# EXPERIMENTAL MEASUREMENTS OF 2-DIMENSIONAL NONLINEAR RESONANCES 

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## Abstract

The measurements on both horizontal and vertical coherent betatron oscillations at nonlinear resonances have been performed with turn-by-turn method. Besides the systematic errors, the magnet misalignments and other imperfections could drive the nonlinear resonances. Experiments were conducted in the region close to the working tunes of the Taiwan Light Source electron storage ring to study their effects. The typical measurement results are reported. The phenomena of these 2-dimensional resonances are investigated.

## 1 INTRODUCTION

The Taiwan Light Source (TLS) electron storage ring of NSRRC is a third generation synchrotron radiation source. In order to provide a low emittance electron beam strong transverse focusing and defocusing quadrupoles are used. The sextupoles are used to correct the large chromatic aberrations generated by quadrupole magnets. The nonlinear effects produced by the chromaticity correcting sextupoles, sextupole fields in dipoles and other small higher order random error multipoles can perturb the beam orbit and give the phase space significantly different from that with linear lattice elements when the betatron tunes are near a resonance condition. A series of experiments have been conducted on the one-dimensional nonlinear resonances in the horizontal plane at TLS[1,2,3], also in other facilities $[4,5,6]$. Since the 2 -dimensional nonlinear resonances could have more effects on the growth in amplitudes of betatron oscillations, experimental measurements were performed recently to investigate their effects.
In a circular accelerator, the betatron oscillations $x(s)$ and $y(s)$ of particles around a closed orbit are given by Hill's equation:
$\frac{d^{2} x}{d s^{2}}+K_{x}(s) x=\frac{\Delta B_{y}}{B \rho} ; \quad \frac{d^{2} y}{d s^{2}}+K_{y}(s) y=-\frac{\Delta B_{x}}{B \rho}$
with $\quad \Delta B_{y}+i \Delta B_{x}=B_{0} \sum_{n=0}^{\infty}\left(b_{n}+i a_{n}\right)(x+i y)^{n}$,
where $b_{n}$ and $a_{n}$ are the normal and the skew multipole components, respectively. $K_{x}(s), K_{y}(s)$ are the quadrupole strength functions, which can be varied in order to adjust the horizontal and the vertical betatron tunes, $v_{\mathrm{x}}$ and $\mathrm{v}_{\mathrm{y}}$. $B \rho=p / e$ is the momentum rigidity and $s$ is the longitudinal particle coordinate. $\Delta B_{x}$ and $\Delta B_{y}$ are linear or nonlinear magnetic multipole field errors. Both $K_{x, y}$ and the anharmonic term, $\Delta B_{x, y} / B \rho$, are periodic functions of $s$ with period of the circumference of circular accelerator. Normally, the higher order anharmonic term, $\Delta B_{x, y} / B \rho$, arising from higher order multipoles is small. When the
betatron tune is near a resonance condition, $m v_{\mathrm{x}}+\mathrm{n} v_{\mathrm{y}}=$ $l$, where $m, n, l$ are integers, respectively, particles in the accelerator can encounter coherent kicks from these multipoles. Its effect will show up as beam diffusion, halo or beam loss. In the phase space map it shows deviation from a simple ellipse.

In this paper, we will present recent experimental measurement results of single resonance at 2-dimensional nonlinear resonance condition. Several problems encountered during the measurements are also discussed.

## 2 EXPERIMENTAL PROCEDURE

The data acquisition system for the turn-by-turn beam position measurement is similar to that used in the previous studies. At present, the electron beam can only be kicked in the horizontal direction by one of the injection kickers, which were also configured for the turn-by-turn experiments. The centroids of kicked beam were measured in both horizontal and vertical direction by two beam position monitors. Various kicking strengths have been applied to kick the electron beam. The signals corresponding to measured beam positions were passed through a hybrid junction and Bergoz's Log-Ratio beam position monitor electronics, Fig. 1, to the VME based transient digitizers, then to a workstation for online analysis. The turn-by-turn beam positions were also saved for further detailed analysis.


Fig. 1: The block diagram of the turn-by-turn beam position measurement system.

Due to the time consuming single bunch beam injection process and an unsolved hardware problem that gives poor signal/noise ratio in the vertical beam position measurement at low beam current, at present, the multibunch beam was used for the experiment. During the experiment the chromaticity was also adjusted to near zero for minimizing the tune spread in the beam bunch. Several 2-dimensional single resonance conditions that were close to the working tunes in regular operation have
been selected for the measurements. During the experiment the betatron tunes were adjusted to near resonance condition by adjusting the strength of one of the quadrupoles manually. It was found that the stop band and also the strength of higher order resonance seemed to be small. The setting of quadrupole power supply was adjusted each step very close to the minimum resolution of hardware.

## 3 RESULT AND DISCUSSION

## $3.13 v_{x}+2 v_{y}=30$

The TLS storage ring is a storage ring with superperiodicity of 6 . At the nonlinear resonance of $3 v_{x}+$ $2 v_{y}=30$, the order of azimuthal harmonic is just a multiple of its superperiodicity. Thus, this resonance deserves special attention, because it should be sensitive to the systematic errors in the lattice elements. In the resonance frame this nonlinear resonance has $J_{1}=J_{x} / 3$ and the constant of motion $\mathrm{J}_{2}=\mathrm{J}_{\mathrm{y}}-2 \mathrm{~J}_{\mathrm{x}} / 3$. It implies that this sum resonance is an unstable resonance and the motion of particles is unbound. Fig. 2 shows the measured beam position spectra of both horizontal and vertical plane. Since the motion of electrons is unbound when the resonance condition meets, one would suppose to see an ever increasing betatron oscillation amplitude and the kicked beam would be lost eventually. During the experiment, it was very difficult to knock all of the electrons out of the chamber through this nonlinear resonance after kicking the beam in the horizontal direction. Because the strong decoherence effect, it was also difficult to see the horizontal turn-by-turn beam positions reached a maximum and back to zero. During the experiment we adjusted the quadrupole strength to tune the horizontal and vertical tune to very close to this resonance condition, then, kicked the beam starting from smaller kicking strength and increased gradually. Due to decoherence the beam positions would decohere and reach to about zero some turns after the start. As the kicking strength increased, one saw the turn number of beam position approaching zero amplitude decreased. But when the sum resonance occurred, one would see the turn number of beam position approaching zero amplitude increased, and the amplitude of beam position at each turn increased also. It indicated that some of the beam was moving away from the center of orbit due to the resonance. Also, the tune could be measured to justify that the resonance occurred. In Fig. 2, one of the typical raw data is shown. The beam positions representing the sum resonance are clearly seen from the start to about $600^{\text {th }}$ turn. An FFT result of this first 600 turns of beam positions shows $v_{x}=0.2400$ and $v_{y}=0.1400$, which makes $3 v_{x}+2 v_{y}=30$. Also, the beam current changed from 72 mA to 17 mA after the kick. The remnant oscillations after $600^{\text {th }}$ turn in the horizontal beam positions in figure 2 should be from the electrons that were not knocked out of the orbit. The two peaks in the FFT spectrum of horizontal beam positions are from this
remnant oscillation. The vertical beam positions after $650^{\text {th }}$ turn shows only the noise when small signals corresponding to the beam positions were detected. Further study of this sum resonance is underway.


Fig. 2: The measured horizontal and vertical turn-by-turn beam positions at resonance of $3 v_{x}+2 v_{y}=30$. The corresponding tune spectrum is shown under its beam position spectrum.

## $3.2 v_{x}-2 v_{y}=-1$

The difference resonance $v_{x}-2 v_{y}=-1$ is intrinsically stable and does not lead to instable particle motion. At the resonance the energy exchanges between the horizontal and vertical plane. A set of typical measured turn-by-turn beam positions in both horizontal and vertical plane with a horizontal kick and their corresponding tune (FFT) spectra are shown in Fig. 3. In the figure the horizontal turn-by-turn beam positions decohere rapidly in about 700 turns due to a large tune spread within the multi-bunched electron beam. Even though, one sees a dip in the horizontal position spectrum at around $400^{\text {th }}$ turn, and a corresponding nonlinear coupling peak appears clearly at about the same turn numbers in the vertical position spectrum. For the larger turn numbers the horizontal beam positions strongly suppressed by the decoherence effect. But, the remnant oscillations still can be seen with small amplitude. In the vertical plane the beam positions show clearly the oscillations after the energy transformed from the horizontal plane due to the nonlinear coupling. The corresponding sidebands are also shown clearly in the vertical tune spectrum. From the sidebands of the tune spectrum one can estimate the island tune is about 0.0016 . If the FFT spectrum of horizontal beam positions from $750^{\text {th }}$ turn to $2250^{\text {th }}$ turn are obtained, two small sidebands appear at both sides of the main peak, $\mathrm{Qx}=0.2674$, and
with larger intensity than the main peak. The island tune obtained from these sidebands seems smaller than that from Fig. 3. This seems to result from a smaller detune effect.


Fig. 3: The measured horizontal and vertical turn-by-turn beam positions at resonance of $v_{x}-2 v_{y}=-1$. The corresponding tune spectrum is shown under its beam position spectrum.

## $3.34 v_{x}-v_{y}=25$

The horizontal beam position spectrum and its corresponding FFT spectrum taken at $4 v_{x}-v_{y}=25$ resonance line are shown in Fig. 4. It shows oscillations produced by this resonance. This resonance is a difference resonance. Thus, its invariant actions $\mathrm{J}_{\mathrm{x}}+4 \mathrm{~J}_{\mathrm{y}}=$ constant. Changes in $J_{x}$ could only result in a very small change in $\mathrm{J}_{\mathrm{y}}$. At present, our hardware did not have enough resolution to let us see the betatron oscillation in the vertical plane.


Fig. 4: The horizontal turn-by-turn beam positions and the tune spectrum at resonance of $4 v_{x}-v_{y}=25$.

## $3.52 v_{x}-3 v_{y}=2$

The horizontal beam position spectrum and its corresponding FFT spectrum taken at $2 v_{x}-3 v_{y}=2$ resonance are shown in Fig. 5. Its vertical beam positions are also not seen clearly due to the hardware resolution.

## 4 CONCLUSION

The effects of 2 dimensional single nonlinear resonances were measured recently at TLS. Besides using the present acquired data to study their mechanism, we
are also investigating better methods to improve our beam position measurement. At present the multi-bunch beam has been used, which had a larger beam size and could also have a larger intrinsic tune spread due to the transverse non-linearity. Thus, even the chromaticity was adjusted to about zero, the decoherence effect was still large, especially when the beam was kicked to a larger amplitude. This made the interpretation of the experimental data very difficult. In our later experiment, we are considering running the experiments with a single bunch beam or with a multi-bunch beam of fewer buckets. Also, a plan to install a vertical pinger is considered in order to excite the electron beam in the vertical direction. The data acquisition electronics is also under major improvement recently in order to achieve a much better resolution and accuracy in beam position measurement in both horizontal and vertical plane. After these improvements, we hope the 2 dimensional resonances can be studied much easier.


Fig. 5: The horizontal turn-by-turn beam positions and the FFT spectrum at resonance of $2 v_{x}-3 v_{y}=2$.

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