3D METALLIC LATTICES FOR ACCELERATOR APPLICATIONS

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Abstract

We present the results of our recent research on 3D metallic lattices operating at microwave frequencies, with applications to advanced accelerating structures, radiation sources based on the Smith-Purcell (SP) effect, and SP-based microbunch diagnostics. Electromagnetic waves in a simple 3D cubic lattice formed by metal wires are investigated using HFSS. The bulk modes in the lattice are determined using single cell calculations, with phase advances in all three directions. The Brillouin diagram for the bulk modes of the cubic wire lattice is calculated. The recently predicted "plasmon" and "photon" modes are identified in the simulations. Surface modes at the vacuum/wire lattice interface are identified, and their dispersion relation (the frequency vs. the wave number along the interface) is studied. The surface mode profiles clearly demonstrate that the wire lattice acts as a negative dielectric constant material.

INTRODUCTION

Photonic crystals, particularly, three-dimensional metallic and dielectric lattices have been intensively studied during past years because they open new possibilities for guiding and control of light propagation [1]. In the microwave frequency range, photonic (electromagnetic) crystals have applications as accelerator structures and microwave tube circuits. Recently, we have conducted experiments on a 17 GHz photonic band gap accelerator structure [2]. This paper presents a basic study of a photonic crystal shaped as a 3D array of intersecting metal wires shown in Fig. 1. Such crystal has an important accelerator application as beam diagnostic: the Smith-Purcell radiation of the beam traversing above the crystal [3] can be measured and used to characterize the beam profile.

The "artificial plasma" model was described by Pendry et. al. [4]. As demonstrated in [4], a 3D wire lattice can model isotropic plasma with the plasma frequency

$$\omega_p = \frac{c}{d}\left(\frac{2\pi}{\ln\dfrac{d}{a}}\right)^{\frac{1}{2}} \qquad (1)$$

which depends on the wire radius a and lattice period d; c is the speed of light. Equation (1) is derived within the quasi-static approximation assuming both the wire radius

and lattice period much smaller than the wavelength, and also $a \ll d$. For frequencies $\omega < \omega_p$ the dielectric constant $\varepsilon = 1 - \omega_p^2/\omega^2$ is negative.

A surface mode decaying into both vacuum and the lattice can be observed at the crystal interface. The properties of the surface mode follow from the semi-empirical description of the wire lattice crystal as plasma. Particularly, the surface plasma resonance frequency is $\omega_p/\sqrt{2}$ at an infinitely large wave vector component along the plasma surface. This resonance corresponds to the plasma dielectric constant ε=-1. Because in all envisioned applications a charged beam is propagating outside of the wire lattice, understanding the surface wave properties (never rigorously verified for realistic wire lattices) is crucial.

In this paper we will calculate numerically the bulk modes in the 3D simple cubic lattice of wires. These simulations can be done for an arbitrary ratio of the wire radius (or the square wire width) to the lattice period. We also will characterize the surface waves on the interface of the simple cubic wire lattice. We find that the simple "artificial plasma" model of the wire lattice is not adequate for describing surface waves.
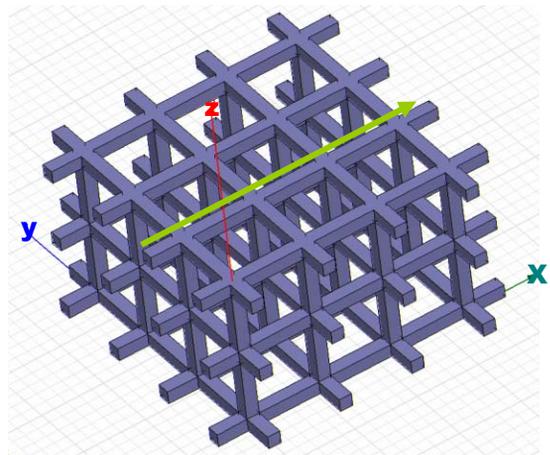

Figure 1: Metallic photonic crystal built as a cubic lattice of intersecting square wires. The green arrow represents an electron beam passing above the lattice.

BULK MODES

Fully electromagnetic simulations are carried out using High Frequency Structure Simulator (HFSS) developed by Ansoft Corp. A cubic cell of the 3D wire lattice

consists of three intersecting square wires of the width w and length d as shown in Fig. 2. The eigenmode solver of HFSS is utilized to determine the bulk modes. The ideal metal boundary conditions are set at the wire surfaces. The phase advance boundary conditions are set on three pairs of cube faces. Phase advances $\phi_i = k_i d$ (where $i=x,y,z$) are related to the wave number $\mathbf{k}=k_x\mathbf{e}_x+k_y\mathbf{e}_y+k_z\mathbf{e}_z$. All three phase advances are varied to calculate the complete Brillouin diagram.

According to the common notations for a simple cubic lattice, there are four points in the wave number $\mathbf{k}$ space which are vertices of the Brillouin zone: $\Gamma(0,0,0)$, $X(\pi/d,0,0)$, $M(\pi/d,\ \pi/d,0)$, and $R(\pi/d,\ \pi/d,\ \pi/d)$. In the HFSS simulations we vary the phase advance between one pair of faces from 0 to 180 deg. to go from Γ to X, then, keep the phase advance of 180 deg. between the planes $x=-d/2$ and $x=d/2$ and vary the phase advance between the planes $y=-d/2$ and $y=d/2$ from 0 to 180 deg., and so on. The reduced Brillouin zone is traversed as follows: Γ-X-M-Γ-R-X.
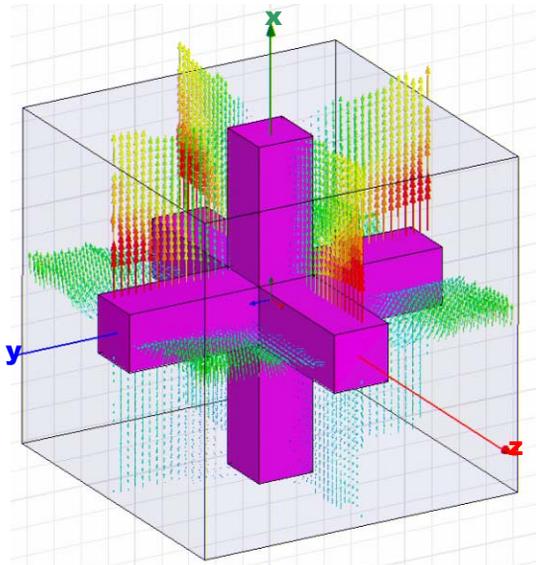

Figure 2: Cell of wire lattice, square cross section of width w=0.1 cm, period d=0.58 cm. The plasmon mode at 20.85 GHz is calculated for the phase advance of 60 deg. in the x direction, and 0 in the y and z directions. **E**-vectors are plotted in the x, y, z planes. Amplitude is ranged from 1 (red) to 0.1 (blue).

The lattice with the following parameters is analyzed: the lattice period d=0.58 cm and the square wire width w = 0.1 cm. The Brillouin diagram in Fig. 3 shows the frequencies of the modes calculated as functions of the phase advances. The plasma frequency for the lowest frequency propagation band, $f_p = \omega_p/(2\pi)$=20.5 GHz, is calculated at the Γ point of the Brillouin zone.

Figure 2 shows the electric field vectors and intensity map on the x, y, and z planes. It is an example of the lowest frequency mode in the wave vector region Γ-X. This mode at the frequency of 20.85 GHz is calculated at

the phase advances of (60 deg., 0, 0). This mode is classified as a plasmon: longitudinal mode with the electric field predominantly in the x direction of propagation. Emergence of a plasmon in a 3D wire lattice was recently described [5]. There are two degenerate photon modes [6] which are polarized transversely to x. If the wave vector varies in the region Γ-M, i.e. the phase advances in the x and y directions are equal, there is a longitudinal plasmon mode and two photon modes. One of the photon modes has the electric field in the x-y plane, and the other one has only the E_z component. The frequencies of the two photon modes are different, and degeneracy breaks which is indicated in the Brillouin diagram (Fig. 3). In the region Γ-R, there is a longitudinal, low frequency plasmon mode and two degenerate photon modes.
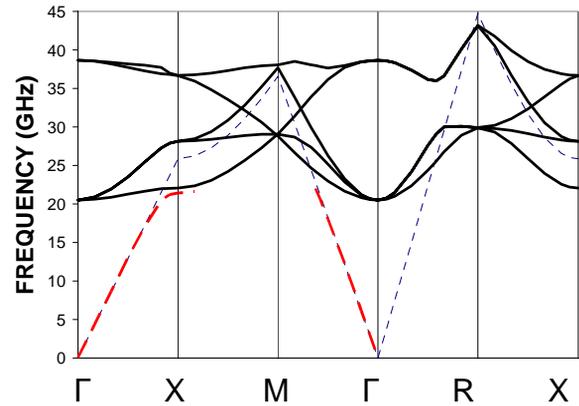

Figure 3: Brillouin diagram for the metallic wire lattice. The period d=0.58 cm, wire width w=0.1 cm. The free space dispersion lines are shown as blue dashed lines. The lower frequency mode is a plasmon mode. Red dashed line - the dispersion of the surface mode.

SURFACE MODE

The lattice interface is periodic in the x and y directions. We calculate the frequency of a surface mode as a function of the wave vector varying in the Γ-X-M-Γ region. According to the "artificial plasma" model [4], at small wave numbers, close to the Γ point, the phase velocity of the surface mode is equal to the speed of light. At large wave numbers, the frequency of the surface mode at the isotropic plasma boundary is predicted to asymptote to $f_p/\sqrt{2}$. For a 3D wire lattice with an infinitely small period d, the infinitely large wave number corresponds to the X point. Therefore, the surface mode frequency at the X point is $f_p/\sqrt{2}$ =14.49 GHz in the "artificial plasma" model.

To investigate electromagnetic surface waves that exist at the wire lattice/vacuum interface, the structure shown in Fig. 4 was analyzed using HFSS. The structure consisting of 5 vertically stacked cells accurately models a semi-infinite in z wire lattice. The structure is periodic in the x and y directions. Therefore, a semi-infinite

structure can be substituted by its single period shown in Fig.4. Phase-shifted periodic boundary conditions are set at the side faces. Similar to the bulk mode simulations, phase advances are varied between the pairs of side faces. Wires are assumed to be perfectly conducting. Tangential electric and magnetic fields are set to zero at the top and bottom boundaries of the simulation domain, respectively.

The surface mode frequency is calculated as a function of the phase advances ϕ_x and ϕ_y. Phase advances are varied from $\Gamma(0,0)$ to X(180 deg., 0), then to M(180 deg., 180 deg.), and back to Γ. The results are plotted in Fig.3 (dashed red line) as the Brilloun diagram.

Figure 4 plots the electric field distribution in the surface plasmon mode calculated at the frequency of 20.45 GHz for the phase advances of 150 deg. in the x- and 0 in the y direction. The surface mode is a slow wave with a phase velocity smaller than the speed of light. The surface mode found in our simulations does not asymptote to $f_p/\sqrt{2}$ =14.49 GHz. In fact, at the X point its frequency is significantly higher. The dispersion curve extends up to the plasma frequency of 20.5 GHz at which it intersects the dispersion curve of the bulk plasmon mode (Fig. 3), and similarly in the Γ-M region.
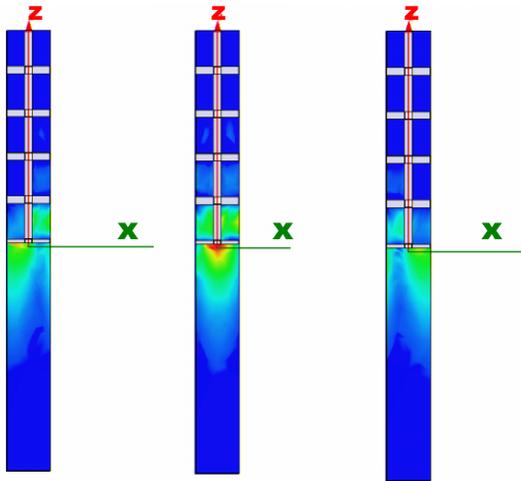

Figure 4: Stack of cubic cells and field intensity. The surface mode localized at the interface. The period d=0.58 cm, wire width w=0.1 cm. The surface mode is calculated at frequency of 20.45 GHz for the phase advance of 150 deg. in the x direction. The snapshots at ωt=-60 deg., 0, and 60 deg., respectively, show opposite directions of wave propagation in vacuum and the lattice. The field amplitude is ranged from 1 (red) to 0.1 (blue).

The reason for the disagreement between our simulations and the "artificial plasma" model is that the 3D wire lattice is medium with spatial dispersion, which is not taken into account in the isotropic plasma model.

Figure 4 shows the snapshots taken at different phases: ωt=-60 deg., 0, and 60 deg. The phase advance of

150 deg. is set in the x direction. The frequency of the surface mode is 20.45 GHz. The surface mode propagates in the x direction from left to right in vacuum below the crystal. The surface wave in the crystal propagates in the opposite direction. This indicates that the effective dielectric constant of the crystal is negative.

CONCLUSIONS

Bulk modes in a simple cubic 3D lattice built of metallic wires are rigorously calculated using HFSS. The lower frequency plasmon and photon modes are determined. The surface plasmon mode is found at the interface of the 3D wire lattice. The dispersion relation of the surface plasmon is found to be significantly different from that predicted by the "artificial plasma" model.

ACKNOWLEDGMENTS

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