COMPARISON OF OFF-LINE IR BUMP AND ACTION-ANGLE KICK MINIMIZATION*

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Abstract

The interaction region bump (IR bump) nonlinear correction method has been used for the sextupole and octupole field error on-line corrections in the Relativistic Heavy Ion Collider (RHIC). Some differences were found for the sextupole and octupole corrector strengths between the on-line IR bump correction and the predictions from the action-angle kick minimization. In this article, we compare the corrector strengths from these two methods based on the RHIC Blue ring lattice with the IR nonlinear modeling. The comparison confirms the differences between resulting corrector strengths. And the reason for the differences is found and discussed.

ACTION-ANGLE KICK MINIMIZATION

To minimize the action change for each order of IR nonlinear field error, it is equivalent to minimize the following quantities simultaneously [1],

$$\oint_L dsC_z c_n + (-1)^{n+1} \oint_R dsC_z c_n, \tag{1}$$

where L and R mean the left and right sides of the interaction region, z stands for x or y plane, c_n stands for the normal or skew field errors b_n or a_n, n is the field error order. C_z is the weight factor. Table 1 lists the weight factors for different field error b_n and a_n.

Table 1: Weight factors for action-angle kick minimization

Order	C_x	C_y	$k_n l$ $/k_{ns} l$	$(-1)^{n+1}$
b_2	$\beta_x^{3/2}$	$\beta_x^{1/2}\beta_y$	$k_2 l$	-1
b_3	β_x^2	β_y^2	$k_3 l$	1
b_4	$\beta_x^{5/2}$	$\beta_x^{1/2}\beta_y^2$	$k_4 l$	-1
b_5	β_x^3	β_y^3	$k_5 l$	1
a_2	$\beta_x\beta_y^{1/2}$	$\beta_y^{3/2}$	$k_{2s} l$	-1
a_3	$\beta_x^{3/2}\beta_y^{1/2}$	$\beta_x^{1/2}\beta_y^{3/2}$	$k_{3s} l$	1
a_4	$\beta_x^2\beta_y^{1/2}$	$\beta_y^{5/2}$	$k_{4s} l$	-1
a_5	$\beta_x^{5/2}\beta_y^{1/2}$	$\beta_x^{1/2}\beta_y^{3/2}$	$k_{5s} l$	1

The action-angle kick minimization assumes that the phase advances in the horizontal and vertical planes across the interaction point are close to π. And it ignores the phase advance in the triplet because of the small β^*. It assumes the beam is round beam, therefore, only the leading resonances in horizontal and vertical planes are corrected.

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IR BUMP CORRECTION

The IR bump correction method [2, 3, 4, 5] is an elegant way for the operational IR nonlinear corrections in a real machine. It creates a local horizontal or vertical orbit bump across the interaction region. The small tune shifts due to the bump are measured with a high resolution phase lock loop(PLL) tune measurement system. Since the relations between the tune shifts and the bump amplitudes are different for different orders of nonlinear errors, the IR bump correction is performed order by order by minimizing the polynomial fitting coefficients of the tune shifts.

For example, to correct the sextupole errors in the IR, we minimize the linear terms of the tune shifts from the horizontal IR bump with respect to the bump amplitude. To correct the octupole errors, we minimize the quadratic terms of the tune shifts from the horizontal IR bump with respect to the bump amplitude.

COMPARISON

Sextupole Correction

For the horizontal IR bump, the linear terms of the tune shifts are contributed from the sextupoles. We use two sextupole correctors bo7-sx3 and bi8-sx3 to minimize these linear term tune shifts in the Blue IR8. The three dipole kickers bo7-th4, bi8-th3 and bi8-th5 bump produce the desired IR bumps.

Table 2 gives the linear terms from the off-line IR bump simulations. The second block gives the residual linear term of the tune shifts from the IR bump in the IR8. The third block gives the two correctors' contributions to the linear terms with $k_2 l = 0.001$. Based on Table 2, the correction strengths for bo7-sx3 and bi8-sx3 are calculated to cancel the residual linear terms.

From IR bump simulation, the correction strengths for bo7-sx3 and bi8-sx3 are -4.54×10^{-3} m^{-2} and 2.74×10^{-3} m^{-2}, respectively. While from the action-angle kick minimization analytical calculation, based on the nonlinear optics model and Eq.(1), the integrated correction strengths for bo7-sx3 and bi8-sx3 are -3.99×10^{-3} m^{-2} and 2.97^{-3} m^{-2}, respectively. There is about a 10% difference in the correction strength of bo7-sx3.

Table 2: IR bump simulation for sextupole correction in Blue IR8.

Conditions	Plane	Linear term Coefficient
only b_2 errors	x	10.08×10^{-5}
	y	-9.26×10^{-5}
bo7-sx3 $k_2 l = 0.001$ m^{-2}	x	5.65×10^{-6}
	y	-1.43×10^{-5}
bi8-sx3 $k_2 l = 0.001$ m^{-2}	x	-2.74×10^{-5}
	y	1.01×10^{-5}

Octupole Correction

We use the two octupole correctors bo7-oct2 and bi8-oct2 to minimize the quadratic terms of the tune shifts from the horizontal IR bump in Blue IR8. The three dipole kickers bo7-th4, bi8-th3 and BI8-TH5 bump produce the desired IR bumps. Table 3 gives the residual quadratic terms from the octupole errors from the IR bump simulations. The correctors bo7-oct2 and bi8-oct2's contributions to the quadratic terms with $k_3 l = 0.001$ are given, too.

Table 3: IR bump simulation for octupole correction in Blue IR8.

Conditions	Plane	Quadratic Term coefficient
only b_3 errors	x	-1.68×10^{-7}
	y	6.40×10^{-8}
bo7-oct2 $k_3 l = 0.001$ m^{-3}	x	1.58×10^{-7}
	y	-8.74×10^{-8}
bi8-oct2 $k_3 l = 0.001$ m^{-3}	x	7.94×10^{-8}
	y	-1.43×10^{-7}

From the IR bump correction simulation, the integrated correction strengths for bo7-oct2 and bi8-oct2 are 0.121 m^{-3} and -0.029 m^{-3}, respectively. While from the action-angle kick minimization analytical calculation, based on the IR nonlinear modeling, the integrated correction strengths for bo7-sx3 and bi8-sx3 are 0.0768 m^{-3} and -0.023 m^{-3} , respectively. There are about 30% difference in the correction strengths of bo7-oct2.

ANALYSIS

From the off-line IR bump correction and the action angle minimization analytical calculation, we found that there are about 10% difference for the sextupole corrector strengths, and about 30% difference for the octupole corrector strengths. They verified the discripancies found in the sextupole and octupole correction strengths from the operational IR bump corrections and the off-line action-angle kicker minimization analytical calculations.

Here we check the ratios of the linear and quadratic terms of two individual sextupole's and octupoles, respectively. For sextupoles, the horizontal linear term should be proportional to $\beta_x^{3/2}$ for two sextupoles if they have the same integrated strength. From IR bump simulations, the ratio of the linear term of the horizontal tune shifts from two sextupoles bo7-sx3 and bi8-sx3 with the same integrated strength 0.01 m^{-2} is

$$\begin{aligned} &\Delta\nu_x|_{bo7-sx3} : \Delta\nu_x|_{bi8-sx3} \\ &= 5.29 \times 10^{-5} : 27.79 \times 10^{-5} \\ &= 1 : 5.25 \end{aligned} \tag{2}$$

The ratio of the $\beta^{3/2}$ of the two sextupoles are:

$$\begin{aligned} &\beta_x^{3/2}|_{bo7-sx3} : \beta_x^{3/2}|_{bi8-sx3} \\ &= 479.03^{3/2} : 1297.89^{3/2} \\ &= 1 : 4.46 \end{aligned} \tag{3}$$

Therefore, from the IR bump simulation, the ratio of the linear tune shift terms is not proportional to the ratio of $\beta^{3/2}$ for two sextupoles with the same integrated strength. This only reason for the inequality is

$$\begin{aligned} &x_{co}|_{bo7-sx3} : x_{co}|_{bi8-sx3} \\ &\neq \beta_x^{1/2}|_{bo7-sx3} : \beta_x^{1/2}|_{bi8-sx3}. \end{aligned} \tag{4}$$

This guess is verified by the following orbit bump check at these two sextupole correctors. From the IR bump simulation with MADX,

$$\begin{aligned} &x_{co}|_{bo7-sx3} : x_{co}|_{bi8-sx3} \\ &= 1 : 1.955, \\ &\beta_x^{1/2}|_{bo7-sx3} : \beta_x^{1/2}|_{bi8-sx3} \\ &= 1 : 1.646. \end{aligned} \tag{5}$$

If we substitute the orbit ratio of x_{co} instead of the ratio of $\beta_x^{1/2}$,, we get the horizontal tune shift contribution ratio from the two sextupoles:

$$\begin{aligned} &(x_{co}\beta_x)|_{bo7-sx3} : (x_{co}\beta_x)|_{bi8-sx3} \\ &= 1 : 5.30, \end{aligned} \tag{6}$$

which is much closer to the linear term tune shift ratio we get from the IR bump simulation.

Similarly, we check the quadratic term tune shift ratios of two individual octupoles bo7-oct2 and bi8-oct2 with the same integrated strength 0.001 m^{-3}. From the IR bump simulation, we get

$$\begin{aligned} &\Delta\nu_x|_{bo7-oct2} : \Delta\nu_x|_{bi8-oct2} \\ &= 1.58 \times 10^{-7} : 7.94 \times 10^{-8} \\ &= 1.996 : 1 \end{aligned} \tag{7}$$

However,

$$\begin{aligned} &\beta_x^2|_{bo7-sx3} : \beta_x^2|_{bi8-sx3} \\ &= 1042.11^2 : 577.37^2 \\ &= 3.261 : 1 \end{aligned} \tag{8}$$

The ratio of the tune shifts are not equal to the ratio of β_x^2 as assumed from Table 1, either. The ratio of orbit amplitudes at the two octupoles is:

$$\begin{aligned} &x_{co}|_{bo7-oct2} : x_{co}|_{bi8-oct2} \\ &= 1.051 : 1 \end{aligned} \tag{9}$$

Substituting x_{co} ratio instead of $\beta_x^{1/2}$ ratio to calculate the tune shifts due to octupoles, we obtain the horizontal tune shift contribution ratio from the two octupoles:

$$\begin{aligned}(x_{co}^2\beta_x)|_{bo7-sx3} : (x_{co}^2\beta_x)|_{bi8-sx3} \\ = 1.995 : 1\end{aligned} \tag{10}$$

which is almost the same as that from the IR bump simulation.

DISCUSSION

From the above calculation, we find the difference of the correction strengths from the IR bump correction and the action-angle kick minimization comes from the fact that the horizontal orbit is not exactly proportional to the $\beta_x^{1/2}$. The source for this difference is that the phase advance over the interaction region is not exactly equal to π and there is a small phase advance in the triplet.

Presenting the phase advance as $\Delta\Psi = \pi + \Delta\psi$ in another IR side, one can get the ratio of orbit positions on the left and right sides of the interaction region as

$$\begin{aligned}\frac{x_{co,L}}{x_{co,R}} &= \frac{\sqrt{\beta_{x,L}}\sin(\Psi_0)}{\sqrt{\beta_{x,R}}\sin(\Psi_0+\Delta\Psi)} \\ &\simeq -\frac{\sqrt{\beta_{x,L}}}{\sqrt{\beta_{x,R}}}(1-\cot(\Psi_0)\Delta\psi).\end{aligned} \tag{11}$$

Although $\Delta\psi$ is small, the $\cot\psi_0$ can reach 15 units for RHIC IR bump. It leads to considerable difference between the ratio of the orbits and the ratio of $\sqrt{\beta}_x$. Then the correction strengths from the IR bump correction are not the same as that from the action-angle kick minimization analytical prediction.

Action-angle kick minimization ignores the small phase change in the IR bump on both sides of the IP, while the IR bump correction method takes into account the phase shifts. Base on the Hamiltonian perturbation theory, sextupoles could introduce $Q_x = p$, $3Q_x = p$, $Q_x \pm 2Q_y = p$ resonances. To fully correct all the resonances, we should minimize all the following resonance strengths [7], that is,

$$\left\{\begin{array}{lcl}\sum_j k_2 l\beta_x^{1/2}\beta_y e^{i\Psi_x} & \longrightarrow & 0 \\ \sum_j k_2 l\beta_x^{3/2} e^{i\Psi_x} & \longrightarrow & 0 \\ \sum_j k_2 l\beta_x^{3/2} e^{i3\Psi_x} & \longrightarrow & 0 \\ \sum_j k_2 l\beta_x^{1/2}\beta_y e^{i(\Psi_x-2\Psi_y)} & \longrightarrow & 0 \\ \sum_j k_2 l\beta_x^{1/2}\beta_y e^{i(\Psi_x+2\Psi_y)} & \longrightarrow & 0\end{array}\right. . \tag{12}$$

Octupoles induce $4Q_x = p$, $4Q_y = p$, $2Q_x = p$, $2Q_y = p$, $2Q_x \pm 2Q_y = p$ resonances. To correct all the resonances, we should minimize all the following resonance strengths, that is,

$$\left\{\begin{array}{lcl}\sum_j k_3 l\beta_x^2 e^{i4\Psi_x} & \longrightarrow & 0 \\ \sum_j k_3 l\beta_y^2 e^{i4\Psi_y} & \longrightarrow & 0 \\ \sum_j k_3 l\beta_x\beta_y e^{i2\Psi_x} & \longrightarrow & 0 \\ \sum_j k_3 l\beta_x^2 e^{i2\Psi_x} & \longrightarrow & 0 \\ \sum_j k_3 l\beta_x\beta_y e^{i2\Psi_y} & \longrightarrow & 0 \\ \sum_j k_3 l\beta_y^2 e^{i2\Psi_y} & \longrightarrow & 0 \\ \sum_j k_3 l\beta_x\beta_y e^{i(2\Psi_x+2\Psi_y)} & \longrightarrow & 0 \\ \sum_j k_3 l\beta_x\beta_y e^{i(2\Psi_x-2\Psi_y)} & \longrightarrow & 0\end{array}\right. . \tag{13}$$

So the two methods, IR bump correction and the action-angle kick minimization, have different approximations in the betatron phase advance. Action-angle kicker minimization ignores the not exact π phase jump across the IP and the samll phase change in the one side triplet. The IR bump correction method uses a local orbit bump to minimize the introduced polynomial terms of tune shifts. However, the tune shifts from IR bump are proportional to $\sin^n\Phi$, instead of $e^{in\Phi}$ from Eqs. (12) - (13).

The action-angle kick minimization is used for IR nonlinear field correction off line up to b_{10} at CERN [8]. The IR bump correction is applicable for the on linear IR nonlinear field correction. It has been verified and used in the RHIC IR nonlinear corrections. Limited by the bump amplitude and the tune measurement resolution, it is generally used for the lower order nonlinear field error corrections.

CONCLUSION

The correction strengths from the off-line IR bump correction simulation are compared to that from the action-angle kick minimization. It verifies that there are some discrepancies in the correction strengths from these two methods. This is caused by the fact that the not exact π phase advance between the two sides of the interaction region, which makes that the bump orbit not exactly proportionally to the $\beta^{1/2}$. Both methods make different approximations in the betatron phase advances. The action-angle kick minimization is used for IR nonlinear field correction off line, while the IR bump correction is applicable for the on-line IR nonlinear field correction.

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