# SIMULATION STUDY OF COUPLED-BUNCH INSTABILITIES DUE TO RESISTIVE WALL, IONS, OR ELECTRON CLOUD

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### *Abstract*

We simulate the interaction of a bunch train with either an external wake field, (semi-)trapped ions in a field-free region or in a dipole field, or an electron cloud, on successive turns, using a simplified algorithm with only a small number of macro-particles. We present simulated mode spectra and rise times for the ensuing coupled-bunch instabilities, and show that observations at the KEKB HER are consistent with a horizontal instability driven by carbon-monoxide ions in a region without magnetic field.

## 1 INTRODUCTION

A horizontal instability may limit the beam current in the KEKB HER. The instability growth rate depends nonlinearly on the beam current, shows almost a threshold behavior, and is about a factor of 2 faster than the corresponding instabilities in the vertical plane [1]. The measured unstable multi-bunch frequencies, as viewed at a fixed location, were found to be close to the revolution harmonics 7 and 1 [1], and, for another set of data, roughly equal to 17 [2]. In [3] we explored if this instability could be a manifestation of an electron cloud instability in the electron ring. However, the fairly low order of the unstable modes is not easily explained by an electron-driven instability. It appears more compatible with an instability caused by ions (the ion 'oscillation' frequency is much lower than that of the electrons). In addition to ions, low frequencies are also expected for the resistive-wall instability. We have developed a simple simulation model for coupled-bunch instabilities driven by ions, resistive wall, and electron cloud, respectively. For each driving mechanism, the simulation provides growth rates and mode patterns. In the case of ions, we can also vary the ion mass and the magnetic field. After a brief review of the experimental data, we describe the simulation model and present example results.

## 2 EXPERIMENTAL DATA

Figure 1 shows an example measurement. The left picture in Fig. 1 displays raw data (after subtracting the average BPM offset for each bunch) of the 1280 horizontal bunch positions measured on the 2000th turn after turning off the transverse feedback. A gap of 80 missing bunches is visible at the end. We see about ten oscillations, whose amplitudes increase along the train. The right picture illustrates the mode spectrum of the instability computed over 4095 turns, by applying a complex fast Fourier transform to the BPM data. This spectrum shows the lower betatron sidebands as a function of the revolution harmonic. Only the sidebands of the 80 lowest revolution frequencies (from a total of 1280) are displayed here, since the higher-order modes are not excited. The peak corresponds to the lower sideband of the 11th revolution harmonic. This indicates that the 'ion tune' or 'wake tune', *i.e.*, the local oscillation frequency of the wake normalized to the revolution harmonic, is about 11. The number 11 lies between the numbers 7 and 17 determined in [1] and [2].

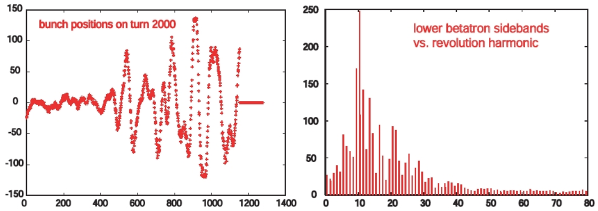

Figure 1: Left: measured horizontal position along the bunch train after 2000 turns; right: amplitude of lower betatron sidebands as a function of the revolution harmonic.

## 3 SIMULATION MODEL

In the simulation, we consider $n_{\rm step}$ uniformly spaced locations around the ring. The train of $n_{\rm bunch}$ bunches passes each of these locations, on $n_{\rm turn}$ successive turns. Each bunch creates a 'wake', which can act on the following bunches in the train or on later turns.

In the case of an ion instability, at each location the 'wake' left behind by a passing bunch is represented by a 'macro-ion'. A similar model was first employed by S. Heifets [4]. The effective charge weight of a newly created macro-ion is $q = \sigma_{\rm ion}\rho_{\rm gas}C/n_{\rm step}$, where $\rho_{\rm gas}$ is the residual-gas density and $\sigma_{\rm ion}$ the ionization cross section, which we take to be equal to 2 Mbarn for CO molecules at multiple GeV beam energies. In the simulation, to enhance the instability, we consider a vacuum pressure up to $10^{-6}$ Torr (the real average pressure is less than 1 nTorr).

The ions are generated at a random transverse position within $\pm 1\sigma_{x,y}$ from the bunch centre. During the passage of a bunch, a macro-ion is accelerated (it receives kicks $\Delta x'_{\rm ion}$ and $\Delta y'_{\rm ion}$) and the passing ($j$th) bunch is deflected as well, by an amount $\Delta x'_j = -\Delta x'_{\rm ion} q A_{\rm ion} r_e/(r_p N_b)$, where $A_{\rm ion}$ is the ion mass in units of the proton mass, and $N_{\rm bunch}$ the bunch population. An analogous relation applies to the vertical plane. The kicks $\Delta x'_{\rm ion}$ and $\Delta y'_{\rm ion}$ are computed from the transverse distance between the macro-ion and the bunch centroid, using the Bassetti-Erskine formula [5] for the electric field of a Gaussian distribution. After a bunch passage the macro-ion either drifts, without magnetic field, or, inside a bending magnet, it performs a cyclotron oscillation in the horizontal plane, until the next bunch arrives. The ion motion in a dipole field is

$$x = x_0 + \kappa(CSx'_0 + S^2 z'_0),\ x' = (C^2 - S^2)x'_0 + 2SCz'_0\ ,$$

$$z = z_0 + \kappa(CSz'_0 - S^2 x'_0),\ z' = (C^2 - S^2)z'_0 - 2SCx'_0\ ,$$

where the quantities with subindex $0$ are the initial coordinates just after passage of a bunch, and we have used the abbreviations $\kappa \equiv 2E/(eBc)$, where $B$ is the dipole field and $E$ the beam energy, $C \equiv \cos(L_{\rm sep}/\kappa)$, and $S \equiv \sin(L_{\rm sep}/\kappa)$. Between the $n_{\rm step}$ interaction points, the bunches perform a free betatron oscillation. The ions can survive for many bunches or even several turns. Light hydrogen are overfocused within the train and are lost in the vertical direction. Heavier ions, such as CO, are overfocused only in the gap at the end of the train. In a dipole field, the ions can be stabilized by cyclotron motion. The typical decay constant $n_{\rm decay}$ for each case was determined by a separate simulation, where we tracked groups of different ions in various field configurations during repeated passages of the KEKB bunch train [6]. The decay constants so obtained vary between one hundred and several thousand bunch passages (up to a few turns). In [4], each passing bunch generated a new macro-ion at each of the $n_{\rm step}$ locations. To limit the total number of macro-ions, we here merge the charges, positions and momenta of all (macro-)ions that would be generated at the same location into those of a single ‘super macro-ion’. Taking into account ion losses, at each bunch passage the effective contribution from old ions to the ‘super macro-ion’ is reduced by a factor $n_{\rm decay}/(n_{\rm decay}+1)$. The total number of macro-ions around the ring is always equal to $n_{\rm step}$. The charge of a ‘super macro-ion’ initially increases until it saturates at a value $qn_{\rm decay}$. The above simplification may not correctly represent the ion dynamics at large amplitudes, but we still expect to obtain a fairly accurate image of the excited multi-bunch mode patterns and a reasonable estimate of the instability rise times.

The simulation of the resistive-wall instability proceeds analogously. We employ the same type of ‘macro-particles’ to store the offset of all bunches passing a certain location and their longitudinal position along the train as well as the turn number. For each bunch a resistive-wall wake-field deflection is computed as

$$\begin{bmatrix}\Delta x'_j\\ \Delta y'_j\end{bmatrix} \approx \begin{bmatrix}0.5\\ 0.8\end{bmatrix}\sum_{i<j}\frac{2r_eN_b}{\gamma\pi b^3}\sqrt{\frac{c}{\sigma_0\ (s_i-s_j)}}\begin{bmatrix}x_i\\ y_i\end{bmatrix},$$

where $(s_i - s_j)$ is the longitudinal distance between two bunches, $\sigma_0 \approx 5.4 \times 10^{17}\ {\rm s}^{-1}$ the conductivity of copper, $b$ the vertical beam-pipe half gap, and the coefficients $0.5$ or $0.8$ for an elliptical chamber follow from [9]. We should add the contributions from all previous bunches, but in practice we often truncate the sum after two turns. The convergence can be checked easily. Unlike for the ion case, the ‘macro-particles’ are static and do not move between bunch passages. Here all the bunches are initially offset transversely by the same constant value of $10^{-3}\sigma_{x,y}$. The constant offset is chosen, because it is close to the asymptotic multi-bunch pattern of the resistive-wall instability, which is driven at low frequency.

Table 1: KEKB HER electron-beam parameters.

| variable | symbol | value |
|---|---|---|
| beam energy | $E$ | 8 GeV |
| rms bunch length | $\sigma_z$ | 6 mm |
| transv. rms beam sizes | $\sigma_{x,y}$ | 687, 73 $\mu$m |
| average beta function | $\beta_{x,y}$ | 15 m |
| bunch spacing | $s_b/c$ | 8 ns (4 buckets) |
| bunch population | $N_b$ | $3.5 \times 10^{10}$ |
| total no. of bunches | $n_b$ | 1200 |
| missing bunches (gap) | $n_{\rm gap}$ | 80 |
| ring circumference | $C$ | 3016 m |
| half aperture | $h_{x,y}$ ($b$) | 52, 28.5 (25) mm |

Finally, in the case of the electron cloud, we consider a short-range wake coupling only consecutive bunches, and zero electron memory from turn-to-turn. This case is similar to the resistive-wall case, but the number of macro-particles needed is much smaller, since only a single preceding bunch contributes to the wake, $\Delta x'_j \approx (r_e N_b/\gamma) W_x x_{j-1}$. The initial transverse bunch centroid positions are chosen randomly within $\pm 10^{-3}\sigma_{x,y}$. For the electron-cloud case, we assume a bunch-to-bunch wake field of strength $W_{x,y} \approx 10^5\ {\rm m}^{-2}$, about a factor 10 smaller than typical for positron or proton beams [7].

The other simulation parameters are listed in Table 1.

## 4 RESULTS

Figure 2 shows the simulated horizontal bunch positions along the train, and Fig. 3 the oscillations of a few selected bunches as a function of turn number, for various cases exhibiting an instability. In the resistive-wall case, the oscillation amplitudes of all bunches grow with a similar rate, while, both with CO ions in a field-free region and with an electron cloud, the growth rate increases towards the end of the train. The case of H ions in a dipole shows a faint instability, with three ‘bursts’ along the train. For two other examples (not shown), *i.e.*, hydrogen ions without magnetic field, and carbon monoxide ions in a dipole field, the beam appears stable. Figure 4 presents the mode spectra computed for the same 4 simulations as above. The case of carbon-monoxide ions in a field-free region (top left picture) is the only one that — nearly perfectly — matches the observation. The unstable mode number of 13 is almost the same as measured (11). This number is more than 2 times smaller than expected from the angular ion oscillation frequency near the center of the beam $\omega_{i;x} \approx \left(2N_b r_p c^2/\left(L_{\rm sep}\sigma_x(\sigma_x+\sigma_y)A\right)\right)^{1/2}$, which for CO would yield an ‘ion tune’ of 28. We attribute this frequency reduction to the finite size of the beam and the ion cloud, and to the nonlinearity of the force acting between them. The 15% difference between 13 (simulated) and 11 (measured) can be explained by a 30% difference in bunch population between this particular experimental data set ($N_b \approx 2.7 \times 10^{10}$) and the simulation. In the top left picture of Fig. 4, also the mode 2 is strongly excited, which is not seen in the experimental data of Fig. 1, but resembles those presented in Ref. [1]. This mode likely is a consequence of the nonlinear beam-ion force, causing detuning with amplitude and saturation.

None of the other 3 cases yields a simulation result which even remotely resembles the observed spectrum. However, the mode spectrum simulated for the electron-cloud wake is similar to that observed in the KEKB LER positron ring without a solenoid field [1].

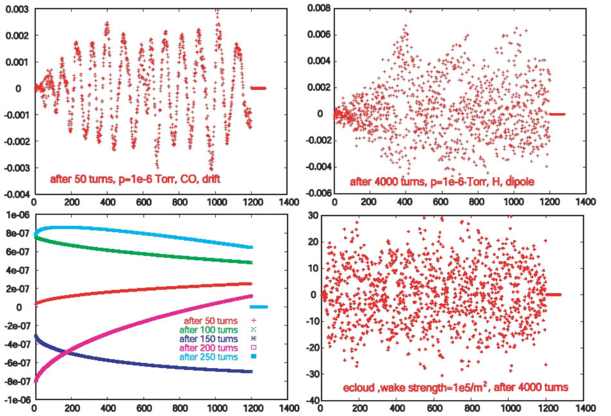

Figure 2: Simulated horizontal position along the bunch train for carbon monoxide without magnetic field after 50 turns (top left), for hydrogen in a dipole field after 4000 turns (top right), for the resistive wall instability after various numbers of turns (bottom left) and for the electron cloud after 4000 turns (bottom right).

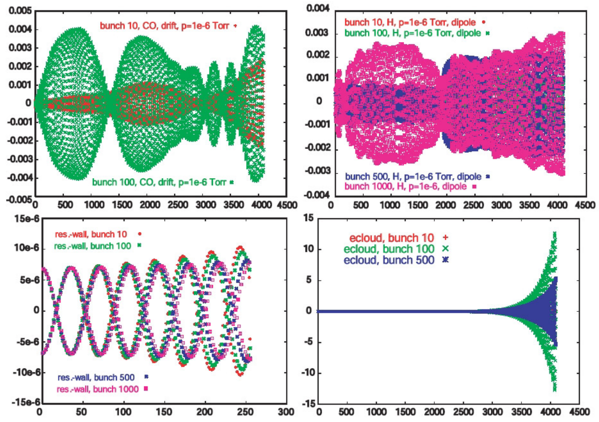

Figure 3: Simulated horizontal position of different bunches as a function of turn number, for the same four cases as in Fig. 2.

From Fig. 3 we can estimate the growth rates at the end of the train. To this end, for the ion instabilities, we scale with the square of the bunch position and extrapolate linearly from 1 $\mu$Torr to the actual pressure of 1 nTorr. We then find rise times of 1 ms for carbon monoxide ions without field, 5 ms for the resistive wall, 2 ms for the electron cloud, and 4 s for hydrogen ions inside a dipole. The growth rates for the resistive wall and electron cloud are consistent with analytical estimates. The horizontal growth rate for carbon-monoxide ions fits the observation. Additional simulations suggest that it is larger than the vertical growth rate, which would resolve another ‘puzzle’.

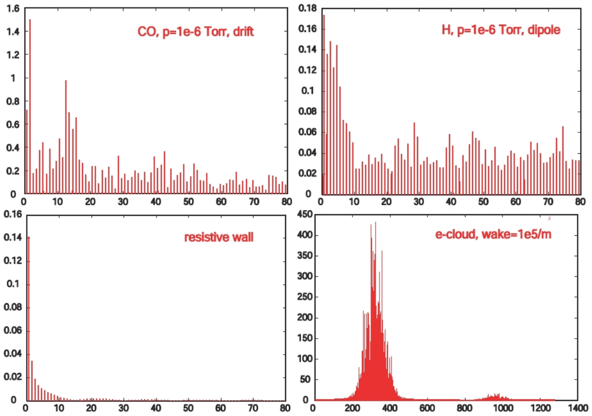

Figure 4: Amplitudes of lower betatron sidebands as a function of the revolution harmonic for the same four cases as in Fig. 2. The last picture (for the electron-cloud wake) shows the full spectrum; in the other three cases, where higher modes are not excited, only the harmonics 1–80 are displayed.

## 5 CONCLUSION

A fast computer simulation was written to model coupled-bunch instabilities driven by various sources. Application to the KEKB HER suggests that the observed horizontal instability is caused by carbon monoxide ions in field-free regions. For this case the simulated mode patterns and growth rates almost exactly match the observations. On the other hand, electron cloud, resistive wall, hydrogen ions, or ions trapped inside dipoles are all incompatible with the measurements.

## 6 REFERENCES

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