCTION**

In storage rings the longitudinal beam dynamics of a bunched beam can be described with the Haissinski equations [2] as a distortion of the potential well as long as the energy distribution of particles remains Gaussian and is independent of the beam intensity. Above a threshold current longitudinal bunch shape variations are accompanied by energy widening and are called 'turbulent bunch lengthening' or 'microwave instability'. Based on the description of the particle distribution within a bunch in the two dimensional longitudinal phase space in terms of azimuthal and radial modes it is believed that the mixing of modes causes the instability. Whether and at which beam intensity modes are mixed so strongly that an instability sets in depends on the interaction of the particles. The interaction is either expressed by the longitudinal delta function wakefield or equivalently by the longitudinal coupling impedance. Until recently the contribution of the vacuum chamber and the influence of leading particles on trailing particles was seen as the dominating effect. However, with shorter and shorter bunches, the CSR interaction and the impact of the trailing on leading particles becomes more and more important for the stability of a bunched beam.



Fig. 1: Appearance of the radiation bursts in time and frequency domain.

This paper presents observations made at the  $3^{rd}$  generation light source BESSY, a 1.7 GeV electron storage ring. Experiments were performed with bunches having a rms-length in the range from 2 ps up to 40 ps [3] by operating the storage ring lattice with reduced momentum compaction factor. The stability of single bunches was determined by looking at the time dependent emission characteristics of radiation around 10 cm<sup>-1</sup>. Parts of the experimental results are compared to analytical and numerical calculations.

### **OBSERVATIONS**

#### Bursts of Radiation

At the infrared beamline [4] 60x40 mrad<sup>2</sup> synchrotron radiation is focussed onto the input window of an InSb-FIR detector model HDL-5 from QMC Instruments Ltd. This detector is most sensitive around 20 cm<sup>-1</sup> and is capable of resolving the passage of a single circulating bunch every 800 ns. The detector is internally coupled to an AC-amplifier so that only the input power variations of the radiation can be seen. For the results presented here the output of the detector was connected either to a digital scope or to a spectrum analyser. Figure 1 shows a comparison of the signals in time and frequency domain for single bunch currents above a clear, bunch length dependent threshold. In this case the zero current bunch length was 14.5 ps. Individual bursts of radiation are very short, can occur in groups, and appear more or less randomly. The average burst rate, the timing jitter between the bursts, or the randomness of the emission of bursts is more clearly seen in the frequency domain. At low beam currents the average rate is around 1 kHz and there is strong correlation between bursts and many multiples of the fundamental burst frequency. At medium intensity the pulses are less correlated and the spectrum becomes broad. The characteristics of the time domain data is very similar to the observations at the ALS [5].



Fig. 2: Longitudinal beam spectra taken at 10 GHz as a function of single bunch current with  $F_{syn}$ =7.4 kHz

## Longitudinal Beam Spectra

The synchrotron sideband spectra were acquired with a Rohde & Schwarz spectrum analyser FSEK30 connected to a stripline. Measurements were performed as a function

<sup>\*</sup> Work supported by the Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie and by the Land Berlin

of beam current at 10 GHz and are shown in Fig. 2. More and more, and strongly shifted, harmonics of the synchrotron frequency appear as the beam current goes up. The first synchrotron sideband is always visible and presumably excited by phase noise of the RF system. The appearance of the quadrupole mode seems to be correlated with the radiation bursts.

### Comparison of Burst rates and Spectra

Fig. 3 shows a comparison of longitudinal sidebands and radiation bursts observed simultaneously with two spectrum analysers. Opposite to what was claimed for the ALS [5] sidebands at a fraction of the synchrotron frequency ( $F_{syn}$ =7.4 kHz) are observed and identical to the repetition rates of radiation bursts. The conclusion is that the density modulations inside the bunch of electrons (microbunching) responsible for the bursting emission of radiation in the THz range are correlated with bunch shape variations showing up at10 GHz.



Fig. 3: Comparison of bursts of THz radiation and low frequency longitudinal sidebands at 10 GHz. The longitudinal stripline spectra are shifted by 10 dB each.

The measurement was taken during the natural decay of the intensity over many hours. Even though a spectrum is acquired every minute and the intensity does not change much during this time the burst's average repetition rate as well as the sideband frequencies can change slowly or can jump suddenly to a new value. At low beam intensity there seem to be typical rational relations between the burst frequencies before and after a jump: 2 to 3 and 3 to 4. This behaviour is very reproducible and better seen in Fig. 4. Below a threshold beam current no bursts of radiation can be detected. In the case shown in Fig. 3 and 4 two superconducting wavelength shifters were operated at 7 and 6 Tesla and the bursts set in at a single bunch current of 4.5 and 3.8 mA. Without these strong field insertion devices the threshold is 2.3 mA presumably due to the smaller natural energy spread in this case.

With shorter bunches the results are very similar, however, the first line appearing in the spectra of the bursts is close to three times the natural synchrotron frequency and also visible in the longitudinal spectra at 10 GHz. These lines shift strongly upwards with beam current. Well above the instability threshold the overall behaviour is rather similar and independent of the bunch length: Regions of more or less correlated bursts with repetition times around 1 ms ( the natural longitudinal damping time is 8 ms) and eventually at very high intensity random and chaotic bursts. By inserting a cut-on filter at 10 cm<sup>-1</sup> in front of the IR-detector it could be verified that the more regularly appearing bursts are connected to radiation below the cut-on whereas the chaotic region is related to radiation above it.



Fig. 4: Spectra of the FIR radiation bursts as a function of single bunch current.

# COMPARISON OF OBSERVED AND CALCULATED THRESHOLDS

Thresholds are found either by looking at the appearance of bursts on the oscilloscope, appearance of lines in the spectra of the bursts, or by the sudden power increase observed with a lock-in amplifier with the revolution frequency as a reference [6]. The results of the experiments together with analytical and numerical calculations are presented in Fig. 5 as a function of the synchrotron frequency which is proportional to the bunch length. 1 kHz corresponds to a  $\sigma$  of 1.9 ps.



Fig. 4: Comparison of observed and calculated threshold currents as a function of the bunch length.

The predictions of the analytical theory [1] are shown as straight lines and the agreement with the observations and the calculations based on the numerical solution of the Vlasov-Fokker-Planck [7] equation is excellent. The analytical instability model extends a coasting beam result to bunched beams and it turns out that good agreement with the observations is obtained if the wavelength of the perturbation is chosen equal to the rms bunch length. This conditions seems more stringent than the shielding. In general, coasting beam results are applicable to bunched beams if the bunch length is much larger than this wavelength.

Recently the interaction of a bunch with its own radiation field including shielding was investigated by the numerical solution of the VFP equations [7]. Similar calculations were performed for the BESSY case based on the original approach [8] and with the radiation wakefield between perfectly conducting parallel plates as given by Murphy, et al. as their equ. (9.10) [9]. In Fig. 6 an attempt is made to visualise the delta function wakefield. Note that the interaction goes forward as well as backwards. In the calculations this wakefield is used to determine the wakepotential of the bunch. Venturini [7] does this with the help of the shielded CSR impedance. His and our results concerning the threshold do agree especially if one takes into account that he assumes a distance between the plates 5 mm smaller than in reality (H=3.5 cm).



Fig. 6: Wakefield as used in the numerical solution of the VFP equations. In case of the free space radiation the wake acts only in the forward direction (shown in green and to the right) with the asymptotic dependence displayed in red. If the radiation takes place in between perfectly conducting parallel plates then the wakefield (yellow) drops to zero a few mm in front of the electron and additional oscillating contributions occur behind the electron (to the left).

The results of our calculations are collected in Fig. 7 as a function of the scaled intensity,  $\Gamma$ , where c is the speed of light,  $Z_0$  is the vacuum impedance of  $120\pi \Omega$ , R is the dipole bending radius, To·I is the total charge, V'rf is the derivative of the accelerating voltage, and  $\sigma_{\rm zo}$  is the rmsbunch length.  $\delta_0$ ,  $\delta$  and  $\sigma_0$ ,  $\sigma$  are the natural and the actual rms-width of the distribution function in momentum and time. These quantities, the shift of the centre of gravity, COG,  $<To>/\sigma_o$ , corresponding to the synchronous phase shift, and the asymmetry (skewness) of the distribution were determined from the statistical moments of the distribution functions that solve the VFP equation. Results for very short bunches which is identical to assuming no shielding are in agreement with the solution of the Haissinski equation given by Bane, et al. [10]. The other results have to be taken with some caution since the positive shift of the COG is related to an energy gain of the bunch as the current increases which is impossible in reality. This point has to be clarified before more detailed comparisons of the numerical results with the observations can be made more faithfully. Nevertheless the current at which the energy spread starts to increase is taken as the onset of the instability and are in very good agreement with the observations and displayed in Fig. 5 as red squares.



Fig. 7: Results of the numerical solution of the VFP equation for the wakefield due to shielded CSR.

The numerical solution of the VFP equation is valid also in the unstable region of intensities. There are indications that the observed time dependence of the more regular as well as the chaotic bursts do show up in these calculations. Studies in this direction including attempts to model the longitudinal spectra will be made in the future and as soon as the problems with the chosen wakefield have been removed.

In conclusion, a rich variety of observations on the dynamics of bunches in storage rings can be made at 10 GHz and well above into the THz region. The theoretical understanding of bunched beams well into the region of microwave instability is still incomplete.

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