6-D BEAM DYNAMICS IN AN ISOCHRONOUS FFAG RING

F. Lemuet*, F. Méot†, G. Rees‡

Abstract

Numerical ray-tracing tools for 6-D tracking in FFAG accelerators have been developed. They are applied to the simulation of muon acceleration in the newly introduced *isochronous* type of FFAG ring designed for 16-turn, 8 to 20 GeV muon acceleration in the Neutrino Factory.

INTRODUCTION Fixed field alternating gradient (FFAG) synchrotron science is subject to a regain of interest [1] in the context of muon acceleration for the neutrino factory [2], with an increasing number of projects in various domains as hadrontherapy, high power beams, and other acceleration of ion beams [3].

Stepwise ray-tracing is considered, from the very beginning [4], a good technique to simulate particle motion in FFAG lattices, and allows drawing all necessary periodic machine parameters from single- or multi-turn tracking.

The numerical means involved here are based on such methods using the computer code Zgoubi [5]. These have earlier been successfully applied to scaling FFAG lattices, as described in Ref. [6] which can be referred to for details.

They are now applied to the study of an *isochronous* FFAG, based on a 5-magnet cell isochronous at all energies, as proposed for fast acceleration of muons in the neutrino Factory, from 8 to 20 GeV in 16 turns [7].

ISOCHRONOUS LATTICE By contrast with scaling FFAG lattices that feature constant tunes during acceleration, the *isochronous* FFAG is of the non-scaling type : tunes are allowed to change in the course of acceleration - detailed considerations on the concept of “non-scaling” can be found in recent literature [3].

Compared to non-scaling FFAGs based on linear magnets which are isochronous on a central energy (the phase slippage is zero only at first order in momentum), the *isochronous* type of lattice is based on non-linear optics, which allows fulfilling the requirement for isochronism $\gamma_t = \gamma$, at all γ. This insures best use of the RF, by avoiding the beam slipping in phase relative to the fixed RF frequency (201.2 MHz), at all energies, as in a cyclotron.

Isochronous designs have been sought (details can be found in [7]) that minimize the apertures of the superconducting magnets, employed to reduce the size of the ring. Three different magnet types are used in a cell of five magnets and 10.2 m length (Fig. 1). At 20 GeV, top energy, the cell acts like a bFDFb triplet with reverse b bends and positive bends in the F and D focusing units, while at the lower energies, the magnet gradients change gradually so

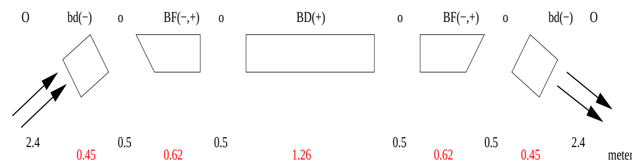

Figure 1: Isochronous cell.

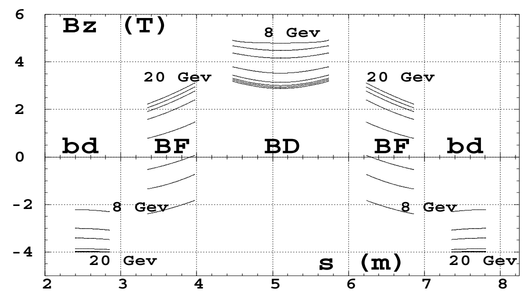

Figure 2: Magnetic field on closed orbits at various energies.

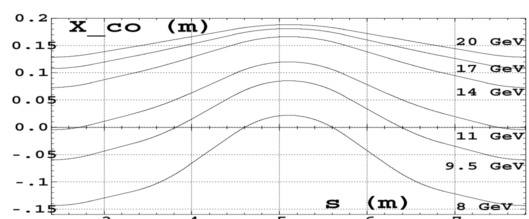

Figure 3: Closed orbits at various energies.

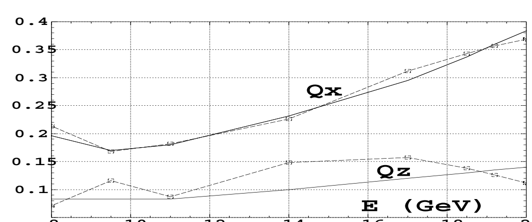

Figure 4: Cell tunes vs. energy obtained either from matrix method or (markers) from ray-tracing.

that, at 8 GeV, injection energy, the cell approximates a dFBFd triplet, with reverse bends in d and F units and positive B bends (Fig. 2). Orbit separations in the bd, BF and BD magnets can be seen in Fig. 3, with the lowest orbit spacing provided for the magnets of highest field and greatest length, the BD units. Each 4.8 m straight section has a 4.0 m free space between magnet cryostat ends, a length which is sufficient for the injection, extraction, vacuum, diagnostic and RF cavity systems. The gradual change of the cell tunes with energy is shown in Fig. 4.

---

* CERN AB/ABP (franck.lemuet@cern.ch)
† DAPNIA, CEA-Saclay (fmeot@cea.fr)
‡ RAL, UK (ghrees@ukonline.co.uk)

0-7803-8859-3/05/$20.00 ©2005 IEEE

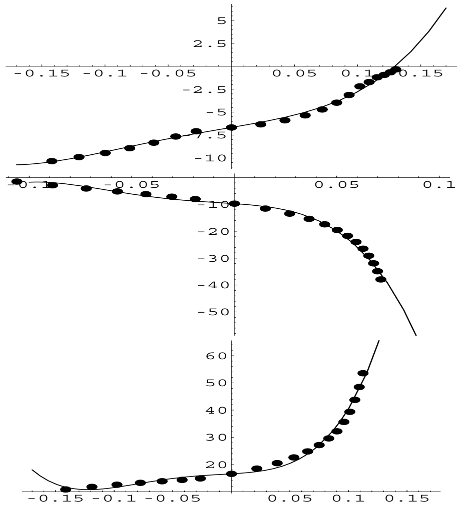

Figure 5: Field strengths (m^{-2}) as a function of transverse coordinate in, respectively, bd (top), BD (middle) and BF (bottom). The dots are the design data, the solid lines represent the ray-tracing 4th degree interpolation polynomials.

The circumference of the 3×41 cell ring is about 1254.6 m ; the multiple 3, in the number of cells, allows the possibility of resonant dispersion excitation studies.

FIELDS Dipole fields on a reference orbit, together with magnet gradients as a function of x-coordinate (Fig. 5), are the products of the above design studies, performed using matrix transport methods. 3-D field models then need be drawn for the ray-tracing purposes, which is performed as follows.

In the “d” and “D” type magnets, assumed rectangular, parallel face, the gradients are approximated using 4th degree polynomials,

$$K_{bd}(x) = -1.161 + 52.79\,x + 3349\,x^2 + 24707\,x^3 - 5.7710^6\,x^4$$
$$K_{BD}(x) = -1.930 - 168.4\,x - 10056\,x^2 - 2.110^6\,x^3 - 1.7210^8\,x^4$$

These strengths are then integrated to get the multipole coefficients of the field, $G^{(2q)}(s)$, as used in the ray-tracing code which assumes classical multipole dependence of the form (see Ref. [5] for details) $\vec{B} = \vec{\mathrm{grad}} V_n$ with

$$V_n(s,x,z) = \quad (n!)^2 \left(\sum_{q=0}^{\infty} \frac{(-)^q\, G^{(2q)}(s)(x^2+z^2)^q}{4^q q!(n+q)!} \right) \times \left(\sum_{m=0}^{n} \frac{sin(m\frac{\pi}{2}) x^{n-m} z^m}{m!(n-m)!} \right)$$

The “F” magnet is of sector type, with 360/123/2=1.463 degrees sector angle, with radial field dependence

$$B_{BF}(r) = \mathbf{b_{BF0}} + 16.56\,r + 12.61\,r^2 + 86.43\,r^3 + 2987\,r^4 + 13647\,r^5$$

from what, extrapolation off mid-plane yields the 3-D $\vec{B}$ model necessary for ray-tracing. This modelling also allows introducing field fall-offs (“fringe fields”).

This method yields good agreement between the design (matrix transport) machine parameters and the ray-tracing ones, like closed orbits, tunes, etc. Final adjustment of the ray-tracing parameters to the design ones, at better than the percent level, is performed by automatic fitting, using such variables as the dipole component of the d, F and D fields, and closed orbit positions.

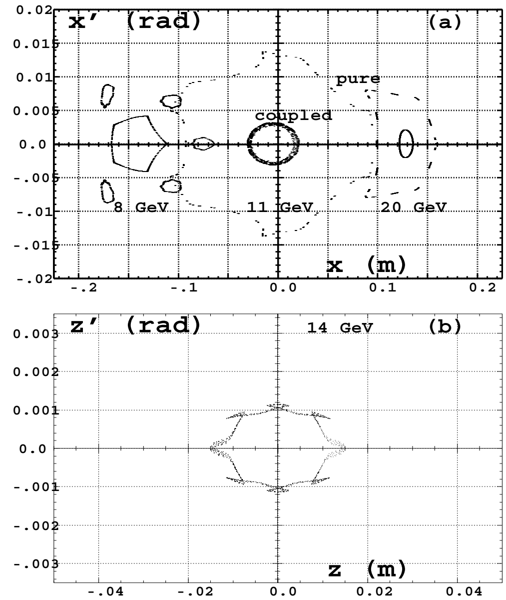

Figure 6: Stability limits. **(a)** in horizontal phase-space, at 8, 11 and 20 GeV, either pure (the largest amplitude) or with paraxial z component (“coupled”, the smallest amplitude). **(b)** vertical phase-space, at 14 GeV, with initial (x, x') taken on closed orbit.

STABILITY LIMITS Next to setting up ray-tracing data, the transverse motion stability limits are sought, by multi-turn ray-tracing at fixed energies, with two main goals : check the symplecticity of the motion over the all energy span (eventually considered to be satisfactory, see below), find the maximum stable amplitudes (which yields the maximum affordable initial beam emittances in further transmission computations). Sample results are shown in Fig. 6 : it can be seen (a) that the horizontal limit is strongly decreased by the presence of (even small) vertical motion ; the vertical motion (b) features coupling, because it induces horizontal motion around the closed orbit.

AMPLITUDE DETUNING Figs. 7, 8 give an idea of the (ν_x, ν_z) extent of the beam - this is the goal of further studies to work it out from an extensive Fourier analysis over the machine acceptance. The non-linear fields induce strong amplitude detuning, liable to have limiting effects on dynamic acceptance, and to induce beam loss at traversal of systematic resonances during acceleration.

BEAM TRANSMISSION A 10^4 particle beam is now launched for 16 turn acceleration (123 cells/turn), from 8 to 20 GeV. The particles injected are distributed at random with bell-shape transverse densities within the

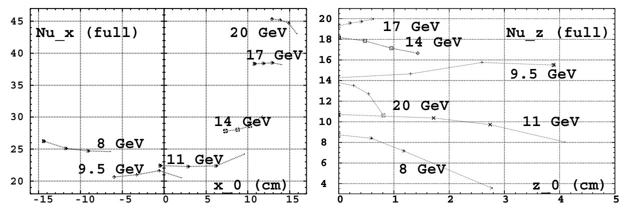

Figure 7: Amplitude detuning and its energy dependence.

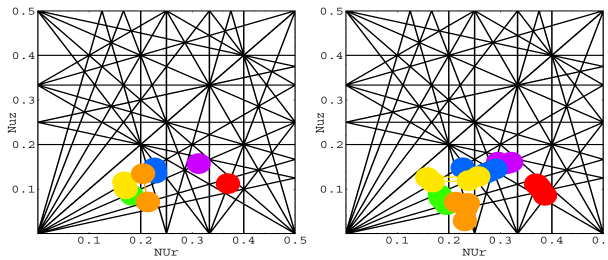

Figure 8: Representation of Fig. 7 in the tune diagram.

8 GeV machine acceptance (as discussed in the "STABILITY LIMIT" Section) and with 10^{-3} *rms* Gaussian momentum spread. The acceleration rate is 18.3 MV per cavity, one cavity every three cells. The phase slippage is not taken into account here : all particles receive the same longitudinal kick at cavity traversal.

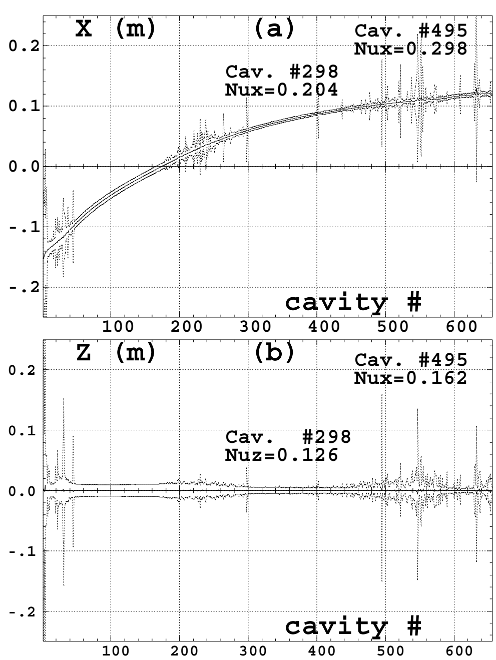

Figure 9: Beam centroid position and envelopes.

Fig. 9 displays tracking results under the form of the *rms* value of the x and of the z coordinates of the particle population, centered on the beam centroid, $\overline{y} \pm \sqrt{\overline{y^2} - \overline{y}^2}$ (y=x (Fig. a) or z (Fig. b)). Blow-up of σ_x and of σ_z can be observed at various energies in the course of acceleration, they correspond to particles experiencing large x or z coordinate, which are eventually lost : beam transmission ensues, more or less efficient depending in particular on initial emittances, resonance crossing speed, etc. Correlations with the cavity number (there are 41×16 cavity crossings) and with tunes are indicated in two cases of *rms* size blow-up - these occur at traversal of the $\nu_x - 2\nu_z = 0$ line ; additional details can be found in Ref. [8], deeper analysis is underway, including phase-space analysis, this will be subject of further reports.

Goal in this kind of tracking studies concern producing beam transmission, acceptance at injection, etc. These are crucial parameters in the optimization of muon accelerators in the Neutrino Factory.

Studies foreseen next are, • pursue resonance study, • use 201 MHz acceleration including RF phase, • introduce fringe fields (some assessments have been performed already, no strong effect is seen for the moment), • field and positioning defect studies, including again dynamic aperture and full cycle transmission.

CONCLUSION Similar simulations have been performed [8] on a 10-20 MeV electron model of an *isochronous FFAG* [7]. They are the subject of a poster at the present PAC05 Conference and will be published.

REFERENCES

[1] See, e.g., (a) The rebirth of the FFAG, M. Craddock, CERN Courrier 44-6 (2004) ; (b) FFAG synchrotrons, F. Méot, ICFA-HB2004 Workshop, Bensheim, 18-22 Oct. 2004.

[2] A feasibility study of a neutrino factory in Japan, KEK report, Feb. 2001.

[3] FFAG04 workshop, KEK, Tsukuba, http://hadron.kek.jp/FFAG/FFAG04_HP/index.html.

[4] O CAMELOT ! A Memoir Of The MURA Years (Section 7.1), F.T.Cole, Proc. Cycl. Conf, April 1994.

[5] (a) The ray-tracing code Zgoubi, F. Méot, NIM A 427 (1999) 353-356 ; (b) Zgoubi users' guide, F. Méot, S. Valero, CEA DAPNIA SEA-97-13/FERMILAB-TM-2010 (1997).

[6] Developments in the ray-tracing code Zgoubi for 6-D tracking in FFAG rings, F. Lemuet, F. Méot, NIM A (2005).

[7] An isochronous ring for muon acceleration, G.H. Rees, FFAG04 workshop, KEK (2004), http://hadron.kek.jp/FFAG/FFAG04_HP/menu.html.

[8] Beam transmission in isochronous FFAG lattices, F. Lemuet, F. Méot, FFAG Workshop 2005, Fermilab, April 2005, http://bt.pa.msu.edu/ffag/main.html.