MEASUREMENT OF SEXTUPOLAR RESONANCE DRIVING TERMS IN RHIC

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Abstract

Theory predicts that resonance driving terms can be determined by harmonic analysis of BPM data recorded after applying single kicks. In recent experiments at the CERN SPS this technique has been successfully applied to measure coupling and sextupolar resonance terms around the ring. A similar experiment has been carried out in RHIC, BNL, to prove the feasibility of this measurement in this more complex machine. Promising results of the experiment are presented, including a direct measurement of sextupolar resonances and a comparison to the model.

INTRODUCTION

The turn–by–turn single particle motion in normalised coordinates to first order in the non–linearities is given by [1]

$$\begin{aligned}\hat{x}(N) - i\hat{p}_x(N) &= \sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{x_0})} \\ &\quad -2i\sum_{jklm} j f_{jklm}(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}} \\ &\quad \times e^{i[(1-j+k)(2\pi\nu_x N+\psi_{x_0})+(m-l)(2\pi\nu_y N+\psi_{y_0})]}\end{aligned} \tag{1}$$

where I_x and I_y are the horizontal and vertical actions, ψ_{x_0} and ψ_{y_0} are the horizontal and vertical initial phases, ν_x and ν_y are the horizontal and vertical tunes including the amplitude dependent detuning and the factors f_{jklm} are the generating function terms. These are related to the Hamiltonian terms h_{jklm} by the following expression,

$$f_{jklm} = \frac{h_{jklm}}{1 - e^{-i2\pi[(j-k)\nu_x+(l-m)\nu_y]}} \ . \tag{2}$$

Note that the term f_{jklm} drives the resonance $(j-k, l-m)$. The Hamiltonian terms are defined by the following expansion of the non–linear Hamiltonian,

$$\begin{aligned}H &= \sum_{jklm} h_{jklm}(2I_x)^{\frac{j+k}{2}}(2I_y)^{\frac{l+m}{2}} \\ &\quad \times e^{-i[(j-k)(\psi_x+\psi_{x0})+(l-m)(\psi_y+\psi_{y0})]} \ ,\end{aligned} \tag{3}$$

where ψ_x and ψ_y are the horizontal and vertical angle variables. Eqs. (1) and (2) suggest that a FFT of the turn–by–turn complex signal can be used to measure the generating function and the Hamiltonian terms. The spectral line $(1-j+k, m-l)$ depends only on the term f_{jklm}. By line (m,n) we mean the spectral line with frequency $m\nu_x + n\nu_y$. To illustrate these relations the third order resonance is studied. The Hamiltonian and the generating function terms driving the third order resonance are h_{3000} and f_{3000} respectively. The spectral line produced in the horizontal motion by this resonance is therefore (-2,0). In a real machine the complex signal is constructed from two pick-ups with 90° phase advance. If the phase advance between the two pick-ups is not exactly 90° a linear transformation can be applied to the data from both pick-ups to obtain the corresponding set of data with 90° of phase advance. Only one pick-up cannot be used to measure resonance driving terms unambiguously since the spectral lines (m,n) and (-m,-n) cannot be disentangled.

The Hamiltonian and the generating function terms depend on the longitudinal location where they are calculated. To understand how they vary along the ring the values of a Hamiltonian term at both sides of a source of non–linearity are compared. Prior to this element the term is h^1_{jklm} and after it is h^2_{jklm}. The non–linearity contributes to the first case with the quantity k_{jklm} and to the second case with the quantity $e^{-i2\pi[(j-k)\nu_x+(l-m)\nu_y]}k_{jklm}$ because the element has moved to the end of the lattice. Therefore the relation between the two Hamiltonian terms is expressed as

$$h^2_{jklm} = h^1_{jklm} + (e^{-i2\pi[(j-k)\nu_x+(l-m)\nu_y]} - 1)k_{jklm} \ , \tag{4}$$

the equivalent relation between the generating function terms is given by

$$f^2_{jklm} = f^1_{jklm} - k_{jklm} \ . \tag{5}$$

These relations state that the amplitude of these terms changes abruptly at the location of the sources. Their amplitudes remain constant along sections free of sources. This feature is very important since it allows the localisation of multipolar kicks. Applications of this feature in the CERN SPS can be found in [2] and [3].

In a real machine the beam is not a single particle but a particle distribution and processes like the beam decoherence change the Fourier spectrum of the turn–by–turn motion. The effect of the decoherence due to amplitude detuning has been described in [2]. The relevant conclusion is that the spectral line (m,0) of a decohered signal is reduced by a factor of |m| compared to the single particle case. In particular, the two spectral lines (-2,0) and (2,0), produced by sextupolar fields, from the decohered motion are decreased by a factor of 2 compared to the single particle case. In order to compare the results from the experiment to single particle simulations the corresponding factor is applied to the experimental results.

DESCRIPTION OF THE RHIC MODEL

The tracking program SixTrack [4] is used for the analysis. The RHIC tracking model is constructed from an ideal lattice, characterized by a β-function of 10 m at all interaction points. This configuration was used in the experiment. Magnetic field errors are introduced in the arc dipoles and

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quadrupoles. At injection, these field errors dominate the dynamic aperture.

For the dipoles, cold measurements at 660 A in 58 magnets are used to determine the average and rms values for the geometric field errors [5, 6]. The dipole current at injection is 470 A. Normal and skew components are measured up to 22-poles. The measured mean and rms values are used to assign random errors in the lattice. Mean and rms values for the sextupole component, dominated by persistent currents, are determined in a separate measurement, and also used to assign random errors in the lattice.

For the quadrupoles, only geometric errors are considered. Mean and rms values are measured in 380 magnets at 30 A and room temperature. A good correlation is found between warm and cold measurements in these magnets [5, 6]. No additional persistent current error contributions are included in the quadrupoles.

Tunes and chromaticities are set to the values observed in the experiment. Closed orbit errors are disregarded. In the experiment one interaction region sextupole corrector was changed to excite the third order resonance.

DESCRIPTION OF THE EXPERIMENT

The measurement of resonance driving terms was carried out in gold operation at injection. Relevant parameters are displayed in Tab. 1. In all cases the transverse injection oscillations of a single bunch were observed turn-by-turn. The horizontal oscillation amplitude was varied by changing the strength of the injection septum. The oscillation amplitudes were increased in steps until most of the beam was lost in the injection process. 12 beam position monitors (BPMs) in either plane recorded 1024 turns after the injection.

Five sets of data were taken. In each set the horizontal oscillation amplitude was varied. The first set was taken with the unchanged machine. For the other sets, the single interaction region sextupole was powered to 0.09, 0.03, -0.03 and -0.09 m^{-2} respectively. This sextupole should drive first and third order resonances.

The sextupole that was changed in the experiment is at a location with lattice functions $(\beta_x, \beta_y) = (143\ \mathrm{m}, 50\ \mathrm{m})$. For comparison, the 72 focusing arc sextupoles are at locations $(\beta_x, \beta_y) = (45\ \mathrm{m}, 11\ \mathrm{m})$ with a strength of $+0.18\ \mathrm{m}^{-2}$; 72 defocusing arc sextupole are at locations $(\beta_x, \beta_y) = (11\ \mathrm{m}, 44\ \mathrm{m})$ with a strength of $-0.39\ \mathrm{m}^{-2}$. The arc sextupoles correct for the natural chromaticities of $(\xi_{x,n}, \xi_{y,n}) = (-55, -57)$ and the persistent current chromaticities $(\xi_{x,b_2}, \xi_{y,b_2}) = (-38, +36)$ [7].

EXPERIMENT VERSUS MODEL

Sextupolar fields introduce three spectral lines in the Fourier spectrum of the horizontal motion: (-2,0), (2,0) and (0,0). The first one is related to the third order resonance and the other two are related to the first order resonance. An example of the Fourier spectrum obtained from the experimental data and using the SUSSIX [8] code is shown in Fig. 1. In this figure the tune line and the three sextupolar spectral lines are seen plus the spectral line (-1,0) which is due to quadrupolar and octupolar resonances.

Table 1: Machine and beam parameters for the experiment.

parameter	unit	value
ring	...	Yellow
specie	...	Au^{79+}
relativistic parameter γ	...	10.52
ions per bunch N_b	10^9	$0.1 - 0.7$
norm. emittance, 95% $\epsilon_{x,y}$	μm	≈ 10
tunes (ν_x, ν_y)	...	(28.223, 29.235)
chromaticities (ξ_x, ξ_y)	...	$\approx (-2, -2)$
sext. strength K_2L(MAD)	m^{-2}	from -0.09 to 0.09

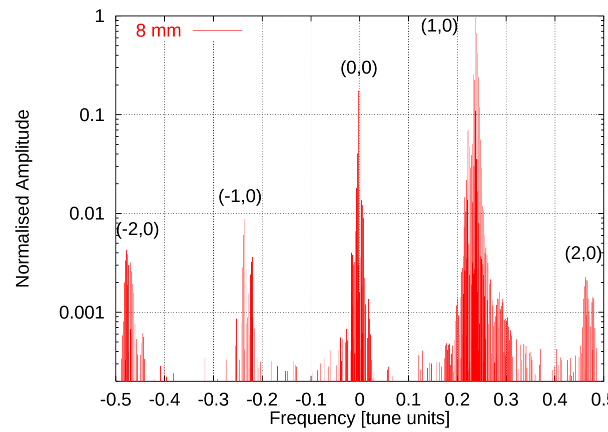

Figure 1: Fourier spectrum of the complex signal from pick-ups $yo5$-$bh10$ and $yo5$-$bh12$ for the unchanged RHIC and for an oscillation amplitude of 8 mm. The label (m,n) means that the frequency is equal to mν_x+nν_y.

To measure the spectral line (0,0) the closed orbit previous to the excitation of the betatron motion is needed. Since in this experiment the betatron motion was excited by injecting with a certain transverse angle, the reference closed orbit is not known. Therefore the line (0,0) cannot be measured. The spectral lines (-2,0) and (2,0), normalised to the tune line, are proportional to the oscillation amplitude and to the resonance terms h_{3000} and h_{1200} respectively, as derived from Eq. (1). For the different sets of data the normalised amplitude of either spectral line is plotted versus the normalized betatron amplitude $a_x/\sqrt{\beta_x}$. The betatron amplitudes have been measured from the first turns of the BPM data. A beta function of 48 m has been assumed at the BPMs to compute the single particle emittance, which is twice the action I_x. Only the data coming from two of the pick-ups could be systematically used for all the different settings. In Fig. 2 the normalised amplitude of the spectral line (-2,0) coming from these BPMs is plotted versus the oscillation amplitude for the five different settings of the interaction region sextupole. The experimental values have been multiplied by the decoherence factor of two to compare to the curves predicted by the model. There is a good agreement except for the case with $-0.09\ \mathrm{m}^{-2}$. To measure the first order resonance the previous procedure

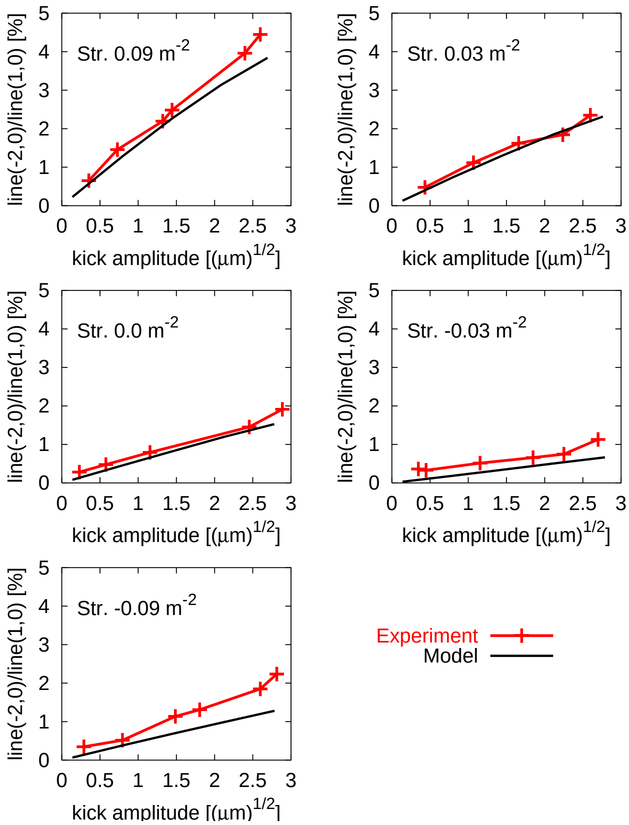

Figure 2: Measurement of the third order resonance. The normalised amplitude of the spectral line (-2,0) is plotted versus horizontal betatron amplitude for five different strengths of the sextupole. Experimental results are multiplied by the decoherence factor of 2 and predictions from the model are shown.

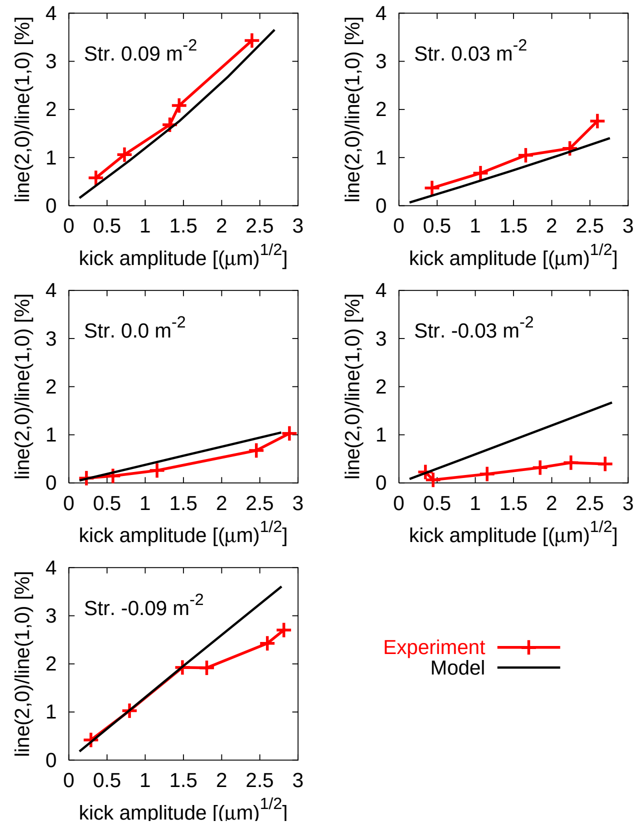

Figure 3: Measurement of the first order resonance. The normalised amplitude of the spectral line (2,0) is plotted versus horizontal betatron amplitude for five different strengths of the sextupole. Experimental results are multiplied by the decoherence factor of 2 and predictions from the model are shown.

is followed for the spectral line (2,0). The resulting plots are shown in Fig. 3. There is a good agreement for the cases with positive and zero strength. The agreement not as good for for the cases with negative strength. In one case (see Fig. 3) disagreement between experiment and model is only at large amplitudes.

CONCLUSIONS

For the first time we were able to demonstrate that sextupole driving terms can be measured in RHIC, an operational superconducting machine. We measured two types of horizontal sextupole resonances in a RHIC experiment and successfully compared them with model calculations of that machine. This opens the possibility of monitoring and correcting the nonlinear content of an operational accelerator. In conjunction with an AC dipole, currently being installed in RHIC, one can use this technique continuously from injection, through the ramp, and on flat top. By using BPMs around the whole machine, one can determine driving terms as function of the longitudinal positions and identify locations of strong non-linear errors or incorrectly powered magnets.

ACKNOWLEDGMENTS

The authors are thankful to A. Jain, S. Tepikian, and J. van Zeijts for support in the preparation of the RHIC model. F. Pilat assisted in the measurements. We are grateful to T. Satogata for his support in the data analysis.

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