# PERFORMANCE CALCULATION ON ORBIT FEEDBACK FOR NSLSII 

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## Abstract

We discuss the preliminary calculation on the performance of closed orbit feedback system for NSLSII, its relation to the requirement on BPM, floor and girder stability, power supply stability etc.

## A SIMPLE MODEL FOR ORBIT MOTION DUE TO QUADS VIBRATION

In a simplified model of the storage ring, when all the magnets are subjected to an uncorrelated random motion, they cause the closed orbit to move. To study this random motion, we first examine the effect of quads vibration on the vertical orbit. When a specific quad labelled as i with


strength $(\mathrm{kl})_{\mathrm{i}}$ experiences a displacement $\Delta \mathrm{y}_{\mathrm{i}}$, it gives a kick in the orbit and causes a displacement $y(z)$ of the orbit at position z. Sum over all the quads i gives:
$y(z)=\sum_{i} \frac{\sqrt{\beta(z) \beta_{i}}}{2 \sin \pi v} \cos \left(\pi v-\left|\varphi_{i}-\varphi(z)\right|\right)(k l)_{i} \Delta y_{i}$,
where $\beta_{i}, \phi_{i}, \beta(z), \phi(z)$ are the beta function and betatron phase at the quad i and at position $z$ respectively, $v$ is the vertical tune. Now we assign the displacement $\Delta y_{i}$ for all the quads with different random numbers from a gaussian distribution with the same rms value of $1 \mu \mathrm{~m}$, and calculate the orbit $y(z)$. For the NSLSII ring model in figure 1, averaged over 200 different sets of random numbers, the result is shown in figure 4 as the blue curve. We plot only two super-periods to see the details. As a comparison we plot the function $10 \sqrt{\beta_{y}} \mu \mathrm{~m}$ ( $\beta_{\mathrm{y}}$ is in meter) and found it is a good approximation. We define the rms value $\sigma_{\Delta y}$ of the resulting orbit movement divided by the rms value of the quads motion as the amplification factor for this model. It is a function of the distance $z$ along the ring. Obviously, the blue curve is just the

amplification factor, it varies between about 20 to 50 for this case. The fact that the curve is nearly periodic means that the 200 samples are nearly enough for convergence already. The beam motion is proportional to this function. For example, if the rms value of the quads vibration is 0.5 $\mu \mathrm{m}$, the orbit motion at the center of the straight section will be about $10 \mu \mathrm{~m}$ according to figure 2 .

## A FEEDBACK SYSTEM BASED ON THE SINGULAR VALUE DECOMPOSITION

A schematic diagram for a feedback system based on singular value decomposition [1](SVD) is shown in figure 3. In the figure, $y$ represents the signal from the BPMs used in the feedback system while $t$ represents the input signal sent to the corrector trims. R represents the
response matrix of the storage ring. SVD factorizes R into a product of matrices of form:

$$
\begin{equation*}
R=\tilde{U} W V \tag{2}
\end{equation*}
$$

where W is a diagonal matrix, while both U and V are orthogonal, i.e., $\tilde{U} U=\widetilde{V} V=1$. This means that in figure 3 the response matrix from the input signal $\mathrm{f}_{\mathrm{c}}$ of $\tilde{V}$ to the output signal $f_{e}$ of $W^{-1}$ is the identity matrix 1 . The PID box represents the feedback circuit that connects each individual channel of $f_{e}$ to a corresponding $f_{c}$ with a high negative gain amplifier. In the low frequency limit, the negative gain of the PID circuit is so high that it forces the $f_{e}$ to nearly zero. This in turn means that the orbit motion generated by the feedback system produces a signal in $f_{e}$ which exactly cancels the sum of the signal in $y$ generated by the noise due to the electron beam motion and the signal in $y$ generated by either the motion of BPMs inside the feedback loop or the noise generated by the BPM themselves. Thus we can write:

$$
\begin{equation*}
W^{-1} U R \widetilde{V} f_{c}=f_{c}=-W^{-1} U y_{0} \tag{3}
\end{equation*}
$$

where $y_{0}$ is the array of the signal from the BPMs in the feedback loop assuming the feedback loop is open. We remark that R may not be a square matrix and may not have a inverse matrix to be used to solve the equation, specially when the number of BPMs and correctors are not equal, hence the repetitive occurrence of $W^{-1} U$ in this equation shows how SVD is useful in solving the problem. When $y_{0}$ is known, this expression can be used to calculated the corrector signals $t=\widetilde{V} f_{c}$, given the response matrix R and its singular value decomposition $\mathrm{U}, \mathrm{V}$, and W . The array $\mathrm{y}_{0}$ is calculated by

$$
\begin{equation*}
y_{0 j}=\Delta y_{0 j}+\sum_{i} \frac{\sqrt{\beta_{j} \beta_{i}}}{2 \sin \pi v} \cos \left(\pi v-\left|\varphi_{i}-\varphi_{j}\right|\right)(k l) \Delta y_{i} \tag{4}
\end{equation*}
$$

where we have used eq.(1) to calculate the contribution from the noise generated by the quads as the second term on the right hand side, while the first term $\Delta y_{0 j}$ is the signal generated by the $j$ 'th BPM itself due to the vibration of the BPM itself and the electronic noise in the BPM. We remark that $\Delta y_{0 j}$ does not represent real orbit motion. Once the corrector strength vector $t$ is calculated, we can calculate the real orbit motion when the feedback loop is closed:

$$
\begin{align*}
& y(z)=\sum_{i} \frac{\sqrt{\beta(z) \beta_{i}}}{2 \sin \pi v} \cos \left(\pi v-\left|\varphi_{i}-\varphi(z)\right|\right)(k l)_{i} \Delta y_{i} \\
& +\sum_{k} \frac{\sqrt{\beta(z) \beta_{k}}}{2 \sin \pi v} \cos \left(\pi v-\left|\varphi_{i}-\varphi(z)\right|\right) t_{k} \tag{5}
\end{align*}
$$

where the first term is beam motion due to the vibration as calculated in the Equation 1 with the index i running through all the quads, while the second term is the beam motion due to the feedback signal $t$, with the running index k going through all the correctors.

For each set of gaussian random numbers for the quads vibration $\Delta y_{i}$ and $B P M$ vibration $\Delta y_{0 j}$ all with rms value of $1 \mu \mathrm{~m}$, we used Equation (4) to calculate open loop BPM signal $y_{0}$, then use Equation (3) to calculate the corrector vector $t$, and finally use Equation (5) to calculate orbit with the feedback loop closed. After averaging over 200 random samples, we obtain the residual rms beam motion as shown in the figure 2 , represented by the red curve. In this specific example, we use 4 BPMs and 4 correctors in each super-period. The location of them is marked by dark green and pink spots in figure 4 respectively. Two BPMs and two correctors are in the long straight sections next to the two quads QD1, hence they are close to the insertion devises. Two BPMs are in the long straight section but next to the short dipoles BS. Two correctors are in the short straight sections, also next to the short dipoles BS. Hence they are at the position with high vertical beta function. Notice that to be able to see the residual orbit we multiplied it by a factor 10 . The height of the pink spots represents the rms strength of the correctors in unit of $\mu \mathrm{rad}$. It is seen from figure 2 that the feedback loop reduces the beam motion at the center of the long straight section ( $\mathrm{z}=0$ ) from $20 \mu \mathrm{~m}$ to $0.7 \mu \mathrm{~m}$. The rms corrector strength is on the order of $0.7 \mu \mathrm{rad}$. To study how the performance of the feedback system depends on the number and position of BPMs and correctors, we carry out similar calculation for different configurations, one of the best is the one in figure 1. The amplification factor as we discussed in section 1, in this case of closed feedback loop and the feedback system configuration described by figure 1 , is then 0.7. That is, if we assume all the quads and BPM mounted on the girders have uncorrelated random motion of $0.7 \mu \mathrm{~m}$ and the BPM electronic noise is negligible, the residual motion at the centre of the long straight section is $0.49 \mu \mathrm{~m}$. Then the requirement for the vertical beam motion of $0.6 \mu \mathrm{~m}$ mentioned in section1 is satisfied. In figure 3, we plot the ratio of the vertical beam motion over the rms beam size as a function of z in the ring.


Figure 3
It is seen that although the condition of beam motion $10 \%$ of beam size at $z=0$ is satisfied (about $9 \%$ ), it is almost satisfied but still slightly higher at $z=15 \mathrm{~m}(12 \%)$. Hence we need to design the girder system and the floor construction such the ground movement rms value below
$0.7 \mu \mathrm{~m}$. In this calculation we ignored the fact that for low frequency ground motion, the movement of different components mounted on the girders may be correlated since the sound wavelength at low frequency may be larger than the girder dimension. Actually, simulation for correlated movement of quads mounted on same girder shows reduced amplification factor since the quads when moving together tends to cancel each other [2]. We also ignored the noise caused by the ripples in the power supply corrector magnet current. We also neglected the effect of stray field, such as the booster noise. However, we find that the fast feedback system is very efficient in reducing this type of effect, which is similar to the effect due to the quads vibration. Compared with this effect, the effect due to the vibration of BPMs is much more difficult to suppress. Actually, it is very difficult to reduce the beam motion to much less than the amplitude of the BPMs vibration amplitude. The beam motion due to quads and BPM vibration, the focusing effect in the insertion devices is found to be negligible. For example, a 5 m long undulator with $\mathrm{K}=1$ and period 3 cm is found to have $\mathrm{kl}=0.003 / \mathrm{m}$ at 3 GeV , this is negligible when compared with the typical kl value $0.3-1 / \mathrm{m}$ for the quads.

## ORBIT MOTION DUE TO POWER SUPPLY NOISE

For beam motion due to power supply noise, we add a third term to the equation similar to equation (4) to take into account the field errors $\Delta \mathrm{t}_{\mathrm{k}}$ of the correctors. In a digital feedback system, this error is determined by the voltage corresponding to the last bit of the power supply and the power supply current noise itself [2, 3]. To determine the requirement on the size of the last bit of the power supply, we need to separate this effect from the effect of the ground vibration in our simulation. For this purpose, we ignore the vibration or noise of the BPM and the quads and let the vector $\Delta \mathrm{t}_{\mathrm{k}}$ takes a Gaussian random distribution with rms value of 1 nrad . The resulting rms orbit motion averaged over 200 samples is shown in figure 4. Again, we use $16 \sqrt{\beta_{y}} \mathrm{~nm}$ to approximate the beam motion represented by the blue curve, but unit now

is nm . It is clear from this plot that to have the motion smaller than 300 nm rms at $\mathrm{z}=0$, the last bit rms error of the power supply must be less than 10 nrad kick. That is, for a trim with maximum strength of 1 mrad , the error of
the power supply last bit should be less than 10 ppm in order to have the beam motion caused by it to be less than $0.3 \mu \mathrm{~m}$, in the case of the feedback system used in figure 4. Since 0.29 of the size of the last bit is equal to its rms value, the last bit size is less than $10 \mathrm{ppm} / 0.29=33 \mathrm{ppm}$.

For the horizontal orbit motion, the calculation is similar, and gives specification to the tolerance on the noise of the dipole power supplies as well as correctors.

## REQUIREMENT ON FLOOR STABILITY

According to our analysis here, to satisfy the orbit stability requirement, the rms motion of the magnets and BPMs mounted on the girder must be less than $0.7 \mu \mathrm{~m}$. The short term (within an hour) ground motion is found to be about $0.5 \mu \mathrm{~m}$ peak to peak ( $\mathrm{rms} 0.12 \mu \mathrm{~m}$ ) near the future NSLSII site. It is difficult to use feedback system to correct the orbit to sub-micron level with the long-term (longer than a week) ground motion which is larger than a few $\mu \mathrm{m}$. But since beam lines can be realigned or recalibrated, and most experiments require short and medium term (between an hour and a week) sub-micron stability, this seems acceptable [2]. So the most stringent requirement is for the medium term. The closed orbit feedback system will pin the orbit to fixed positions relative to the BPMs, while the beamline samples have a distance from the BPMs unless they are in the feedback loop. According to the ATL law [4], these beamline samples will move relative to those BPMs used in the feedback loop. The relevant motion is then their motion relative to the two BPMs closest to them within the feedback loop. Assume the distance is typically 10 meters, by scaling from the NSLSII diameter of about 200 m to 10 m , and scaled from half year to one week, that the motion is reduced from about $100 \mu \mathrm{~m}$ to $4 \mu \mathrm{~m}$ by a factor of 25 . This is still much larger than our required stability of $0.6 \mu \mathrm{~m}$. Thus it is important to design the concrete slab such that the beam line and the two closest BPMs are on the same slab with stability better than 0.6 $\mu \mathrm{m}$ within a desired period. For example, if the requirement is no realignment within a week, then the slab must not move more than $0.6 \mu \mathrm{~m}$ within a week.

## REFERENCES

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