ON-LINE MONITORING OF THE LINEAR COUPLING FOR THE LHC

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Abstract

Before correction, the difference coupling coefficient in LHC is expected to reach $|c_-| \approx 0.2$ at injection and vary by 30 to 40% from injection to top energy. These high values arise mostly from the field imperfections in the superconducting dipoles, the feed-down from the parasitic sextupoles and the strongly-focused low-β insertions. A measurement method which lends itself to continuous monitoring and active feedback is therefore investigated. Our approach is based on the beam response to an excitation in the other plane (coupling BTF). The analytical approach [1] shows that a proper measurement protocol allows extracting the global and local complex coefficients of both the difference and sum coupling resonances. In order to prevent any emittance blow-up, the beam is excited at small amplitude outside its eigen-frequencies with smooth transitions (AC-dipole principle). A first experiment in the SPS [2] confirms the approach and its robustness.

INTRODUCTION

Betatron coupling causes an oscillation in one plane to be transferred to the other plane. This transfer depends not only on the excitation of the linear coupling resonances but as well on the local tilt of the eigen-modes and, to a lesser extent, on the perturbation of the β-functions. This paper presents first the beam response to a sinusoidal excitation in the presence of coupling. It is used to define measurement protocols yielding the coupled β-functions and the global and local coupling coefficients. The systematic errors in the methods are analysed. First experimental results using a non-nominal set-up are presented.

BEAM TRANSFER FUNCTION (BTF) IN TWO DIMENSIONS

At at given abscissa $s_0 \equiv 0$ of the lattice and at each turn n, the beam is assumed to be excited by a dipolar oscillating force of the form

$$\begin{pmatrix}\Delta x'(n)\\ \Delta y'(n)\end{pmatrix} \equiv \cos(2\pi Q_D\, n + \phi) \times \begin{pmatrix}x'_D\\ y'_D\end{pmatrix}\,, \qquad (1)$$

with x'_D and y'_D being the amplitudes of the AC kick, ϕ its phase with respect to the beam at turn $n = 0$ and Q_D a frequency given in tune units (i.e. normalized to the revolution frequency). We expect a response at the drive frequency, varying linearly with the excitation amplitude in the approximation of harmonic oscillators (i.e. linear optics approximation). Using complex notations and according to Ref. [1], the beam response can be written as follows:

$$\begin{pmatrix}x(n;s)\\ y(n;s)\end{pmatrix} = e^{2i\pi Q_D n + i\phi} \times \mathcal{A}(Q_D;s)\,\begin{pmatrix}x'_D\\ y'_D\end{pmatrix}\,, \qquad (2)$$

where

- s denotes the curvilinear abscissa around the ring taking the AC-dipole as the origin. $x(n;s)$ and $y(n;s)$ are the horizontal and vertical oscillations of the beam at turn n and abscissa s.
- $\mathcal{A}(Q_D;s)$ is a complex 2×2 matrix which, at the first order in the strength of the skew quadrupole errors of the lattice, is given by

$$\mathcal{A}(Q_D;s) \equiv \begin{pmatrix}\mathcal{A}_{xx}(Q_D;s) & \mathcal{A}_{xy}(Q_D;s)\\ \mathcal{A}_{yx}(Q_D;s) & \mathcal{A}_{yy}(Q_D;s)\end{pmatrix}\,, \qquad (3)$$

with

$$\begin{cases}\mathcal{A}_{xx}(Q_D;s) \stackrel{\text{def}}{=} -\dfrac{\sqrt{\beta_x(0)\beta_x(s)}}{4}\left[\mathcal{A}_x^+\, e^{i\mu_x(s)} - \mathcal{A}_x^-\, e^{-i\mu_x(s)}\right]\\ \mathcal{A}_{yy}(Q_D;s) \stackrel{\text{def}}{=} -\dfrac{\sqrt{\beta_y(0)\beta_y(s)}}{4}\left[\mathcal{A}_y^+\, e^{i\mu_y(s)} - \mathcal{A}_y^-\, e^{-i\mu_y(s)}\right]\\ \mathcal{A}_{x,y}^{\pm} \stackrel{\text{def}}{=} \dfrac{e^{i\pi(Q_D\mp Q_{x,y})}}{\sin\left[\pi(Q_D\mp Q_{x,y})\right]}\,,\end{cases} \qquad (4)$$

$$\begin{cases}\mathcal{A}_{xy}(Q_D;s) \stackrel{\text{def}}{=} \dfrac{\pi}{8}\,\sqrt{\beta_y(0)\,\beta_x(s)}\,\times\\ \Big\{\mathcal{A}_y^+\left[\bar{\mathcal{A}}_x^+ \mathrm{C}_-(Q_x{-}Q_D;s)e^{i\mu_x} - \bar{\mathcal{A}}_x^- \overline{C}_+(Q_x{+}Q_D;s)e^{-i\mu_x}\right] +\\ \quad \mathcal{A}_y^-\left[\bar{\mathcal{A}}_x^- \overline{C}_-(Q_x{+}Q_D;s)e^{-i\mu_x} - \bar{\mathcal{A}}_x^+ \mathrm{C}_+(Q_x{-}Q_D;s)e^{i\mu_x}\right]\Big\}\\ \mathcal{A}_{yx}(Q_D;s) \stackrel{\text{def}}{=} \dfrac{\pi}{8}\,\sqrt{\beta_x(0)\,\beta_y(s)}\,\times\\ \Big\{\mathcal{A}_x^+\left[\bar{\mathcal{A}}_y^+ \overline{C}_-(-Q_y{+}Q_D;s)e^{i\mu_y} - \bar{\mathcal{A}}_y^- \overline{C}_+(Q_y{+}Q_D;s)e^{-i\mu_y}\right] +\\ \quad \mathcal{A}_x^-\left[\bar{\mathcal{A}}_y^- \mathrm{C}_-(-Q_y{-}Q_D;s)e^{-i\mu_y} - \bar{\mathcal{A}}_y^+ \mathrm{C}_+(Q_y{-}Q_D;s)e^{i\mu_y}\right]\Big\}\,,\end{cases} \qquad (5)$$

$$\mathrm{C}_\pm(q;s) \stackrel{\text{def}}{=} c_\pm - 2ie^{-i\pi q}\sin(\pi q)\, c_{\pm_<}(s)\,. \qquad (6)$$

- $\beta_{x,y}(s)$ are the β-functions at abscissa s (beam observation), $\mu_{x,y} \equiv \mu_{x,y}(s)$ are the betatron phase advances from $s = 0$ (AC dipole location) to s, $Q_{x,y}$ denote the betatron tunes, the coefficients $c_{\pm_<}(s)$ are the $(1,\pm 1)$ linear resonance driving terms integrated from $s = 0$ to s,

$$c_{\pm_<}(s) = \frac{1}{2\pi}\int_0^s ds_1 \sqrt{\beta_x(s_1)\beta_y(s_1)}\, K_{\text{skew}}(s_1) e^{-i\left[\mu_x(s_1)\pm\mu_y(s_1)\right]},$$

and $c_\pm \equiv c_{\pm_<}(s{=}C)$ represents the usual sum and difference coupling coefficients.

The beam is now assumed to be excited in one of the two transverse planes, say in the vertical plane (that is $x'_D \equiv 0$ in Eq. (1)). According to Eq. (2), the Fourier transform of the beam signal at the excitation frequency Q_D is given by

$$\begin{cases}\hat{x}\,(Q_D;s) \equiv \dfrac{1}{N}\displaystyle\sum_{n=0}^{N-1} x(n;s)\, e^{-2i\pi Q_D n} \stackrel{N\to\infty}{\approx} \dfrac{y'_D}{2}\, e^{i\phi}\mathcal{A}_{xy}\,(Q_D;s)\\ \hat{y}\,(Q_D;s) \equiv \dfrac{1}{N}\displaystyle\sum_{n=0}^{N-1} y(n;s)\, e^{-2i\pi Q_D n} \stackrel{N\to\infty}{\approx} \dfrac{y'_D}{2}\, e^{i\phi}\mathcal{A}_{yy}\,(Q_D;s),\end{cases} \qquad (7)$$

N being the number of turns used for the BPM acquisition.

MEASUREMENT PROTOCOL FOR THE β-FUNCTIONS

The "direct" transfer function $\mathcal{A}_{yy}$ is most sensitive to the β-function. The term $\mathcal{A}_y^+$ is dominant when the excitation frequency Q_D lies in the vicinity of the vertical betatron tunes. Starting from Eq. (4), $\mathcal{A}_{yy}$ may be re-expressed by a dominant term and an "error" $\epsilon(s)$:

$$\mathcal{A}_{yy}(Q_D;s) = -\frac{\sqrt{\beta_y(0)\beta_y(s)}\,[1+\epsilon(s)]}{4\sin[\pi(Q_D-Q_y)]}\,e^{i\pi(Q_D-Q_y)+i\mu_y(s)} \quad (8)$$

$$\text{with} \quad \epsilon(s) \equiv -\frac{\sin[\pi(Q_D-Q_y)]}{\sin[\pi(Q_D+Q_y)]}e^{2i\pi\,Q_y-2i\mu_y(s)}\,. \quad (9)$$

Combining Eq.'s (7) and (8) and assuming $\epsilon=0$, we get

$$\beta_y(s) = |\hat{y}\,(Q_D;s)|^2\,/\mathbf{K}^2 \quad (10)$$

$$\text{with} \quad \mathbf{K} \stackrel{\text{def}}{=} \frac{y'_D}{8\,|\sin[\pi(Q_D-Q_y)]\,|}\sqrt{\beta_y(0)}\,. \quad (11)$$

We now assume that the β-beating measured at the BPM's has no systematic component, i.e.

$$\frac{1}{N_{BPM}}\sum_{j=1}^{N_{BPM}}\frac{\beta_y(s_j)}{\beta_y^{(0)}(s_j)} \approx 1\,,$$

with N_{BPM} being the number of available BPM's and $\beta_y^{(0)}(s_j)$ the nominal vertical β function at BPM number j. Combining the above condition with Eq. (10), we get

$$\frac{1}{N_{BPM}}\sum_{j=1}^{N_{BPM}}\frac{|\hat{y}\,(Q_D;s_j)|^2}{\beta_y^{(0)}(s_j)} = \mathbf{K}^2\,, \quad (12)$$

leading to an estimate of the β-function at BPM number i:

$$\beta_y(s_i) \approx \frac{N_{BPM}\;|\hat{y}\,(Q_D;s_i)|^2}{\sum_{j=1}^{N_{BPM}}|\hat{y}\,(Q_D;s_j)|^2\,/\beta_y^{(0)}(s_j)}\,. \quad (13)$$

Neglecting the contribution $\epsilon(s)$ induces an intrinsic measurement error of the order of

$$\frac{\Delta\beta_{x,y}}{\beta_{x,y}} \sim |\epsilon| \sim \frac{\sin[\pi(Q_D-Q_{x,y})]}{\sin[\pi(Q_D+Q_{x,y})]} \sim 4-8\%\,,$$

using the SPS experiment parameters $Q_{x,y}=0.18/0.15$ and $Q_D=0.19/0.13$. The function $\epsilon(s)$ (Eq. (9)) oscillates at twice the betatron frequencies and therefore cannot be disentangled from a real β-beating.

This systematic error can be suppressed by exciting the beam at two distinct frequencies [1], preferably on both sides of the betatron tune. The data processing requires the knowledge of the phase ϕ (Eq. (1)) of the AC dipole excitation with respect to the beam, not accessible at the time of the experiment.

MEASUREMENT PROTOCOL FOR THE LINEAR COUPLING

As shown in Eq. (5), the coupling transfer function $\mathcal{A}_{xy}$ can be split into two well-distinct components. Assuming the machine to operate close to the coupling resonance, i.e. $Q_x-Q_y \ll 1$, and the excitation frequency to lie in the vicinity of the vertical betatron tune, the main contributions is the one proportional to the amplitude $\mathcal{A}_y^+ \propto 1/\sin[\pi(Q_D-Q_y)]$:

$$\begin{aligned}\mathcal{A}_{xy}(Q_D;s) &\approx \frac{\pi}{8}\sqrt{\beta_y(0)\,\beta_x(s)}\,\mathcal{A}_y^+ \times \\ &\Big[\bar{\mathcal{A}}_x^+\mathrm{C}_-(Q_x-Q_D;s)\,e^{i\mu_x(s)} - \bar{\mathcal{A}}_x^-\overline{C}_+(Q_x+Q_D;s)\,e^{-i\mu_x(s)}\Big] \\ &= \frac{\pi}{8}\times\frac{e^{i\pi(Q_x-Q_y)}\sqrt{\beta_y(0)\,\beta_x(s)}\,e^{i\mu_x(s)}}{\sin[\pi(Q_D-Q_y)]\,\sin[\pi(Q_D-Q_x)]}\times \\ &\quad[\mathrm{C}_-(Q_x-Q_D;s)+\lambda(s)\overline{C}_+(Q_x+Q_D;s)]\,,\end{aligned} \quad (14)$$

$$\text{with} \quad \lambda(s) \equiv -\frac{\sin[\pi(Q_D-Q_x)]}{\sin[\pi(Q_D+Q_x)]}\,e^{-2i\pi Q_x-2i\mu_x(s)}\,. \quad (15)$$

Combining Eq.'s (7) and (14) and assuming $\lambda=0$, we get

$$\hat{x}\,(Q_D;s) \approx \frac{\pi\mathbf{K}\,\sqrt{\beta_x(s)}e^{i\phi+i\pi(Q_x-Q_y)+i\mu_x(s)}}{2\,\sin[\pi(Q_D-Q_x)]}\,\mathrm{C}_-(Q_x-Q_D;s), \quad (16)$$

$$\text{or} \quad |\mathrm{C}_-(Q_x-Q_D;s)| = \frac{2\,|\sin[\pi(Q_D-Q_x)]|}{\pi\,\mathbf{K}\,\sqrt{\beta_x(s)}}\,|\hat{x}\,(Q_D;s)|\,, \quad (17)$$

with $\mathbf{K}$ the calibration factor defined in Eq. (11) and estimated via the relation (12), and $\beta_x(s)$ as measured following the previous section .

Under the approximation $\lambda(s)=0$, note that both the phase and the modulus of the coefficient $\mathrm{C}_-(Q_x-Q_D;s)$ could be determined thank to the relation (16). Nevertheless, this would again require that the phase-shift ϕ of the excitation w.r.t. to the beam is well-known at turn $n=0$.

If the coupling errors are random around the machine or localized but spaced by $\mu_x \sim \mu_y \sim \pi$ in both transverse planes (which was the case for the SPS when the skew quadrupoles were switched on to generate coupling), the sum and difference resonance driving terms are of the same order of magnitude and therefore the sum coupling resonance induces a measurement error of the order of

$$\frac{\Delta\,|\mathrm{C}_-|}{|\mathrm{C}_-|} < \frac{|\Delta\mathrm{C}_-|}{|\mathrm{C}_-|} \sim \frac{|\Delta\mathrm{C}_-|}{|\mathrm{C}_+|} = |\lambda| = \left|\frac{\sin[\pi(Q_D-Q_x)]}{\sin[\pi(Q_D+Q_x)]}\right|\,.$$

With the betatron tunes of the SPS $Q_{x,y}=0.18/0.15$ and a beam excitation in the vertical (resp. horizontal) plane at a frequency of $Q_{D_y}=0.13$ (resp. $Q_{D_x}=0.19$), we get

$$\frac{\Delta\,|\mathrm{C}_-|}{|\mathrm{C}_-|} \lesssim 19\%-14\%\,, \text{which is significant.} \quad (18)$$

As explained in Ref. [1], to get rid of this measurement error, the coupling transfer function $\mathcal{A}_{xy}(Q_D;s)$ must be measured at two consecutive BPM's, say BPM j and $j+1$, under the condition that the sources of coupling between these two BPM's remains negligible (e.g. two BPM's spaced by a simple drift), that is

$$\mathrm{C}_\pm(q;s_j) \stackrel{\text{def}}{=} c_\pm - 2ie^{-i\pi q}\sin(\pi q)\,c_{\pm<}(s_j) \approx \mathrm{C}_\pm(q;s_{j+1})\,,$$

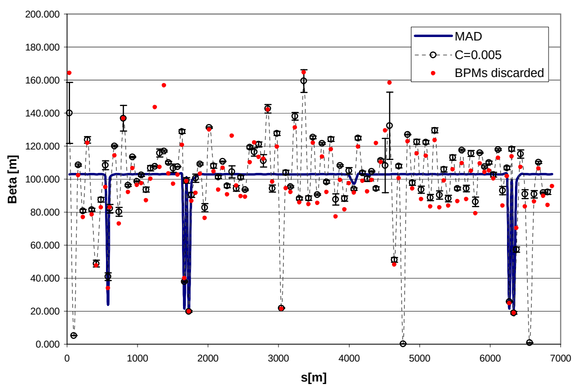
leading to

$$\int_{s_j}^{s_{j+1}} ds'\,\sqrt{\beta_x(s')\,\beta_y(s')}\,K_{\text{skew}}(s')\,e^{-i[\mu_x(s')\pm\mu_y(s')]} \ll c_\pm\,.$$

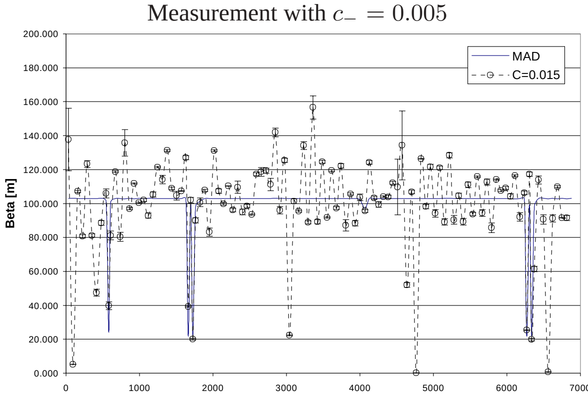
The data processing however requires the knowledge of the betatron phase advance between the AC dipole and the two BPM's, which turns out to be accessible if and only if the phase ϕ (Eq. (1)) of the AC excitation is known. As said previously this quantity was not available at the time of experiment and the perturbation $\lambda(s)$ was neglected in the off-line analysis.

MEASUREMENT RESULTS ON THE CERN SPS

Measurements were carried out at three excitation levels of the SPS skew quadrupole chain corresponding to $|c_-| = 0.005$, 0.01 and 0.015. For each excitation level, the closest tune approach was measured and the beam responses to an excitation in the horizontal plane at $Q_{D_x} = Q_x + 0.01$ and subsequently in the vertical plane at $Q_{D_y} = Q_y - 0.02$ were recorded over 1000 turns. The oscillation amplitude was of the order of one σ. Due to the impossibility of measuring the phase ϕ and the absence of two consecutive BPM's separated by a drift, the cancellation of the systematic errors $\epsilon(s)$ and $\lambda(s)$ was not carried out.
The β-functions are calculated per Eq. (13) and shown in Fig. 1 together with the MAD model. The error bars are estimated from the reproducibility of the measurements (three different sets of multi-turn data taken in each case). The measured β-beating is of the order of $\pm 20 - 30\%$ peak to peak. It should be real since significantly higher than the "β-beating like" systematic error $\epsilon(s)$ introduced in Eq. (9)) and estimated at 5-10% . As expected, the β-functions is weakly dependent on coupling at this level. A few BPM's are either wrong or badly calibrated.



Measurement with $c_- = 0.005$



Measurment with $c_- = 0.015$

Figure 1: *β-functions measured in the SPS for two different values of the difference coupling coefficient c_-.*

The local coupling parameters $C_-(Q_{x,y} - Q_{D_{y,x}}; s)$ measured from Eq. (17) are shown on figure Fig. 2. Given the cell phase advance of $\pi/2$, the observed beating from

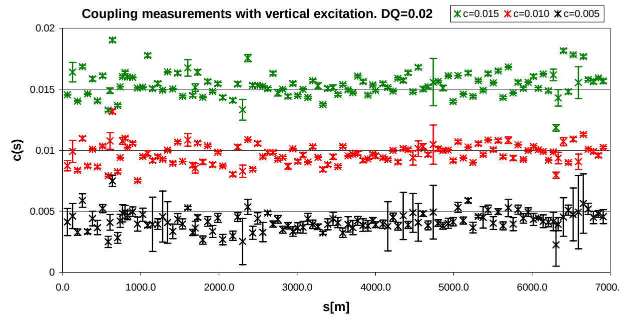


Figure 2: *Local coupling coefficient $|C_-(Q_D - Q_x)|$ versus machine azimuth for three excitation levels of the skew quadrupoles measured by the closest tune approach.*

BPM to BPM is consistent with the expected systematic error $\lambda(s)$ given in Eq. (15). In principle, this systematic error must cancel when averaged over all BPM's of the ring. In addition, the average of the local coupling parameters $C_-(Q_{x,y} - Q_{D_{y,x}}; s)$ is close to the global coupling coefficient with an estimated bias of less than 15% (Eq. (6) which vanishes for vanishing coupling). The random error on the average is found much below the 10^{-3} level, showing an excellent agreement with the closest tune approach.
The local coupling parameter should in principle exhibit a jump at each coupling source. In the SPS, the six different skew quadrupoles are all in phase. Following Eq. (6), the expected jump amplitude does not exceed $2\pi q\ |c_-|/6 \lesssim 0.05\ |c_-|$ for $q \equiv Q_{x,y} - Q_{D_{x,y}} = 0.04 - 0.05$, that is a factor 3 to 4 below the local measurement error $\lambda(s)$ estimated in Eq. (18). Finally, contrary to the β-functions, it can be proven and it is indeed observed by comparing Fig.'s 1 and 2, that badly calibrated BPM's do not affect the measurement of the local coupling coefficients $C_-(q; s)$.

CONCLUSION

The on-line coupling measurement reached a relative accuracy of 15% due to expected systematic errors and of a few 10^{-4} due to other errors. The latter is our expectation when two BPM's separated by a drift and the synchronization between excitation and observation are available. The measurements, each over 1000 turns, caused no measurable emittance blow-up. The method appears thus well suited for a continuous monitoring and feedback on the difference coupling resonance. Given the large natural coupling in LHC and its anticipated fast variations, this possibility may become essential to prevent a confusion of the beam diagnostics, e.g. of the tune feedback in case the tunes would cross. A second application is the monitoring in real-time of local coupling, notorious to be relevant at the feedback systems, the collision points and/or the cleaning insertions.

REFERENCES

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