CHROMATICITY AND IMPEDANCE EFFECT ON THE TRANSVERSE MOTION OF LONGITUDINAL BUNCH SLICES IN THE TEVATRON *

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Abstract

The Transverse turn-by-turn evolution of a bunch slice are examined considering chromatic and impedance effects. A quasi-analytical approximation is developed using perturbative expansion of Hills equation with a wake field. This approximation is compared to turn-by-turn measurements taken in the Tevatron and from this linear and second order chromaticity, and Impedance are calculated as well as beam stability thresholds.

INTRODUCTION

Head-Tail instabilities driven by wake fields are a significant contribution to instabilities and beam loss at all stages of operation in the Tevatron. These instabilities require running the Tevatron with a large chromaticity at High Energy Physics store. High chromaticities however reduce beam lifetime and thus the total integrated luminosity delivered to the experiments. With improvements in the speed of available digital oscilloscopes it is now possible to experimentally measure the transverse evolution of longitudinal bunch slices down to resolutions below .4 nsecs. We can now attempt to directly compare existing or new models with experiment and arrive at a more exact characterization of the transverse wake. While attempting to accurately model turn-by-turn bunch slice evolution we have found it necessary to include wake field effects to account for the observed beam behavior. In the process we consider an alternate perturbative method from the standard solution to Sacherer's integral equation. We attempt to arrive at an analytical approximation using a direct perturbative method employing "strained parameters" to eliminate the secular terms and thus evaluate Hills equation in the presence of a wake field. This approximation is compared to multiparticle simulations and both to the actual experimental data. In the process we find the dominate contribution to the transverse wake in the Tevatron arising from resistive wall.

EXPERIMENTAL SET-UP

Using the 1-meter long strip line detector installed at F0 in the Tevatron the proton signal is captured using a Tektronics TDS7000 series oscilloscope. The A-B and A+B signals are measured with a resolution of 0.4 ns across 20 ns for 1049 turns. Since the recorded signal is the sum the image current traveling with the beam and the reflected image of the beam from the downstream end we first deconvolute the single image by subtracting out the reflected image. This is accomplished digitally using knowledge of the length of strip line and the velocity of the beam. After reconstruction of the signal the transverse position is then determined by taking the ratio of the sum and difference signal times a factor give by the geometry of the strip-line (27). All data was taken with octupoles off at Tevatron injection energy of 150 GeV.

ANALYSIS

Of particular interest was an attempt to explain the difference in the turn-by-turn coherence and decoherence structure between the head and tail of the bunch after a $\pm 1mm$ kick. As you can see in Fig 1 the head and the tail of the bunch separated here by 8 nsecs display very different behavior in the beam envelope. A simple analytical model

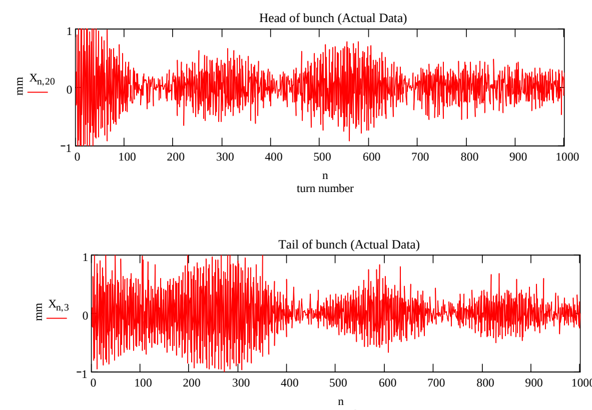

Figure 1: Transverse beam motion after 1 mm kick with .4 nsec longitudinal bunch slices of Head (top) and the Tail (bottom) each 4 nsecs from the bucket center. Chromaticity measured to be $\xi = 4.0 \pm 1$ and σ_τ= 3 necs.

with only linear chromaticity cannot account for this difference. Other effects such as space charge and synchrotron tune spread while significant representing tune shifts on the order of 7×10^{-4} operate symmetrically on the bunch and therefore could not account for this behavior. If we extend the simple analytical model to include 2nd order chromaticity in manner of S. Fartoukh and R. Jones [1], we can begin to see some asymmetry between the head and the tail as shown in Fig. 2. However since 2nd order chromaticity at the strength necessary to produce this effect will produce significant damping we can not explain a coherence, recoherence pattern where a small recoherence is followed by a large recoherence. We concluded that this effect must be due primarily to transverse wake fields. This is clearly

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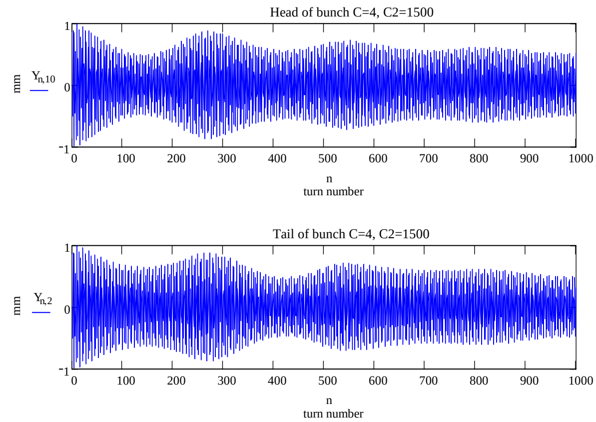

Figure 2: Analytical model with linear chromaticity set to $\xi = 4.0$ and nonlinear $\xi' = 1500$ and σ_τ= 3 necs. Transverse motion of Head (top) and the Tail (bottom) each 4 nsecs from the bucket center.

verified by comparison with multi-particle simulations using code developed by A. Burov [3]. Tracking 1000 particles over 1000 turns under the influence of a resistive wall wake we found good agreement between the experimental data and model as can be seen in Fig. 3 - 4 We found

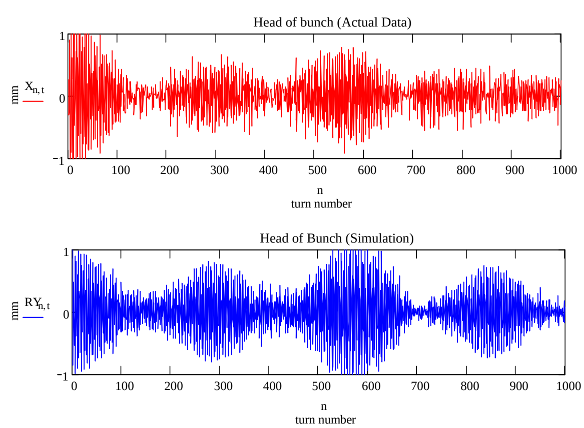

Figure 3: Transverse turn-by-turn data of Head after a 1 mm kick 4 nsecs ahead of bucket center. Actual turn-by-turn data (top) compared with 1000 particle simulation with resistive wall wake $W_1 = 4.4 \times 10^5 \frac{1}{cm^2}$ and chromaticity $\xi = 3.733$ and σ_τ = 3 nsecs.

the inclusion of a resistive wall wake field strength from $4.4 \times 10^5/\mathrm{cm}^2 - 7 \times 10^5/\mathrm{cm}^2$ necessary to fit the experimental data. For these simulations we considered only the effects of transverse resistive wall wake and linear chromaticity. The inclusion of 2nd order chromaticity might improve things by adding more landau damping.

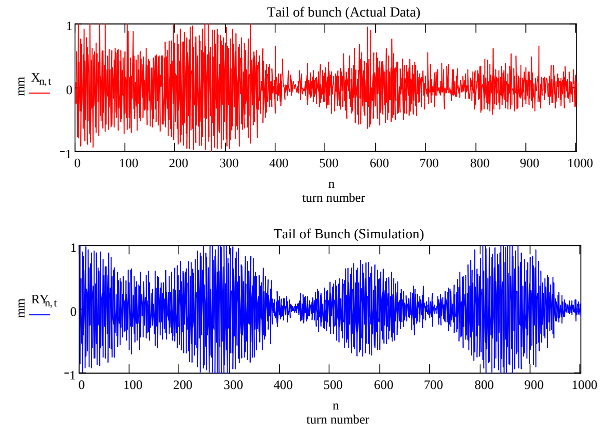

Figure 4: Transverse turn-by-turn data of Tail after a 1 mm kick 4 nsecs behind bucket center. Actual turn-by-turn data (top) compared with 1000 particle simulation with resistive wall wake $W_1 = 4.4 \times 10^5 \frac{1}{cm^2}$ and chromaticity $\xi = 3.733$ and σ_τ = 3 nsecs.and σ_τ = 3 nsecs.

PERTURBATIVE CALCULATION OF THE EFFECTS OF RESISTIVE WALL WAKE

The differential equation which governs the evolution of the transverse motion of a particle in a bunch under the influence of a wake field can be given in the most general form,

$$\frac{d^2Y(z,\delta,s)}{ds^2} + \omega^2(\delta)/c^2 Y(z,\delta,s) = -\frac{r_0}{C\gamma}\int_{-\infty}^{\infty}\int_z^{\infty} d\delta dz' \rho(z',\delta) Y(z',\delta,s) W_\perp(z-z'), \quad (1)$$

where $r_0 = e^2/m_0c^2$, C is the circumference of the ring, $W_\perp(z)$ is the transverse wake field, $N = \int dz'\rho(z')$ is the number of particles in a bunch. We use s as the longitudinal coordinate and z defines the longitudinal motion in the bunch given by,

$$\begin{aligned} \delta(s) &= \frac{-\omega_s}{\eta c} r\sin(\omega_s s/c + \phi) \\ z(s) &= r\cos(\omega_s s/c + \phi). \end{aligned} \quad (2)$$

$\delta = \Delta p/p$ is the relative momentum difference, ω_s the synchrotron angular frequency ϕ the phase of the synchrotron motion, η the slippage factor. In the absence of a wake field we can find solutions to the left hand side of Eq. (1),

$$Y_0(z,\delta,s) = Ae^{\pm i\Phi(z,\delta,s)} \quad (3)$$

where,

$$\Phi(z,\delta,s) = \omega_\beta s/c + \frac{\xi\omega_0 z}{\eta c}(1-\cos(\omega_s s/c)) + \frac{\xi\delta}{Q_s}\sin(\omega_s s/c), \quad (4)$$

Here A is the constant amplitude and Q_s is the synchrotron tune and ξ is the chromaticity. If we define $\alpha = \frac{-r_0 W_1 N c^2}{C\gamma\omega_\beta^2}$

we can argue that for stables beams $|\alpha| << 1$ since the wake field perturbation to the tune is small. If we expand the dependent variable in α as has been done for case of linear accelerators [4] and try solve we will find this solution is valid only so long as $Y_0 > \alpha Y_1$ the appearance of secular terms in Y_1 guarantees that this condition will not hold in the long term for Y_1 will grow with s. This problem can be dealt with using a "strained parameter" method [2]. Secular terms which appear in the integral for Y_1 and be transferred to the frequency if the time variable s is expanded as well $s = t(1 + \alpha\psi_1)$. With both these expansion our solution to first order becomes,

$$Z(\tau_0, s) =$$
$$A\left(e^{\pm w(\tau_0, s/(1+\alpha\psi_1))} - \alpha(\omega_\beta/c)e^{\pm w(\tau_0, s/(1+\alpha\psi_1))}\times\right.$$
$$\left.\int^{s/(1+\alpha\psi_1)} \frac{e^{\mp w(\tau_0,x)}G_0(\tau_0,x) - 2\psi_1}{\mp 2i}dx\right) \quad (5)$$

Where we choose ψ_1 so that any secular terms will be cancelled. Here G_0 is,

$$G_0(\tau_0, s) = \int_{-\infty}^{\infty} \rho(\delta_0)F_0(\tau_0\cos(\omega_s s/c) + \frac{\delta_0\eta}{\omega_s}, s)d\delta_0 \quad (6)$$

and F_0 is for a Gaussian distribution,

$$F_0(\tau, s) = Ae^{-\frac{\chi^2}{2}\sigma_\tau^2\sin^2(\omega_s s/c) + i\omega_\beta s/c}\times$$
$$\int_\tau^\infty d\tau' \frac{e^{-\frac{\tau'^2}{2\sigma_\tau} + i\chi\tau'(1-\cos(\omega_s s/c))}W_0(c\tau - c\tau')}{\sqrt{2\pi}\sigma_\tau}. \quad (7)$$

Here we use $\chi = \frac{\omega_0 Q'}{\eta}$ and $\sigma_\delta = \frac{\omega_0 Q_s\sigma_\tau}{|\eta|}$ and $\tau = z/c$. Now we can evaluate G_0 for any given wake function. In this case we look at resistive wall wake which assumes the form,

$$\mathrm{W}(\tau) = \frac{\mathrm{W}_0}{\sqrt{|\tau|}} \quad (8)$$

So our integral for $F_0(\tau, s)$ involves the following integral,

$$\int_\tau^\infty \frac{e^{\frac{-\tau'^2}{2\sigma_\tau^2} + i\chi\tau'(1-\cos(\omega_s s/c))}}{\sqrt{|\tau - \tau'|}}d\tau' \quad (9)$$

the solution of which can be evaluated analytically. However the integral over δ_0 needs to be evaluated numerically. The integral over t can be evaluated by expanding the integrand in a Fourier series and from there the secular terms can be identified and thus ψ_1 evaluated for each τ_0. Comparison of the semi-analytical approximation matches up well (Fig. (5 - 6) when compared with the multi-particle simulation. It appears to capture the general features of the beam coherence and decoherences across the whole bunch.

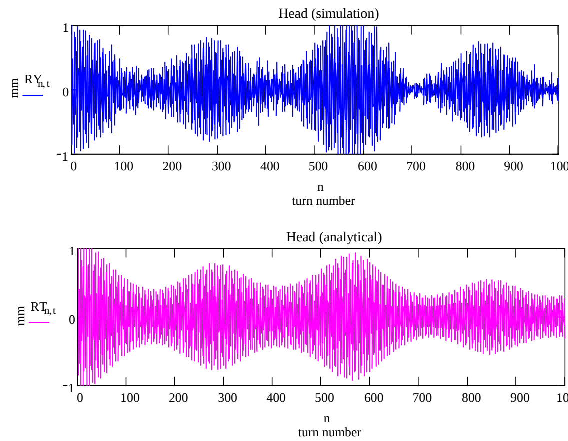

Figure 5: Transverse turn-by-turn data of Head after a 1 mm kick 4 nsecs ahead of bucket center. 1000 particle simulation (top) compared perturbative semi-analytical model (bottom), both with resistive wall wake $W_1 = 4.4 \times 10^5 \frac{1}{cm^2}$ and chromaticity $\xi = 3.733$ and σ_τ = 3 nsecs.

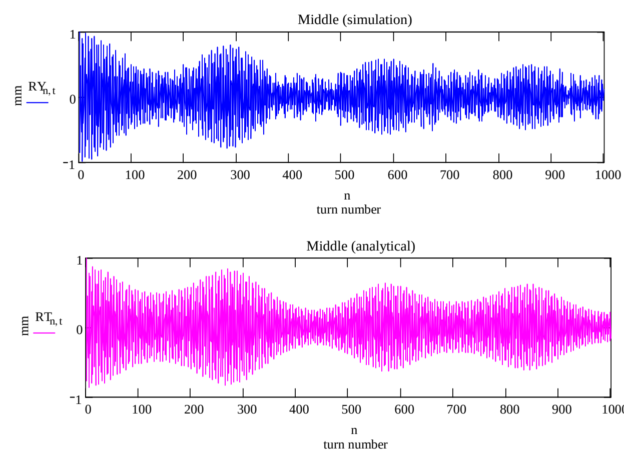

Figure 6: Transverse turn-by-turn data of bunch Center after a 1 mm kick. 1000 particle simulation (top) compared perturbative semi-analytical model (bottom), both with resistive wall wake $W_1 = 4.4 \times 10^5 \frac{1}{cm^2}$ and chromaticity $\xi = 3.733$ and σ_τ = 3 nsecs.

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