IMPROVEMENT OF THE LONGITUDINAL BEAM DYNAMICS TUNING PROCEDURE FOR THE MSU RIA DRIVER LINAC*

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Abstract

The Rare Isotope Accelerator (RIA) driver linac will use a superconducting, cw linac with independently phased superconducting rf cavities for acceleration and utilize beams of multiple-charge-states (multi-q) for the heavier ions. Given the acceleration of multi-q beams and a stringent beam loss requirement in the RIA driver linac, a new beam dynamics code capable of simulating nonlinearities of the multi-q beam envelopes in the longitudinal phase space was developed. Using optimization routines, the code is able to maximize the linearity of the longitudinal phase space motion and thereby to minimize beam loss by optimizing values for the amplitude and phase of the cavities for a given accelerating lattice. Relative motion of the multi-q beams is also taken into account so that superposition of the beam centroids and matching of their Twiss parameters are automatically controlled. The new tuning procedure and its benefit on the performance of the beam dynamics in the longitudinal plane are discussed in the paper.

INTRODUCTION

Multi-q beam acceleration, especially in the low energy section of the Superconducting (SC) linac, is one of the major challenges for RIA. Though independently phased SC cavities offer more tuning flexibility in the longitudinal plane, no computational optimization tool was readily available. LANA [1] used for end-to-end simulations of the MSU/RIA linac [2] is well suited for precise particle tracking simulations in realistic 3D fields but does not have specific algorithm to optimize the longitudinal acceptance by adjusting the amplitudes and phases of the accelerating cavities. For this reason, a complementary code was developed.

DIMAD [3] has been used to design the transverse focusing lattice of the Michigan State University (MSU) RIA driver linac. It has fitting algorithms to adjust the focusing elements and obtain transverse beam matching and provide alignment and high-order aberration corrections. The new code provides similar capabilities in the longitudinal plane. For this, the second order terms effect on the longitudinal beam envelope are computed and specific tools regarding multi-q beam acceleration are included. Overall, the new code facilitates the cavity tuning procedure and improves the longitudinal performance of the accelerating lattice.

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TUNING CODE ALGORITHM

The energy gain of a particle going through an rf cavity depends non-linearly on the rf phase, and therefore the second order terms on the longitudinal beam envelope must be included in the tuning optimization. In the following, as a simplification, the problem is restricted to the longitudinal variables ($\delta\phi,\delta W$), where $\delta\phi$ and δW refer, respectively, to the phase and energy difference of a particle with respect to a given reference particle.

Assuming that the coordinates of a particle are ($\delta\phi_0,\delta W_0$) at the entrance and ($\delta\phi_1,\delta W_1$) at the exit of a cavity and assuming that for any particle the variation of the phase $\Delta\phi$ and energy ΔW through the structure are non-linear functions of the entrance phase ϕ and entrance energy of the particle, one can separate the linear (L) and non-linear (NL) contributions (e.g. write $\delta\phi_1 = \delta\phi_0 + \delta\phi_L + \delta\phi_{NL}$) and up to second order obtain

$$\begin{aligned}\delta\phi_L &= \frac{\partial\Delta\phi}{\partial\phi}\delta\phi_0 + \frac{\partial\Delta\phi}{\partial W}\delta W_0\\ \delta\phi_{NL} &\approx \frac{1}{2}\left(\frac{\partial^2\Delta\phi}{\partial\phi^2}\delta\phi_0{}^2 + \frac{\partial^2\Delta\phi}{\partial\phi\partial W}\delta\phi_0\delta W_0 + \frac{\partial^2\Delta\phi}{\partial W^2}\delta W_0{}^2\right)\end{aligned} \tag{1}$$

Similar equations hold for δW. The transfer matrix through an accelerating cavity can therefore be approximated by a [5x5] matrix in the vector basis ($\delta\phi,\delta W, \delta\phi^2, \delta\phi\, \delta W, \delta W^2$) with the form

$$R = \begin{pmatrix} R_L & R_{NL} \\ 0 & R_{L^2} \end{pmatrix} \tag{2}$$

where R_L is a [2x2] matrix representing the linear transformation linked to the first order derivatives, R_{NL} is a [2x3] matrix representing the second order corrections linked to the second order derivatives and where R_{L^2} is a [3x3] matrix obtained by "squaring" the terms of R_L

$$R_{L^2} = \begin{pmatrix} R_{11}{}^2 & 2R_{11}R_{12} & R_{12}{}^2 \\ R_{11}R_{21} & R_{11}R_{22} + R_{12}R_{21} & R_{12}R_{22} \\ R_{21}{}^2 & 2R_{21}R_{22} & R_{22}{}^2 \end{pmatrix} \tag{3}$$

where $R_{11}, R_{12}, R_{21}, R_{22}$ are the elements of the matrix R_L.

Transforming the particle coordinates using the matrix R of Eq. (2) gives accurate results only if the non-linear terms are small compared to the linear ones. This is understandable since the non-linear components of the vector basis, namely ($\delta\phi^2, \delta\phi\, \delta W, \delta W^2$), are only approximated by "squaring" the linear components. As a consequence, when particles go through multiple elements, the accuracy of the non-linear corrections can degrade significantly.

If the velocity of a particle remains approximately constant through a cavity, explicit analytic expressions for R_L and R_{NL} are known [4]. In cases where the velocity change through the accelerating element is significant, alternative methods can be considered [5].

It is known that the non-linear correction terms primarily modify the speed of rotation of the particles in the longitudinal phase space [6]. Hence, if the beam envelope Twiss parameters at the entrance of a cavity are matched so that the shape of the beam phase space ellipse is similar to the elliptical motion described by a single particle passing through the cavity, the severity of the emittance dilution caused by the non-linear terms is minimized. Therefore, it seems valuable to consider non-linear terms not only for the transformation of individual particles but also for the particle distribution as a whole.

Under Liouvillian linear transformations it is possible to show that a group of particles enclosed within an ellipsoid remains inside an ellipsoid of similar volume [7]. In the 2-D case, the emittance ε (i.e., phase space area=$\pi\varepsilon$) of an ellipse drawn around a group of particles is an invariant of such a transformation and the Twiss parameters α, β and γ (i.e., the coefficients of the ellipse) are not independent but linked through the relation $\beta\gamma-\alpha^2=1$. The matrix representing the transformation of the Twiss parameters through an element can be deduced from the elements of the transfer matrix R_L. Typically, writing X as the vector representing the particle phase space coordinates, one can write the transformation $X=R_LX$ and conclude that the transformation for the Twiss parameters is given by $\sigma_L=R_L\,\sigma_L R_L^t$ where the σ matrix elements are the coefficients of the ellipse ($\sigma_{11}=\beta$, $\sigma_{12}=\sigma_{21}=-\alpha$, $\sigma_{22}=\gamma$). For linear transformations, the equation describing the beam envelope is an invariant and is a second power bivariate polynomial. Assuming x and y are phase space coordinates, the equation is of the form $\gamma x^2+2\alpha xy+\beta y^2=\varepsilon$ (or $X^t\sigma^{-1}X=\varepsilon$ in matrix form).

It is straightforward to extend the invariance of form to non-linear transformations. For example, if second order terms are included in the transformation, the mathematical function that remains invariant is not dependent on the second power of the phase space coordinates but on the fourth power. Assuming x and y the variables of the phase space with a transformation of the form $x=ax+by+cx^2+dxy+ey^2$ and equivalent transformation for y, the equation invariant of form is the fourth power bivariate polynomial

$$\sum_{i,j=0}^{4}\eta_{ij}x^i y^j=0 \qquad \text{with } i+j\leq 4 \tag{4}$$

The η coefficients are therefore the equivalents of the Twiss parameters for non-linear transformations and one has $\eta_{00}=-\varepsilon$, $\eta_{20}=\gamma$, $\eta_{11}=2\alpha$, $\eta_{02}=\beta$ for the coefficients related to the linear transformation. Though the geometrical representation of the η coefficients is complicated, their computation is not and their magnitude can be used to quantify the distortion of the particle distribution ellipse. It is possible when computing the beam dynamics through accelerating cavities to calculate and minimize the η coefficients by using an optimizing procedure with cavity amplitudes and phases as variables. Finding the explicit dependence of the η parameters with respect to the elements of the transfer matrix R written in Eq. (2) is straightforward but too long to be reproduced here. The methodology follows the same logic to pass from the R to the σ matrix in the linear case. The new σ matrix is a [5x5] matrix with twelve linearly independent elements instead of a [2x2] matrix with three linearly independent elements (if the linear transformation is also unitary there are only two independent elements as mentioned above).

The algorithm of the optimizing routine is complicated by the fact that when setting the cavity amplitudes and phases other parameters such as the energy gain and relative position and mismatch of the different charge states envelopes must also to be taken into account. At the beginning of the RIA driver linac, minimization of the longitudinal emittance distortions is the most important. Due to phase damping, the non-linearities become less of an issue as the beam progresses down the linac and the setting of the cavities in the later part of a segment can be dedicated to the control of the multi-q effective emittance (i.e., the ellipse drawn around all charge states as illustrated in Fig. 1).

EXAMPLES USING THE NEW TUNING PROCEDURE

In the first linac segment, where the emittance distortions in the longitudinal phase space are the most problematic, the new code obtained solutions that minimized the longitudinal emittance growth throughout the segment. In Fig. 1, the longitudinal phase at the output of the first linac segment is shown for solutions obtained from an earlier tuning procedure and from the new tuning procedure described above. The overall linac layout used in both cases is the baseline lattice described in detail in [2]. The tuning algorithm helped to reduce the distortions of the longitudinal emittances and improve the relative matching of both charge state envelopes. In Fig. 2, the longitudinal emittance growth factor in the first linac segment of the driver linac is presented for various input values of the longitudinal emittance for a multi-charge states $U^{28+,29+}$ beam. Using the algorithm has improved the linear longitudinal acceptance of the segment. As a direct benefit, the tolerances for rf errors in the first linac segment have been relaxed from 0.25deg in phase and 0.25% in amplitude to 0.5deg and 0.5%, respectively, without loss of performance.

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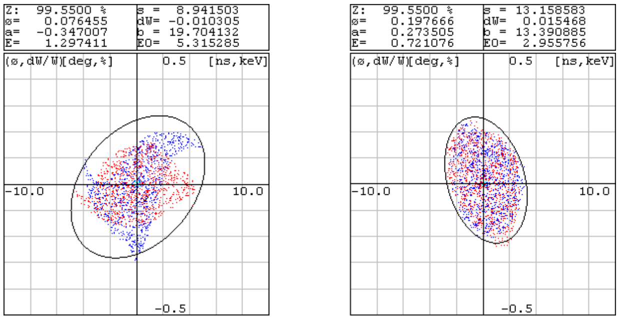

Figure 1: Longitudinal phase spaces at the output of RIA segment1 for $U^{28+,29+}$ beam with old cavity tuning procedure (left) and new tuning procedure using the algorithm (right). The multi-q effective emittance is plotted around all charge states.

The new code was used to evaluate alternative accelerating lattice designs and it was determined that modification of the first four cryomodules of the linac could increase the linear longitudinal acceptance by a factor three as illustrated in Fig. 2. In the baseline design, the drifts between cavities in the first linac segment are ~10cm and ~40cm alternatively with a focusing solenoid in the larger gap. Though such a design is simple, compact and was proven to satisfy the stringent beam loss requirements for RIA, it could be beneficial to modify the layout of the first four cryomodules by having all these drifts set to 40cm length.

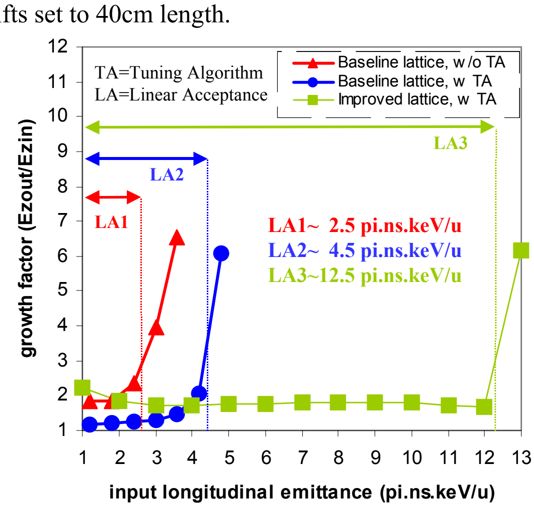

Figure 2: Longitudinal emittance growth factor for $U^{28+,29+}$ beam in segment 1 of RIA SC linac for various input emittance values. The growth factor is the ratio output emittance over input emittance.

At the same time, adding a few more cavities in the first two cryomodules (24 total instead of 18) would give more flexibility to achieve an optimum tuning, especially in case of cavity failure [8]. Because increasing the average drift length between consecutive focusing elements decreases the transverse phase advance, it would nevertheless be necessary to add more focusing solenoids to avoid emittance growth due to first order parametric resonance [9]. Using a focusing solenoid between every cavity in cryomodule number one and number three would be sufficient to avoid any emittance exchange.

The tuning procedure used in the first linac segment was also applied to the second segment with similar benefits. For the last linac segment, the tuning scheme proposed by A. Facco et al. [10] was implemented. In this scheme, the centroids of the different ions of a multi-q beam must be positioned at the entrance of a segment such that the product $q/A\cos\phi$ is the same for all of them. Under this condition, the energy gain in a cavity is the same for all centroids and the relative positions of all ion components remain unchanged along the segment. This scheme is extremely simple and well suited for the higher energy part of the driver linac where the non-linearities and the mismatch between the different ion envelopes in the longitudinal phase space are much less of a problem. To position each charge state component properly, the first few cavities of the segment were used. An optimizing algorithm was implemented in the code to find the amplitudes and phases of those cavities needed to achieve the desired beam centroids positioning.

CONCLUSIONS

A new code was developed at MSU to facilitate and improve the tuning procedure of the accelerating cavities for RIA. Since then, the capabilities have been extended to full 6D motion (envelope and particle tracking) and the code has already been used in other design studies [11,12].

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