BEAM PROPAGATION IN MISALIGNED MAGNETIC ELEMENTS: A MATLAB® BASED CODE

T. F. Silva[1], M. L. Lopes, P. B. Rios, M. N. Martins
Laboratório do Acelerador Linear, Instituto de Física da Universidade de São Paulo
PO Box 66318, 05315-970 São Paulo, SP, Brazil

Abstract

We present a method to calculate kinematical parameters of a beam subject to a misaligned magnetic element. The procedure consists in transforming the kinematical parameters of the beam to the reference frame in which the magnetic element is aligned, propagating the beam through the element, and transforming back to the original frame. This is done using rotation matrices around the X-, Y-, and Z-axes. These matrices are not Lorentz invariant, so the rotations must be relativiscally corrected. We describe the transformation matrices, present a MatLab® based code that uses this method to propagate up to 1000 particles trough a misaligned quadrupole, and show some graphical outputs of the code.

INTRODUCTION

A charged particle, in a beam, can be well described, in matrix formalism, by a six vector:

$$v = \begin{pmatrix} x \\ \theta \\ y \\ \varphi \\ z \\ \Delta p / p \end{pmatrix} \qquad (1)$$

where x and y are the relative position to the center trajectory in horizontal and vertical directions respectively, θ and φ are their derivative with respect to the trajectory position, z is the relative position to the synchronous particle and $\Delta p/p$ is the momentum spread.

In this formalism, each element in the accelerator is represented by a six by six operator (transformation matrix), which, applied over the eigenstate of the particle before the element, results in the eigenstate of the particle after the element [1, 2].

$$v_f = M \cdot v_i \qquad (2)$$

where M is the element matrix.

The basic problem with a misaligned magnet is that the corresponding operator is usually unknown. Since we know the aligned magnet operator, we propose to change the basis of the particle eigenstate to the reference frame where the magnet is aligned, in order to perform its propagation through the element.

This paper describes a method to propagate particles through misaligned magnets, summarized in the scheme presented in figure 1.

BASIS CHANGE

The basis change operator depends on the kind of misalignment the element presents. Purely angular misalignments can be represented by rotation matrices if, instead of $\Delta p/p$, we use $\zeta = dz/ds$, in analogy to $\theta = dx/ds$ and $\varphi = dy/ds$, as the last coordinate. Where s is the synchronous particle longitudinal coordinate.

The classical basis change operators are given by:

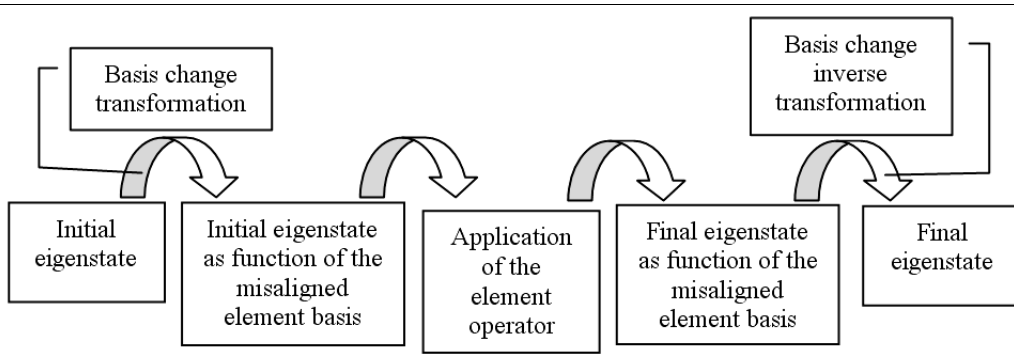

Figure 1 – Schematics of the basis change procedure.

1 tfsilva@if.usp.br

$$R_z = \begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & -\sin\theta & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & \sin\theta & 0 & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R_x = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & -\sin\theta \\ 0 & 0 & \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & \sin\theta & 0 & \cos\theta \end{pmatrix} \tag{3}$$

$$R_y = \begin{pmatrix} \cos\theta & 0 & 0 & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & 0 & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & 0 & 0 & \cos\theta \end{pmatrix}$$

Where the indices indicate the corresponding rotation axes [3, 4].

Following the scheme shown in figure 1, and using the operators presented in (3), we can have the particle eigenstate after passing through a misaligned element, using the expression:

$$v_f = R^{-1} \cdot M \cdot R \cdot v_i \tag{4}$$

Classical rotation matrices imply in classical approximations, so these operators are not Lorentz invariant. In order to use this method for high energy beams, it is necessary to include relativistic corrections.

RELATIVISTIC CORRECTIONS

To make the operators Lorentz invariant, we must include a temporal coordinate (*ict*) in the particle eigenstate.

$$\tilde{v} = \begin{pmatrix} x \\ \theta \\ y \\ \varphi \\ z \\ \zeta \\ ict \end{pmatrix} \tag{5}$$

The element operator must be corrected accordingly as follows:

$$\tilde{M} = \begin{pmatrix} M & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & 1 \end{pmatrix} \tag{6}$$

The same modification must be done for the basis change operators.

$$\tilde{R} = \begin{pmatrix} R & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & 1 \end{pmatrix} \tag{7}$$

This would modify the expression in (4) to:

$$\tilde{v}_f = \lambda \cdot \tilde{R}^{-1} \cdot \lambda^{-1} \cdot \tilde{M} \cdot \lambda^{-1} \cdot \tilde{R} \cdot \lambda \cdot \tilde{v}_i \tag{8}$$

Where λ is the Lorentz transformation matrix in this coordinate system, given by:

$$\lambda = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 & i\beta\gamma \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & i\beta\gamma & 0 & \gamma \end{pmatrix} \tag{9}$$

With γ and β being the usual relativistic parameters [4].

SIMULATION CODE

To simulate these theoretical concepts, we developed a MatLab® [5] based code that uses (8) to propagate 1000 particles through a misaligned quadrupole lens.

The code generates an output that includes a data array that can be manipulated, and a set of maps to help the data analysis (phase space and position maps).

To illustrate the code output, we present results for a 5 MeV electron beam passing through a 5 cm long quadrupole with a 10 T/m gradient. Two different

0-7803-8859-3/05/$20.00 ©2005 IEEE

misalignment cases are presented: a) 10º around the Z-axis (longitudinal); b) 10º around the X-axis (transversal). Those misalignments are exaggerated in order to ease the visualization.

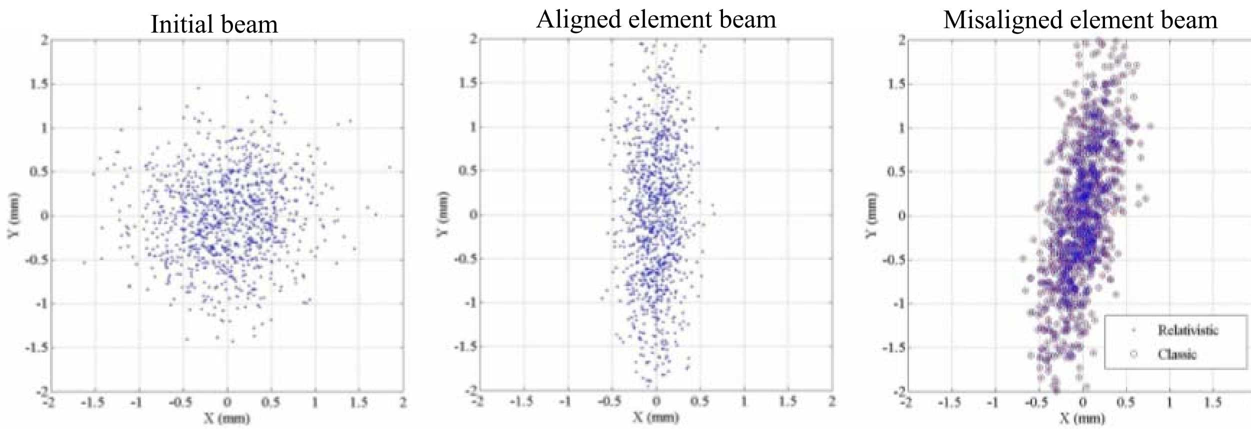

Figure 2 a) – 10º misalignment around the Z-axis.

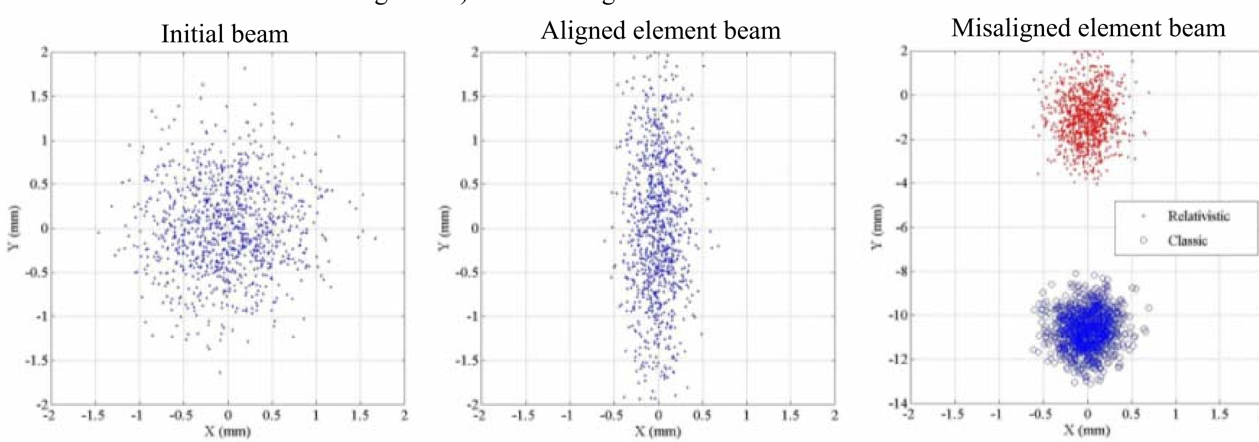

Figure 2 b) – 10º misalignment around the X-axis (note the Y-axis scale).

As it can be observed, classical and relativistic calculations are coincident to misalignments around de Z-axis, due to the fact that this is the only Lorentz invariant rotation.

On the other hand, misalignments corresponding to rotations around the X- or Y- axes, which are not Lorentz invariant, present a significant difference between the classic and relativistic results.

ACKNOWLEDGEMENTS

The authors would like to thank to IFUSP Professor E. V. Passos, Indiana University Professor S. Y. Lee, his assistant X. Huang and Mr. C. M. Silva, Mr. A. A. Malafronte for useful discussions. This work was supported by Brazilian funding agencies FAPESP and CNPq.

REFERENCES

[1] Wiedemann H. – Particle accelerator physics: basic principles and linear beam dynamics, Springer-Verlag, Berlin, 1993.

[2] Chao, A. W.; Tigner, M. (Ed.) – Handbook of accelerator physics and engineering, World Scientific, Singapore, 1998.

[3] Sakurai, J. J. – Modern quantum physics, Addison-Wesley publishing Inc., ninth edition, Reading, 1985.

[4] Moshe, C. – Group theory and general relativity, McGraw-Hill Inc., Great Britain, 1977.

[5] MATLAB® 7, Copyright 1984-2004, The MathWorks, Inc.