SYNTHESIS OF BEAM LINES WITH NECESSARY PROPERTIES

S.N. Andrianov, SPbSU, Saint-Petersburg, Russia

Abstract

In this paper a new approach to the problem of synthesis of beam lines is discussed. Usually this problem can be overcome by the use of numerical simulation and optimal control theory methods. But this results in sufficiently great number of variable parameters and functions. Obviously, that this degrades quality of a modeling procedure. The suggested approach is demonstrated on a problem of a microprobe design problem. Essence of the problem is that necessary to design a high precision focusing system which satisfies some additional conditions. For solution of this problem we use an algebraic treatment based on Lie algebraic methods and computer algebra techniques. Using the symmetry ideology this approach allows rewriting beam properties to enough simple conditions for control parameters and functions. This leads a set of desired solutions and show results in some most suitable form. Moreover, this approach decreases the number of variable parameters.

INTRODUCTION

It is known that different restrictions imposed on beam lines essentially complicate the corresponding design process. But these demands are mandatory and important for realization of desired physical properties. Fortunately, the most of these properties can be written in the form of some symmetry restrictions.

In this paper a class of systems generating proton beams of micron and submicron scale is considered. These systems are known as microprobes. The main demands for similar machines are connected with terminal beam sizes. But in some cases it is important to impose additional restrictions. The most of solution methods for similar problems are based on methods of the optimal control theory or nonlinear programming (see, for example, [1], [2]). As an example of a system we consider a beam line guaranteeing a round image for round diaphragms, which forming the beam phase portrait (see fig. 1). In another words the system should supply with preservation of the rotation symmetry in the configurational space.

THE MICROPROBE CONFIGURATION

Let us consider the focusing system which consists on four quadrupoles separated by drifts [3]. The preliminary scheme of this system one can see on fig. 1.

Here D_{f_1} and D_{f_2} are round diaphragms with R_1 and R_2 as radii, F_i, D_i $(i = 1, 2)$ are focusing and defocusing
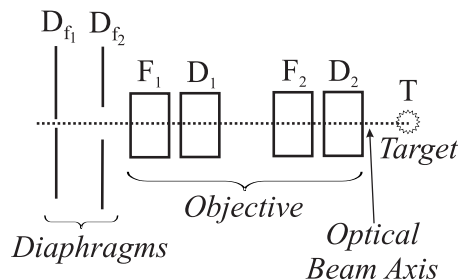

Figure 1: The scheme of a quadruple microprobe.

quadrupoles correspondingly. Free field spaces have the following length correspondingly: a — the fore-distance (from the first diaphragm up to F_1), s_k $(k = \overline{1,4})$ — distances between quadrupoles, g — the working (terminal) distance (the length of terminal drift). The length of the quadrupoles are L_i, $(i = \overline{1,4})$. These geometrical parameters and the focusing forces of the quadrupoles – k_i are the aim of our investigation. The system will be considered as optimal in linear approximation if these parameters will ensure the minimal beam measure on the target T (see fig. 1), and some additional conditions will be fulfilled.

THE LIE METHODS AS BASIC SOLUTION TOOLS

The above formulated property of the round symmetry can be described with the help of the following condition:

$$\mathcal{T}_\alpha \circ \mathfrak{N}_0 = \mathfrak{N}_0 \overset{\mathcal{M}(s_T|s_0)}{\Longrightarrow} \mathcal{T}_\alpha \circ \mathfrak{N}_T = \mathfrak{N}_T, \qquad (1)$$

where $\mathfrak{N}_0$ — an initial beam image in the configuration space $\{x, y\}$, $\mathfrak{N}_T$ — the corresponding image on the terminal target in according to transformation $\mathcal{M} = \mathcal{M}(s_T|s_0)$, generated by the system, $\mathcal{T}_\alpha$ — rotational transformation in the transversal configuration space for an arbitrary angle α around beam optical axis, s — the length measured along the optical axis. The equality (1) leads us to the following equality

$$\mathcal{T}_\alpha \circ \mathcal{M} \circ \mathcal{T}_\alpha^{-1} = \mathcal{M}, \qquad (2)$$

in other words the transformations $\mathcal{T}_\alpha$ and $\mathcal{M}$ commute. Let's write the rotational transformation in the following form: $\mathcal{T}_\alpha = \exp\{\alpha \cdot \mathcal{L}_{\text{rot}}\}$, where $\mathcal{L}_{\text{rot}} = -y\partial/\partial x + x\partial/\partial y - y'\partial/\partial x' + x'\partial/\partial y' = \mathbf{X}^*\mathbb{T}^*\partial/\partial\mathbf{X}$ — a generator for rotational transformation in the plane $\{x, y\}$,

$\mathbf{X} = (x, x', y, y')^*$, $\mathbb{J}$ — a symplectic matrix of the form

$$\mathbb{J} = \begin{pmatrix} \mathbb{O} & -\mathbb{E} \\ \mathbb{E} & \mathbb{O} \end{pmatrix}.$$

The system propagator $\mathcal{M}(s|s_0)$ is presented as a Lie map. Let us denote $\mathcal{L}_{\text{syst}}(s)$ a Lie operator for the our microprobe. For this purpose one can use both chronological exponential operator

$$\mathcal{M}(s|s_0) = \mathrm{T} \exp \left\{ \int_{s_0}^{s} \mathcal{L}_{\text{syst}}(\tau) \right\} d\tau$$

and routine exponential operator (so called Magnus presentation) [4]

$$\mathcal{M}(s|s_0) = \exp \left\{ \mathcal{L}_{\mathcal{G}_{\text{syst}}(s|s_0)} \right\},$$

where the new Lie operator $\mathcal{L}_{\mathcal{G}_{\text{syst}}(s|s_0)}$ can be computed for $\mathcal{L}_{\text{syst}}(s|s_0)$ using simultaneous analogue of the well known CBH–formula.

Using the Lie operator for the beam line $\mathcal{L}_{\text{syst}}(s_T|s_0)$, we can rewrite our equality (2) in the form

$$\exp \left\{ \exp \left\{ \alpha \cdot \mathcal{L}_{\text{rot}} \right\} \circ \mathcal{L}^{\text{syst}}(s_T|s_0) \right\} = \exp \mathcal{L}^{\text{syst}}(s_T|s_0).$$

As $\mathcal{L}^{\text{syst}}$ is generated by the function $\mathbf{G}^{\text{syst}} = \sum_{k=1}^{\infty} \mathbb{G}_k^{\text{syst}}(s_T|s_0)\mathbf{X}^{[k]}$, then we can write

$$\exp \left\{ \alpha \cdot \mathcal{L}_{\text{rot}} \right\} \circ \mathcal{L}^{\text{syst}} = \widetilde{\mathcal{L}}^{\text{syst}},$$

where $\widetilde{\mathcal{L}}^{\text{syst}} = \sum_{k=1}^{\infty} \left(\mathbf{X}^{[k]} \right)^* \left(\widetilde{\mathbb{G}}_k^{\text{syst}} \right)^* \partial/\partial \mathbf{X}$. Using the matrix formalism (see [5], [6]) for matrices generating corresponding Lie operators we can write

$$\mathbb{G}_k^{\text{syst}} \mathbb{J}^{\oplus k} - \mathbb{J}\mathbb{G}_k^{\text{syst} \oplus k} = 0.$$

For the linear case we have $k = 1$ and:

$$\mathbb{G}_1^{\text{syst}} \mathbb{J} - \mathbb{J}\mathbb{G}_1^{\text{syst}} = 0.$$

It is convenient for our aim to present the matrix $\mathbb{G}_1^{\text{syst}} = \mathbb{G}^{\text{syst}}(s_T|s_0)$ as a block matrix

$$\mathbb{G}_1^{\text{syst}} = \begin{pmatrix} \mathbb{G}^{11} & \mathbb{G}^{12} \\ \mathbb{G}^{21} & \mathbb{G}^{22} \end{pmatrix}.$$

The form of $\mathbb{J}$ leads to the following equalities for block matrices $\mathbb{G}^{ik}$:

$$\mathbb{G}^{11} = \mathbb{G}^{22}, \quad \mathbb{G}^{12} = -\mathbb{G}^{21}. \tag{3}$$

For the next calculations we should use the so called Magnus presentation [3] using the matrix formalism. This allows us to write the following equality for the beam line

matrix $\mathbb{G}^{\text{syst}}(s_T|s_0)$ (here we introduce the notation $\mathbb{P} = \mathbb{P}^{\text{syst}}$):

$$\mathbb{G}^{\text{syst}}(s_T|s_0) = \int_{s_0}^{s_T} \mathbb{P}(\tau) d\tau -$$

$$- \frac{1}{2} \int_{s_0}^{s_T} \int_{s_0}^{\tau} \left\{ \mathbb{P}(\tau), \mathbb{P}(\tau') \right\} d\tau' d\tau +$$

$$+ \frac{1}{6} \int_{s_0}^{s_T} \int_{s_0}^{\tau} \int_{s_0}^{\tau'} \left(\{ \{ \mathbb{P}(\tau), \mathbb{P}(\tau') \}, \mathbb{P}(\tau'') \} + \right.$$

$$\left. + \{ \{ \mathbb{P}(\tau''), \mathbb{P}(\tau') \}, \mathbb{P}(\tau) \} \right) d\tau \, d\tau' \, d\tau'' + \dots$$

In general, the matrix $\mathbb{P}(s)$ depends on a control vector function $\mathbf{U}(s)$. In the case of a quadrupole focusing system the vector $\mathbf{U}(s)$ degenerates to a scalar function $u(s) = k(s)$, where $k(s)$ is a generalized focusing force of the microprobe. It is known, that there is the following property for any continuous function $f(t)$, measurable on the interval $[s_0, s_T]$: $\int_{s_0}^{s_T} f(t)dt = \int_{s_0}^{s_T} f(s_T - t)dt$. This leads to the following equation for the matrix $\mathbb{P}(\mathbf{U}, s)$:

$$\int_{s_0}^{s_T} \mathbb{P}\left(\mathbf{U}(\tau), \tau \right) d\tau = \int_{s_0}^{s_T} \mathbb{P}\left(\mathbf{U}(s_T - \tau), s_T - \tau \right) d\tau. \tag{4}$$

Using (4) one can write

$$\mathbb{P}^{11}(\mathbf{U}(s), s) = \mathbb{P}^{22}(\mathbf{U}(s_T - s), s_T - s),$$

$$\mathbb{P}^{12}(\mathbf{U}(s), s) = -\mathbb{P}^{21}(\mathbf{U}(s_T - s), s_T - s),$$

where $\mathbb{P}^{ik}$ — entering block matrices for the matrix $\mathbb{P}$. We should note, that these equalities are not enough for the desired symmetry. For obtaining of additional conditions we can use the special structure of the Magnus's representation. Namely we can use the following property: all terms in the Magnus's representation (with the exception of the first term) are contain only commutators of the corresponding matrices. In our case of quadrupole symmetry matrices $\mathbb{P}^{kk}$ have the form $\mathbb{P}^{11} = \begin{pmatrix} 0 & 1 \\ -k(s) & 0 \end{pmatrix}$ and $\mathbb{P}^{22} = \begin{pmatrix} 0 & 1 \\ k(s) & 0 \end{pmatrix}$. It is not difficult to prove, that we obtain only two types of matrices under integral symbol:

$$f(k(s_1), \dots, k(s_m)) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$g(k(s_1), \dots, k(s_{m-1})) \begin{pmatrix} 0 & 1 \\ \pm k(s_m) & 0 \end{pmatrix}. \tag{5}$$

Here s_i, $i \geq 1$ integration variables for "inner integrals". Let $\mathbb{U}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ be a permutation matrix. Than one can note (taking into account (4) and the form of the matrix $\mathbb{P}^{11}$), that if there is an additional condition for $\mathbb{G}^{11}$:

$$\mathbb{U}_2 \mathbb{G}^{11} - \left(\mathbb{G}^{11} \right)^* \mathbb{U}_2 = 0.$$

This equation leads us to the desired condition (4). At last, using the exponential Magnus's representation one can find an additional condition for a linear matrix propagator $\mathbb{R}^{11}$: $\mathbb{U}_2 \mathbb{R}^{11} \mathbb{U}_2 = \left(\mathbb{R}^{11}\right)^*$, which (taking into account the above given conditions) leads us to the following restriction

$$\left\{\mathbb{R}^{11}\right\}_{11} = \left\{\mathbb{R}^{11}\right\}_{22}. \qquad (6)$$

As a result of the above pointed integral properties one can obtain

$$k(s) = -k(s_T - s) \qquad (7)$$

Eq. (6) and (7) are fully congruent to the desired axially symmetry of the beam line.

COMPUTATIONAL EXPERIMENTS

We should note that the constructed conditions are obtained starting from natural restrictions, imposed on target beam, algebraic and functional properties of the following matrix functions. The resulting equalities are equivalent to two set of the equations:

$$\mathrm{L}_1 = \mathrm{L}_4, \;\; \mathrm{L}_2 = \mathrm{L}_3, \;\; s_1 = s_3 = s, \;\; s_2 = \lambda,$$

$$k_1 = -k_4, \;\; -k_2 = k_3.$$

In this case eq. (6) is equivalent to an algebraic equation for only two forces parameters k_1, k_2 and four geometrical parameters s, λ, L_1 and L_2. In the next simulations we put $\mathrm{L}_1 = \mathrm{L}_2 = \mathrm{L}$.

The condition (6) defines in the parameter space so called load curves, which have several branches and enough intricate form. The corresponding relative location of these curves depends on objective parameters (see, for example, fig. 2). The other parameters a and g should be founded using another conditions. For example, here we can find the working distance g from the condition $\left\{\mathbb{R}^{11}\right\}_{12} = 0$. This equality, as it is well known, corresponds to the "point-to-point" focusing condition. The fore-distance a can be varied in some intervals depending on experimenter possibilities. In particular, the increasing of this parameter leads to uprating of the system demagnification.

It should be noted that information on these curves allows forming a special database. Similar information gives a possibility to choose (using the corresponding interface) a suitable control system, which satisfies the researcher's requirements. For creation of the load curves it is convenient to use nonlinear programming methods, especially in the case of systems with the larger number of lenses. Indeed the usual nonlinear equation solution methods do not lead often to correct results. In particular, we use for this purpose a special method based on an ideology of the predictor–correction approach.

After it the optimal points (corresponding to a minimal beam measure) are searched. In the case of the linear approximation one can use the demagnification value
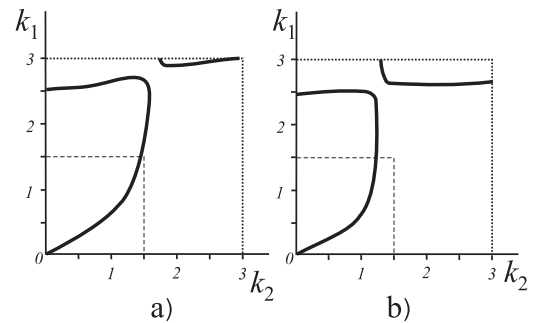

Figure 2: The examples of load curves:
a — $\lambda = 0.005\,\mathrm{m}$, $s = 0.024\,\mathrm{m}$, $L = 0.2015\,\mathrm{m}$;
b — $\lambda = 0.2015\,\mathrm{m}$, $s = 0\,\mathrm{m}$, $L = 0.2015\,\mathrm{m}$;

$\left\{\mathbb{R}^{11}(s_T)|s_0)\right\}_{11}$. Here $s_T - s_0$ is a total length of the system ($s_T - s_0 = a + g + \{$the objective length$\}$). Corresponding parameter values (with the additional auxiliary data) are a database content.

The above suggested approach was applied to so called rectangular (piece-wise) approximation of the control fields. But it can be extended to an arbitrary dependence of $k(s)$ too. This can be realized using parameter presentation or another type of description of the control function $k(s)$ (for example, in the some class of model functions).

The computational experiments demonstrated the high sensitivity of the criterion $\mathbb{R}^{11}$ with respect to possible deflections of the system parameters. This allows indicating zones of the increased sensitivity, that is an important information for designers of similar systems.

In the case of nonlinear approximation the designer can study nonlinear aberrations contribution. In particular, the spherical aberrations of the third order lead to both the geometry changes and the measure of the terminal beam image. Necessarily, these distortions can be corrected using octupole lenses (combined with quadrupole lenses or inserted into the system independently) [7].

REFERENCES

[1] Andrianov S.N., Taryanik V. M., in: Proc. Seventh European Particle Accelerator Conference — EPAC'2000. Vienna (Austria), 1351 (2000).

[2] Andrianov S.N., in: Proc. of the 11-th Int. IFAC Workshop Control Application of Optimization. Vol. 1. Pergamon Press. Oxford. UK. 11 (2001).

[3] Andrianov S.N., Dymnikov A.D., Osetinsky G.M. PTE. Vol. 1. 39 (1982) (in Russian).

[4] Klarsfeld S., Oteo J.A., in: J. Phys. A (Math. Gener.) 1989. Vol. 22. N 21. 4565 (1989).

[5] S.N. Andrianov, in: Mathematics and Computers in Simulation. Vol. 57. Issue 3–5, 147 (2001).

[6] Andrianov S.N., in: Proc of the Fifth Int. Workshop Beam Dynamics & Optimization — BDO'98, SPb, 1998. SPb., 18 (2002) (in Russian).

[7] Andrianov S.N. AIP Conf. Proc. No 405. N.Y., 103 (1997).