# RECTANGULAR DIELECTRIC-LINED TWO-BEAM WAKE FIELD ACCELERATOR STRUCTURE* 

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#### Abstract

A novel dielectric structure is described for a two-beam wake field accelerator which consists of three or four rectangular dielectric slabs positioned within a rectangular conducting pipe. This structure is equivalent to two symmetric, dielectric-lined, three-zone, rectangular waveguides, joined side-by-side. The design mode in the two-beam structure is the $\mathrm{LSM}_{31}$ mode, a combination of two symmetric $\mathrm{LSM}_{11}$ modes in each of the three-zone waveguides. This two-channel mode can be employed to decelerate drive particles in one channel and accelerate test particles in the other, without need for additional power extraction/transfer structures. It is possible to find structure parameters that in principle give a ratio $T$, of acceleration gradient for the test beam to deceleration gradient for the drive beam, even as high as $T=100$.


## INTRODUCTION

In this paper, a novel dielectric structure is described for a two-beam wake field accelerator (WFA) that goes beyond past WFA ideas because of its obtainable high ratio $T$ of acceleration gradient for the test beam, to deceleration gradient for the drive beam. The ratio $T$, which is termed here the "transformer ratio" is close to two in the classic wake field model [1], and is usually not greater than about five, even using sophisticated step-up couplers [2]. For accelerated particles to reach a final energy $W_{f}$ in a two-beam accelerator, the number of accelerator sections $N$ is given by the simple expression $N \geq W_{f} /\left(T W_{d}\right)$, where $W_{d}$ is the energy loss per section by each particle in the drive beam. Thus a higher value of $T$ can lower the number of sections, thereby leading to reduced complexity and cost. Another distinctive feature of the structure described here is that rf power produced by the drive beam is directly coupled into the acceleration channel, without need for an array of coupling structures, as for example in CLIC [3]. Moreover, rectangular geometry allows slots to be cut in the top and bottom walls, centered in each channel, both for suppression of undesired modes and for continuous vacuum pumpout.

## BASIC PRINCIPLE

The dielectric two-beam WFA structure described here is shown in cross-section in Fig. 1. It consists of four

[^0]rectangular dielectric slabs positioned within a rectangular conducting pipe. This structure can be thought of as equivalent to two symmetric dielectric-lined three-zone rectangular waveguides, joined side-by-side.

A single three-zone waveguide supports symmetric and anti-symmetric LSM and LSE modes, where the symmetry is that of axial electric field $E_{z}$ with respect to horizontal coordinate $x$ (perpendicular to the dielectric interfaces). The preferred accelerating mode for the single three-zone structure is $\mathrm{LSM}_{s 11}$ (the subscript $s$ signifies "symmetric"), in which the axial wake field $E_{z}$ reaches an extremum at the center of the structure. In the two-beam WFA structure, the design mode is $\mathrm{LSM}_{31}$, which is made up of two such $\operatorname{LSM}_{\text {s11 }}$ modes that have vanishing electric field components tangential to the interface (dashed line in Fig. 1) and the same axial wavenumber.


Figure 1: Dielectric two-beam WFA structure, with one vacuum channel for a decelerated (drive) beam and the other for an accelerated (test) beam.
Figure 2 shows the calculated dependence of drive beam channel half-width $a_{2}$ and transformer ratio $T$ on drive channel structure half-width $b_{2}$ for a two-beam WFA structure, which operates in the $\mathrm{LSM}_{31}$ design mode at a frequency of 102.816 GHz for a phase velocity equal to the vacuum light speed $c$. The frequency chosen is the third-harmonic of 34.272 GHz , at which the Yale/OmegaP magnicon could serve as the rf source to power the drive beam. The plots in Fig. 2 are for the case $a_{1}=0.5$ mm and $b_{1}=0.75 \mathrm{~mm}$ for the acceleration channel; the common waveguide half-height $d$ is taken to be 20 mm and the dielectric constants are both taken to be 5.7.

It is seen from Fig. 2 that $a_{2}=11.53 \mathrm{~mm}$ and $b_{2}=11.56$ mm when $T=10$, while $a_{2}=38.58 \mathrm{~mm}$ and $b_{2}=38.60$ mm when $T=100$. Of course, the drive and acceleration channels have the same dimension when $T=1$.


Figure 2: Dependence of drive beam channel half-width $a_{2}$ and transformer ratio $T$ on full drive channel half-width $b_{2}$ for a $102.816-\mathrm{GHz}$ two-beam WFA structure. Other parameters are as indicated in the figure.

## NUMERICAL EXAMPLE

A code has been developed for calculation of wake fields excited by rectangular rigid bunches in the structure described. Examples from analysis are presented here for a two-beam WFA structure with $T=10$.
Figure 3 shows the mode spectrum, excited by a train of ten identical $500-\mathrm{MeV}, 100-\mathrm{nC}$ electron bunches, with the first bunch located at $z=100 \mathrm{~cm}$, and with an inter-bunch period of $L_{b}=8.75 \mathrm{~mm}$ (corresponding to a 34.272 GHz injector). The bunch size is taken as $20 \times 10 \times 0.5 \mathrm{~mm}^{3}(1$ $\mathrm{nC} / \mathrm{mm}^{3}$ ), centered in the drive channel. A total of 50 waveguide modes are included in the analysis $\left(1 \leq n_{x}, n_{y} \leq\right.$ 5), 25 LSM and 25 LSE modes, noting that $\mathrm{LSE}_{\mathrm{m} 0}$ modes make no contribution. It is seen that the design $\operatorname{LSM}_{31}$ mode has the largest radiation power, namely 3.422 GW. Competing modes, in descending order of power, are the $\mathrm{LSM}_{13}$ (35.36 GHz, 926 MW$), \mathrm{LSM}_{11}(19.62 \mathrm{GHz}, 40.1$ MW), LSM $_{21}$ ( $88.51 \mathrm{GHz}, 30.5 \mathrm{MW}$ ), $\mathrm{LSM}_{23}$ (107.53 GHz, 17.1 MW), and $\mathrm{LSM}_{35}$ (193.61 GHz, 3.1 MW); others are completely negligible. These powers can be compared with the 1.714 TW beam power.
The radiation power into a given mode can be shown to be proportional to the bunch interference factor $C_{b} \equiv \sin ^{2}\left(N_{b} \xi\right) / \sin ^{2} \xi$, where $N_{b}$ is the number of bunches and $\xi=\pi \lambda_{m n} / L_{b}$ with $\lambda_{m n}$ the wavelength of the $m-n^{\text {th }}$ mode. Radiation into competing (nonsynchronous) modes from a periodic train of bunches will be suppressed if the ratio $\bar{C}_{b}=C_{b} / N_{b}^{2}$ is small enough. For example, the $\operatorname{LSE}_{11}$ mode ( 37.67 GHz ) is completely suppressed by the 10 -bunch train ( $\bar{C}_{b}=0.01$ ), while for a 5-bunch train ( $\bar{C}_{b}=0.5$ ) its power is still $11 \%$ that of the $\mathrm{LSM}_{31}$ design mode. The competing $\mathrm{LSM}_{13}$ mode at 35.36 GHz , has $\bar{C}_{b}=0.7$ for a 10-bunch train, $\bar{C}_{b}=0.23$ for a 20 -bunch train, and 0.007 for a 30 -bunch train.

Figure 4 shows the dependence of axial wake force $F_{\mathrm{z}}$ on $x$ for the design $\mathrm{LSM}_{31}$ mode for a total of 50 modes superimposed, for a test particle located at $y=0$ (the structure mid-plane) and $z=91.25 \mathrm{~cm}$ (namely a distance of 30 wavelengths of the $\mathrm{LSM}_{31}$ mode from the center of


Figure 3: Mode radiation power spectrum, excited by a train of ten rectangular $500-\mathrm{MeV}, 100-\mathrm{nC}$ bunches with a spacing separation of $L_{b}=8.75 \mathrm{~mm}$.


Figure 4: Axial wake field force $F_{\mathrm{z}} v s x$ for the design $\mathrm{LSM}_{31}$ mode and for a superposition of 50 modes.


Figure 5: Axial wake field force $F_{z}$-profiles in acceleration and drive channels. Note scale difference.
the first drive bunch). The acceleration gradient in the acceleration channel is $223.5 \mathrm{MeV} / \mathrm{m}$, while the deceleration gradient is $22.2 \mathrm{MeV} / \mathrm{m}$ in the drive channel. These values imply that the 500 MeV drive beam could travel about 22 m , during which an accelerated bunch
could gain about 5 GeV . It is seen from Fig. 5 that the total $F_{z}$ deviates somewhat from that of the pure $\mathrm{LSM}_{31}$ mode because of multimode interference, where the deviation results mainly from the $\mathrm{LSM}_{13}$ mode.

It should be indicated that the design $\mathrm{LSM}_{31}$ mode is different from all the other interfering modes in that it has no wall current at the center of the top and bottom walls of the drive channel $\left(J_{x}=0\right)$. Therefore, slots may be cut along these positions to suppress undesired modes.

Figure 6 shows the dependence of axial wake field forces in the centers of the drive and acceleration channels for the design $\mathrm{LSM}_{31}$ mode. The first bunch is positioned at $z=100 \mathrm{~cm}$ and the $10^{\text {th }}$ bunch is located at $z=92.12$ cm . The force amplitudes jump after every three periods where the ten drive bunches are injected, but after the $10^{\text {th }}$ bunch they remain constant. If a bunch with a size of 0.6 $\mathrm{mm} \times 0.6 \mathrm{~mm} \times 0.2 \mathrm{~mm}$ is located at the $z=91.25 \mathrm{~cm}$ peak in the acceleration channel, the maximum transverse wake field force in the bunch will be $0.04 \mathrm{MeV} / \mathrm{m}$ while the maximum gradient is about $224 \mathrm{MeV} / \mathrm{m}$.


Figure 6: Axial wake field force $F_{z} v s$ axial distance $z$ in drive and acceleration channels for the design $\mathrm{LSM}_{31}$ mode, when excited by ten $500-\mathrm{MeV}, 100-\mathrm{nC}$ bunches.

## GRADIENT SCALING

The gradient in the acceleration channel falls with an increase in the transformer ratio $T$ for a given bunch charge, since the width of the drive channel must increase with $T$. Figure 7 shows the dependence of gradient $G_{q}$ in the acceleration channel, produced by a unit charge in the drive channel, on transformer ratio $T$ for different half waveguide heights $d$. The plots are drawn by keeping the full acceleration beam channel width $2 a_{1}=1 \mathrm{~mm}$, while setting $\Delta x_{b} /\left(2 b_{2}\right)=0.64, \Delta y_{b} /(2 d)=0.25$, and $\Delta z_{b}=0.2$ mm , where $\Delta x_{b}, \Delta y_{b}$, and $\Delta z_{b}$ are, respectively, the bunch width, height, and length. It is seen that $G_{q}$ increases as $d$ decreases for a given $T$, since the two-beam structure becomes more compact. However, a smaller $d$ leads to less uniformity of axial wake field forces in beam channels and stronger transverse wake field forces, which could be harmful to the stability of the bunch.

Since the ratio $d / a$ in the drive channel falls with an increase in $T$, the drive bunch has considerable effect on the gradient in the acceleration channel. Figure 8 shows the dependence of the gradient $G_{q}$ on the normalized
bunch width $\Delta x_{b} /\left(2 b_{2}\right)$ for $T=10$ and $T=100$ cases, which have the same $d=20 \mathrm{~mm}$. The plots are drawn by keeping $\Delta y_{b} /(2 d)=0.25$. It is seen that the two plots have similar trends, but at different gradient levels. For the $T=10$ case, $G_{q}=0.2128 \mathrm{MV} / \mathrm{m} / \mathrm{nC}$ when $\Delta x_{b} /\left(2 b_{2}\right)=$ 0.1 , while $G_{q}=0.2367 \mathrm{MV} / \mathrm{m} / \mathrm{nC}$ when $\Delta x_{b} /\left(2 b_{2}\right)=0.9$, only different by $11 \%$. For $T=100$ case, $G_{q}=0.0223$ $\mathrm{MV} / \mathrm{m} / \mathrm{nC}$ when $\Delta x_{b} /\left(2 b_{2}\right)=0.1$, while $G_{q}=0.0613$ $\mathrm{MV} / \mathrm{m} / \mathrm{nC}$ when $\Delta x_{b} /\left(2 b_{2}\right)=0.9$, different by a factor of up to 2.75. That is because $d$ is the same and the structure with a smaller $T$ has a smaller drive beam channel width, leading to a larger ratio of $d / a$ and a more uniform axial electric field in the drive channel. Calculations also show that the $\mathrm{LSM}_{21}$ mode and the design $\mathrm{LSM}_{31}$ mode have nearly identical frequencies as $T$ approaches 100 ; here slot $\mathrm{LSM}_{21}$ suppression is required. These considerations show that compromise is needed between bunch stability, mode suppression, transformer ratio, and acceleration gradient, when designing a two-beam WFA structure.


Figure 7: Dependence of gradient $G_{q}$ in the acceleration channel produced by a unit charge in the drive channel on transformer ratio $T$ for different half waveguide heights $d$.


Figure 8: Dependence of gradient $G_{d}$ on normalized bunch width $\Delta x_{b} /\left(2 b_{2}\right)$ for $T=10$ and $T=100$, both with $d=20 \mathrm{~mm}$.

## REFERENCES

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