CALCULATION OF THE MAXIMUM STORED BEAM CURRENT CONSIDERING THE PHASE NOISE OF A GENERATOR RF SIGNAL

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Abstract

Synchrotron motion was numerically simulated to examine the maximum stored beam current, which is limited by beam loading. In addition to the machine operating parameters, the simulation considers the influence of the phase noise of a generator voltage on the maximum stored beam current. The computed result shows that increasing the phase noise level can reduce the maximum stored beam current and the reducing of the maximum stored current, due to an increase in the noise level, escalates with increasing the cavity voltage.

INTRODUCTION

To meet the requirement of users, we will replace two existed conventional cavities with one CESR-III 500 MHz SRF module to increase the stored electron beam from 200 mA to 500 mA and to improve the beam stability. When machine is upgraded, the beam loading ratio will increase from the current value 1.3 to 6.9 [1]. To insure the beam current can reach the design goal, it is necessary for us to examine the influence of the phase noises on the beam current when the machine is operated under such heavy beam loading conditions.

The current theoretical computation of the maximum stored beam current is based on the study of Robison in 1964 [2]. The study indicated that the beam current reaches its maximum limit when the generator rf voltage is pushed to the opposite phase of the beam current by the detuned cavity. The cavity is detuned to compensate for the change, caused by the beam loading, in the cavity matching impedance.

In the theoretical computation, the beam-induced voltage and the generator voltage are treated as a purely sinusoid signals, without noise. In reality, the generator voltage is always accompanied with different kinds of noise, such as harmonic, amplitude and phase noises.

In this paper, the phase noise of a generator voltage is included in the numerical simulation. From the simulation, we can investigate the synchrotron motion of each beam bunch turn by turn. Whether the beam current is above or below the limitation can then be determined from the behavior of each beam bunch in the simulated motion. The maximum beam currents computed for different phase noise levels are demonstrated by an example, which is also used to compare the theoretical and simulated results.

FORMALISM FOR CALCULATION

Difference Equations for Synchrotron Motion

In the study we assume that the electrons in a bunch bucket act as a single rigid macro-particle. The energy and time deviations of a beam bunch, ε and τ, during the synchrotron motion can then be described by the following equations:

$$\frac{d\varepsilon}{dt} = \frac{e}{T_0} V_{acc}(t_s + \tau) - eU_{rad}(\varepsilon) \tag{1}$$

$$\frac{d\tau}{dt} = \alpha \frac{\varepsilon}{E_s} \tag{2}$$

where T_0 is the revolution period of the synchronous particle, e is the electron charge in a bunch, V_{acc} is the accelerating voltage, α is the momentum compaction factor, E is the electron energy, the subscript s presents the quantity for the synchronous bunch. U_{rad} is the radiation loss. The radiation loss in a single turn is given by

$$\begin{aligned} U_{rad}\,[KeV] &= 88.46 \frac{E\,[GeV]^4}{\rho}\,[m] && (3) \\ &= 26.520308 E\,[GeV]^3\,B\,[T] && (4) \end{aligned}$$

where ρ is the radius of curvature of the orbit in the bending magnet and B is the bending magnetic field. Rewriting (1)and (2) for a single turn period and imposing the simplectic condition [3], we can obtain the particle tracking equations for the synchrotron motion:

$$\varepsilon_{j+1} = \varepsilon_j + eV_{acc}(t_s + \tau_j) - eU_{rad}(\varepsilon_j) \tag{5}$$

$$\tau_{j+1} = \tau_j + \alpha T_0 \frac{\varepsilon_{j+1}}{E_s} \tag{6}$$

where the subscript j represents the number of revolutions of a beam bunch. In synchrotron motion, the radiation loss for a synchronous beam bunch is exactly compensated by the accelerating voltage:

$$V_{acc}(t_s) = |\vec{V}_c| \cos(\phi_s) = U_{rad}(\varepsilon = 0) \tag{7}$$

where ϕ_s is the synchrotron phase and $\vec{V}_c$ is the cavity voltage.

Tuning Angle for Beam Loading Compensation

The tuning angle is a measure of the phase difference between the excited rf signal and its generator current for a cavity. From microwave circuit theory, it can be expressed

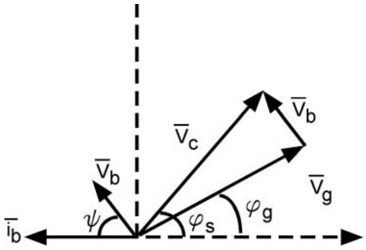


Figure 1: Phasor diagram for beam loading compensation when the cavity is detuned to ψ.

by the cavity loaded quality factor Q_L, the rf signal frequency ω_{rf} and the cavity resonance frequency ω_c as follows:

$$\tan(\psi) = -2Q_L \frac{\omega_{rf} - \omega_c}{\omega_c} \tag{8}$$

When there is beam current in a storage ring, the change in cavity matching impedance, which is changed by beam loading, can be compensated for, to minimize the reflected rf power by tuning the cavity tuning angle ψ to [4]:

$$\begin{aligned} \tan(\psi) &= -\frac{2I_a R_s}{V_c(1+\beta)} \sin\phi_s + \\ &\quad \tan\theta\left[\frac{2I_a R_s}{V_c(1+\beta)} \cos\phi_s + 1\right] \end{aligned} \tag{9}$$

where I_a is the average beam current, R_s is the shunt impedance of the cavity, β is the coupling factor of the cavity and θ is the tuning angle offset when the beam current is zero. The offset is usually set to a negative value to reduce the Robinson instability. Notably in following discussion, the tuning angle is supposed to be tuned to minimize the reflected rf power.

Beam-Induced Voltage and Generator Voltage

The simulation assumes that the initial beam-induced voltage is induced by the beam bunch which is synchronous with the oscillating rf field. The bunch, compared to the rf wavelength, is considered to be zero-dimensional. Based on both assumptions, the initial beam-induced voltage can be expressed as [5],

$$\overrightarrow{V_{b0}} = -\frac{2I_a R_s}{1+\beta} \cos(\psi) \exp(\imath\psi) \tag{10}$$

From the tuning angle expressed in (9), the initial beam-induced voltage in (10) and the phasor diagram shown in Fig.1, the initial generator voltage can be obtained:

$$\overrightarrow{V_{g0}} = \vec{V_c} - \overrightarrow{V_{b0}} \tag{11}$$

As we start to simulate the synchrotron motion of each beam bunch with (5) and (6) from a small initial phase deviation, the initial beam-induced voltage, expressed in (10), begins to decay with time and to be superimposed by the induced voltage of the bunches passed and passing through the cavity. The beam-induced voltage seen by a bunch passing through the cavity can be obtained by adding up the voltages induced by the passed and the passing bunches [6]:

$$\begin{aligned} \vec{V_b}(t_m) = -\frac{\omega_c R_s}{Q_0}\Big\{ &\sum_{n=-\infty}^{n<m} q_n \exp[\imath\,\omega_c(t_m - t_n)] \\ &\cdot \exp\Big(-\frac{t_m - t_n}{T_d}\Big)\Big\} - \frac{\omega_c R_s}{2Q_0} q_m \end{aligned} \tag{12}$$

where Q_0 is the unloaded quality factor of the cavity, T_d is the field decay time constant, and is equal to

$$T_d = 2\frac{Q_L}{\omega_c} \tag{13}$$

Notably according to the fundamental theorem of beam loading [7], only half of the induced voltage acts back on the beam bunch itself, so the last term of (12) contains a factor 1/2.

Here we express the generator voltage as a regular sinusoid signal which has noise sidebands:

$$\begin{aligned} \vec{V_g}(t_m) = |\overrightarrow{V_{g0}}|[1 + \delta_A \cos(\omega_a t_m)] \cdot \exp\{ \\ \imath\,[(\omega_{rf} t_m) + \delta_p \cos(\omega_P t_m) + \phi_g]\} \end{aligned} \tag{14}$$

where ω_a and ω_p are the amplitude and phase modulation frequency respectively, δ_A and δ_p are the magnitudes of the amplitude and phase modulations respectively.

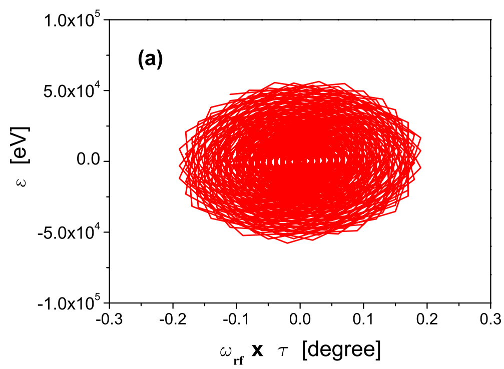
The time period between two near bunches is too short for the feedback circuit to respond for correcting the cavity voltage, so (14) assumes that the variation of the beam-induced voltage during the synchrotron motion can not affect the generator voltage via the feedback circuit. Therefore, the generator voltage in (14) is independent of the beam-induced voltage.

LIMITATION OF STORED BEAM CURRENT

From (5) and (6), we can obtain the synchrotron motion of each beam bunch turn-by-turn. The effect of the beam loading and the phase noise on beam motion acts through the beam-induced voltage in (12) and the generator voltage in (14).

If the beam current is below the maximum limitation, the phase and energy deviations of the beam bunch will be bounded within a limited area as in the simulation of motion, as shown in Fig. (2.a). In contrast, if the beam current is above the limitation, as in Fig. (2.b), both deviations move away from the synchronous point until the beam bunch is lost.

From the calculation shown in Fig. (3), we can find that for a constant cavity voltage, the maximum stored beam



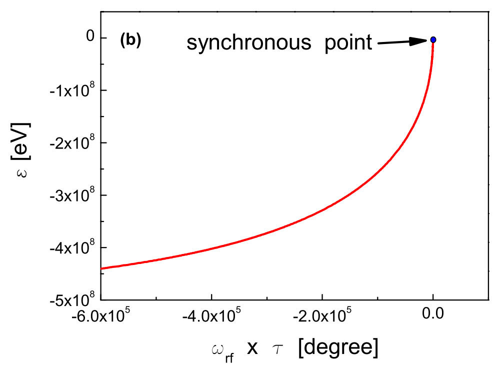


Figure 2: Simulated beam motion described by phase and energy deviations as beam current is (a) under or (b) above the maximum limite

current can be reduced by an increase in the phase noise level of a generator rf signal. The reduction is obvious, particularly when the cavity voltage is high. The theoretical maximum stored beam current,

$$I_{max} = \frac{\sin \Phi_s}{\sin 2(\Phi_s - \theta)} \cdot \frac{V_c}{R_s}(1+\beta) \tag{15}$$

is compared with the numerically simulated value, shown in figure (3), in the case of no noise. The comparison indicates that the maximum limitation predicted from the theoretical computation is always higher than that from the numerically simulated value. A possible reason for the discrepancy is the phases of the passing beam bunch and its induced voltage. The last term in (12) has only a real part, which means that the passing beam bunch and its induced voltage are always in phase regardless of the time deviation of the beam bunch in the motion simulation. The beam bunch will transfer more energy to its induced voltage if we consider this in-phase phenomenon in the energy transfer process. More energy transferred from beam bunches means greater beam loading. This in-phase relationship is not considered in the theoretical computation.

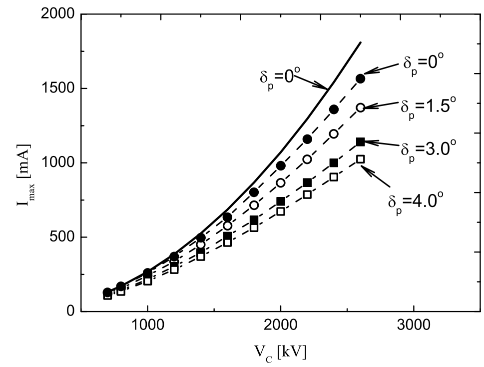


Figure 3: simulated maximum stored beam current versus gap voltage for the generator rf signal with different phase noise levels (symbol-dashed curves), and that predicted from (15) in the case of no noise (solid curve). The parameters used are $E_s = 1.5GeV$, $\omega_{rf} = 500MHz$, $Q_0 = 1.0 \times 10^9$, $R_s/Q_0 = 44.5$, $\beta = 4000$, $T_0 = 200/\omega_{rf}$, $\alpha = 6.768 \times 10^{-3}$, $\omega_p = 720Hz$, $U_{rad}(\varepsilon = 0) = 168keV$, $P_{loss} \equiv V^2/2R_s$.

CONCLUSIONS

This paper has presented a way to calculate the maximum stored beam current, including the influence of the phase noise of the generator rf signal, and has demonstrated that the maximum stored beam current can be reduced by an increase in the phase noise level. The calculation in Fig. (3) suggests that our rf generator must mantain its phase noise of 720 Hz below 3 degrees, if we plan to store the beam current up to 500 mA at the cavity voltage of 1600 kV.

That (15) does not consider the in-phase of the passing beam bunch and its induced voltage may cause the predicted maximum beam current higher than that predicted by the numerical simulation.

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