DISPERSION ANALYSIS OF THE PULSELINE ACCELERATOR*

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Abstract

We analyze the sheath helix model of the pulseline accelerator [1]. We find the dispersion relation for a shielded helix with a dielectric material between the shield and the helix and compare it against the results from 3-D electromagnetic simulations. Expressions for the fields near the beam axis are obtained. A scheme to taper the properties of the helix to maintain synchronism with the accelerated ions is described. An approximate circuit model of the system that includes beam loading is derived.

INTRODUCTION

It has been recently recognized by Briggs that the well-known slow wave structure consisting of a helix wound over an evacuated beam tube, an outer dielectric layer and an outer conductor can be used to accelerate ions [1]. When a pulse is injected into this system by impressing a voltage between the helix and outer conductor a longitudinal electric field will be generated along the helix that persists on the axis if certain conditions are satisfied. The magnitude of the electric field along the helix is roughly given by

$$E_z \cong \frac{1}{v_p}\frac{\partial V}{\partial t} \tag{1}$$

where v_p is the phase velocity of the propagating voltage wave of amplitude V. The helix has non-local coupling due to mutual inductance and capacitance between the windings that gives rise to dispersion. As the ions gain energy from the propagating wave their speed increases and they can lose synchronism with the accelerating field unless the properties of the helical structure are tapered in the appropriate way. There are several operating modes of the helix with respect to whether or not the wave is in synchronism with the particles. In this paper we consider only the mode in which the particles move synchronously with the wave.

FIELD MODEL

Consider the geometry shown in Figure 1. The helix has radius a and the outer conductor has radius b. Between the helix and the outer conductor is a dielectric. The interior of the helix is in vacuum.

* This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

We will use the sheath helix model to find the fields in this structure [2]. In the dielectric layer of relative permittivity ε we have

$$\left[\nabla^2 - \frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right]\begin{pmatrix}E_z\\ B_z\end{pmatrix} = 0 \tag{2}$$

while in the interior vacuum region we have

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\begin{pmatrix}E_z\\ B_z\end{pmatrix} = 0. \tag{3}$$

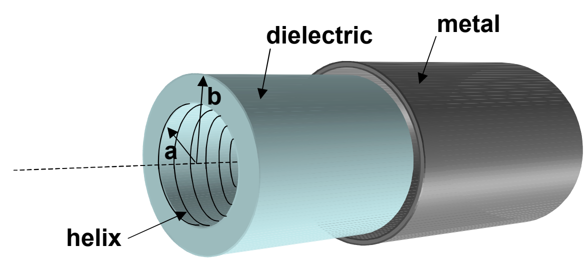

Figure 1: Schematic of helix, dielectric sleeve and outer conductor.

In the sheath helix model the actual fine structure of the helix is ignored which is a reasonable approximation for wavelengths that are long compared to the pitch or spacing of the helical windings. If we let all quantities vary as exp[i(kz-ωt)] the field equations become

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - \left(k^2 - \frac{\varepsilon\omega^2}{c^2}\right)\right]\begin{pmatrix}\tilde{E}_z\\ \tilde{B}_z\end{pmatrix} = 0 \tag{4}$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - \left(k^2 - \frac{\omega^2}{c^2}\right)\right]\begin{pmatrix}\tilde{E}_z\\ \tilde{B}_z\end{pmatrix} = 0 \tag{5}$$

where equation (4) applies to the region a < r < b and equation (5) applies to the region r < a. The tildes indicate Fourier amplitudes. If we make the definitions $p^2=k^2-\varepsilon\omega^2/c^2$ and $\mu^2=k^2-\omega^2/c^2$ then the appropriate solutions of equations (4) and (5) are

$$\begin{pmatrix}\tilde{E}_z\\ \tilde{B}_z\end{pmatrix} = \begin{pmatrix}C\\ D\end{pmatrix} I_o(pr) + \begin{pmatrix}D\\ F\end{pmatrix} K_o(pr) \qquad (6)$$

$$\begin{pmatrix}\tilde{E}_z\\ \tilde{B}_z\end{pmatrix} = \begin{pmatrix}A\\ B\end{pmatrix} I_o(\mu r). \qquad (7)$$

By using the Maxwell curl equations all the other field components may be determined from E_z and B_z. By imposing the condition that E_z and B_r vanish at r = b we find that for a < r < b

$$\tilde{E}_z = C\left[I_o(pr) - \frac{I_o(pb)}{K_o(pb)} K_o(pr)\right] \qquad (8)$$

$$B_z = E\left[I_o(pr) + \frac{I_1(pb)}{K_1(pb)} K_o(pr)\right]. \qquad (9)$$

Next, we impose the conditions the electric field tangential to the helix vanishes

$$\tilde{E}_z \sin\psi + \tilde{E}_\phi \cos\psi = 0 \qquad (10)$$

and that the tangential magnetic field $B_z\sin\psi + B_\phi\cos\psi$ is continuous across the helix [3].

Here the pitch angle $\psi = \cot^{-1}(2\pi a/L)$ where L is the pitch of the helix, the distance between adjacent windings. Imposing the continuity of E_z at r = a allows the determination of all of the unknown 6 coefficients in terms of one remaining coefficient. Taking appropriate ratios of field components allows us to obtain the dispersion relation

$$\left(\frac{\omega a}{c}\right)^2 \cot^2\psi = \frac{\dfrac{\mu a I_0(\mu a)}{I_1(\mu a)} + pa\dfrac{\dfrac{K_0(pa)}{K_1(pb)} + \dfrac{I_0(pa)}{I_1(pb)}}{\dfrac{K_1(pa)}{K_1(pb)} - \dfrac{I_1(pa)}{I_1(pb)}}}{\dfrac{I_1(\mu a)}{\mu a I_0(\mu a)} + \dfrac{\varepsilon}{pa}\dfrac{\dfrac{K_1(pa)}{K_0(pb)} + \dfrac{I_1(pa)}{I_0(pb)}}{\dfrac{K_0(pa)}{K_0(pb)} - \dfrac{I_0(pa)}{I_0(pb)}}} \qquad (11)$$

which agrees with the result obtained by Anicin [4]. A plot of ωa/c vs. ka is shown in Figure 2.

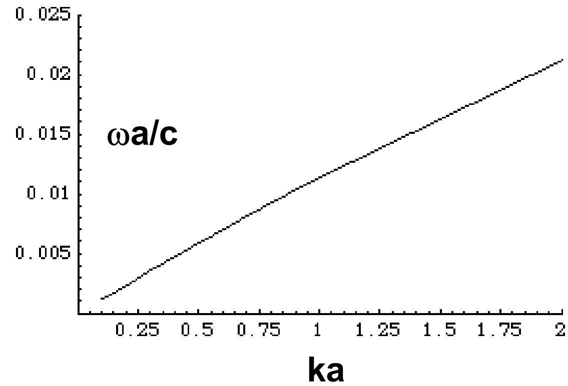

Figure 2: Plot of ωa/c vs. ka for the case ε = 1, ψ = .01 and b/a = 1.5.

Results of a 3-D finite difference time domain electromagnetic code XFDTD show good agreement with equation (11) [5].

Taking the limit of equation (11) as ka approaches zero yields the phase velocity ω/k as

$$v_p = \frac{c}{\sqrt{\varepsilon}}\sqrt{\frac{2\ln(b/a)}{\left(1-\frac{a^2}{b^2}\right)\cot^2\psi + 2\ln(b/a)}}. \qquad (12)$$

From equation (7) and the definitions of μ and v_p we find that

$$\tilde{E}_z = \tilde{E}_{wall}\frac{I_o\left(\frac{\omega r}{\gamma v_p}\right)}{I_o\left(\frac{\omega a}{\gamma v_p}\right)} \qquad (13)$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{v_p^{\,2}}{c^2}}}. \qquad (14)$$

From equation (13) we can see that the on-axis gradient is preserved provided that

$$\frac{\omega a}{\gamma v_p} < 1. \qquad (15)$$

By relating the current in the helix to the fields the all the field components can be determined. We need to calculate the jump in Bz and Bϕ across the helix

$$B_{z>} - B_{z<} = \frac{2I_\phi}{ca} \qquad (16)$$

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$$B_{\phi>} - B_{\phi<} = \frac{2I_z}{ca}. \tag{17}$$

From the sheath helix model we have the boundary condition [2]

$$I_\phi \sin\phi - I_z \cos\phi = 0. \tag{18}$$

From the field solution we have

$$\tilde{E}_r = -\frac{k}{p^2}\frac{\partial \tilde{E}_z}{\partial r} \tag{19}$$

which can be related to the voltage across the line as

$$\tilde{V} = \frac{ik\tilde{E}_z}{p^2} = \frac{ikAI_o(\mu a)}{p^2}. \tag{20}$$

In the low frequency limit A can be related to the current as

$$A = -\frac{i\omega a}{c}\frac{\tilde{I}_z}{ca}\frac{\cot\psi}{\tan\psi}\left(1 - \frac{a^2}{b^2}\right). \tag{21}$$

Using equations (20) and (21) to relate the voltage to the current yields the characteristic impedance (in the low frequency limit) as

$$Z = \frac{Z_o}{4\pi\sqrt{\varepsilon}}\cot\psi\sqrt{\left(1 - \frac{a^2}{b^2}\right)2\ln\frac{b}{a}}. \tag{22}$$

TAPERED LINE FOR SYNCHRONOUS OPERATION

As the ions accelerate in the field of the traveling wave they will accelerate and lose synchronism with the pulse. In order to maintain synchronism the wave speed must remain equal to the ion speed

$$v_p = \frac{1}{\sqrt{LC}} = v_i. \tag{23}$$

In order that the gradient remain constant we must have

$$E_z = \frac{1}{v_p}\frac{\partial V}{\partial t} \Rightarrow \frac{V}{v_p} = constant. \tag{24}$$

As we vary the parameters of the line the impedance will change. The voltage on the line will scale as the square root of the impedance. Thus we must have

$$\frac{(L/C)^{1/4}}{v_p} = constant. \tag{25}$$

Combining equations (23) and (25) gives the tapering

$$L \propto \frac{1}{C^{1/3}} \propto v_i. \tag{26}$$

BEAM LOADING

An approximate circuit model that incorporates the effects of beam loading may be written by assuming that the dispersion in the helix is small so that the propagation of electromagnetic waves is governed by the transmission line equations. We add a term proportional to the line density of the bunch

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \tag{27}$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} + \frac{\partial \lambda_b}{\partial t} \tag{28}$$

where L and C are respectively, the inductance and capacitance per unit length and λ_b is the line charge density of the bunch.

SUMMARY

We have obtained the dispersion relation for a helix wound inside of a dielectric shell that is encased by a conducting cylinder. From the field solution in the sheath helix model we have obtained expressions for the low frequency phase velocity and characteristic impedance as well as the fields. The tapering prescription necessary to maintain synchronous acceleration with constant gradient has been derived and a simple model of beam loading has been presented.

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