THE EFFECTS OF ION MOTION IN VERY INTENSE BEAM-DRIVEN PLASMA WAKEFIELD ACCELERATORS

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Abstract

Recent proposals for using plasma wakefield accelerators in the blowout regime as a component of a linear collider have included very intense driver and accelerating beams, which have densities many times in excess of the ambient plasma density. The electric fields of these beams are widely known to be large enough to completely expel plasma electrons from the beam path; the expelled electrons often attain relativistic velocities in the process. We examine here another aspect of this high-beam density scenario: the motion of ions. In our analysis, for both cylindrically symmetric and flat beams, it is seen that for the proposed "afterburner" scenario the ions completely collapse inside of the electron beam. In this case the ion density is spikes, with a large growth in the beam emittance expected as a result. Particle-in-cell simulations of ion-collapse are presented. Implications of ion motion on the feasibility of the afterburner idea are discussed.

INTRODUCTION

The plasma wakefield accelerator (PWFA), driven in the blowout regime[1], where the beam is denser than the ambient plasma ($n_b > n_0$), has been the subject of much recent experimental and conceptual investigation. In the blow-out regime, the plasma electrons are ejected from the path of the intense driving electron beam, resulting in an electron-rarefied region. This region, containing only ions, possesses linear (in r) electrostatic focusing fields that allow high quality propagation of both driving and accel-erating beams. In addition, the electron-rarefied region has superimposed upon it longitudinal electromagnetic fields, which, because the phase velocity of the axisymmetric wake is nearly c, are independent of r. This wake may accelerate a trailing electron beam just as a traveling wave linac, with strong transverse focusing conveniently supplied by the plasma ions[3].

The condition $n_b > n_0$ is inherent in the blow-out regime. Indeed, for many scenarios of interest, the self-consisted driving beam density, as well as that of the accelerating beam, greatly exceed n_0. Under these circumstances, the beam's electric field is high enough that the ions may move significantly during the beam passage. In fact, for the parameters given by S. Lee, *et al.,* and quoted by Raubenheimer in his discussion of implementing a PWFA afterburner to boost the energy of linear collider, the ions violently collapse, as our analysis below shows. This collapse has serious implications for the preservation of the accelerating beam emittance,

effectively negating the claimed advantage of linear transport in the blowout regime.

Table 1: Beam and plasma parameters for linear collider afterburner, from Refs. 5 and 6.

N_b (drive, accel.)	3×10^{10}, 1×10^{10}
σ_z (drive, accel.)	63 μm , 31 μm
Norm. energy γ	5×10^5 (250 GeV)
Accel. beam $\varepsilon_{n,x(y)}$	4×10^{-6}, 4×10^{-8} m-rad
Drive beam $\varepsilon_{n,x}$	4×10^{-7} m-rad
Ion density n_0	2×10^{16} cm^{-3}
Ion charge state Z	1 (hydrogen)

ROUND BEAM ANALYSIS

Most previous analysis of the PWFA has been carried out under the assumption of cylindrical symmetry in the beam, and thus in the plasma response. We begin on this familiar ground. For the purposes of our analysis, we may invoke some well-known approximations to give the forms of the electrostatic fields that serve to focus both the beams and the ions. The first approximation is that the net force on the beam arises only from the ions' electrostatic fields. The second approximation is that the ions move only under the influence of the beam electrons' electric field, which do not "see" their own electric field because its force is nearly cancelled by their self-magnetic field. As the ions remain non-relativistic, they are unaffected by this magnetic field. These approximations are useful in approaching the analysis of both 2D and 3D effects in the beam-plasma-ion interaction.

We analyze the cases of the driving and accelerating beam in different ways. The driver, as it is not used directly in the linear collider experiment, may be considered to be axi-symmetric. On the other hand, the accelerating beam must, because of the demands of the final focus (beamstrahlung mitigation, etc.), have asymmetric emittances. We make use of form of the self-electric field inside such beams, and concentrate on the relevant vertical ion motion. The parameters we assumed for the driving and accelerating beams, most taken from Ref. 6, and thus derived from Ref, 5, are given in Table 1. For definiteness, we take the emittance of the axisymmetric drive beam, to be the geometric mean of the accelerating beam emittances $\varepsilon_{n,x}(\text{drive}) = \sqrt{\varepsilon_{n,x}\varepsilon_{n,y}}$.

The question of how to create such different driving and accelerating beams in ~100's of fs proximity will remain

unaddressed. We have also assumed that the ionized species is hydrogen, in order to avoid multiple ionization, and uncontrolled plasma formation inside the beam.

We begin with the case of an axisymmetric beam. The matched β-function in plasma is [1]

$$\beta_{eq} = \sqrt{\gamma / 2\pi r_e n_0} = \sqrt{2\gamma} k_p^{-1} \qquad (1)$$

where the density n_0 is the ion density and $r_e = e^2 / m_e c^2 \cong 2.82 \times 10^{-15}$ m. The matched beam area (assuming cylindrical symmetry) is thus

$$\sigma_x^2 = \beta_{eq} = \sqrt{\frac{\gamma}{2\pi r_e n_0}} \varepsilon_x = \frac{\varepsilon_{n,x}}{\sqrt{2\pi r_e n_0 \gamma}}, \qquad (2)$$

where $\varepsilon_{n,x}$ is the normalized horizontal emit-tance, $\varepsilon_{n,x} = \varepsilon_{n,y}$, beam velocity is taken as c.

The peak beam density is, for a bi-Gaussian beam distribution,

$$n_{b,0} = \frac{N_b}{(2\pi)^{3/2}\sigma_z\sigma_x^2} = \frac{N_b}{2\pi\varepsilon_{n,x}\sigma_z}\sqrt{r_e n_0 \gamma}. \qquad (3)$$

The electric field associated with this density near the axis, $r < \sigma_x$, is approximately linear in r,

$$E_r = -2\pi e n_{b,0} r = -\frac{eN_b}{\varepsilon_{n,x}\sigma_z}\sqrt{r_e n_0 \gamma} r. \qquad (4)$$

The ions' radial equation of motion is simply

$$\ddot{r} = \frac{ZeE_r}{Am_a} = -\frac{Ze^2 N_b}{Am_a\varepsilon_{n,x}\sigma_z}\sqrt{r_e n_0 \gamma} r, \qquad (5)$$

or in terms of $\xi = z - v_b t \cong z - ct$,

$$r'' = -\frac{Zr_a N_b}{A\varepsilon_{n,x}\sigma_z}\sqrt{r_e n_0 \gamma} r = -k_i^2 r, \qquad (6)$$

where, $r_a = e^2 / m_a c^2 = 1.55 \times 10^{-18}$ m is the classical radius of a singly charged ion of 1 amu, A is the atomic mass in amu, and $()' = d / d\xi$.

Equation 6 is a simple harmonic oscillator equation with spatial frequency

$$k_i = \sqrt{\frac{Zr_a N_b \sqrt{r_e n_0 \gamma}}{A\varepsilon_{n,x}\sigma_z}}. \qquad (7)$$

The solution of Eq. 7 is thus simply $r(\xi) = r(0)\cos(k_i\xi)$, the total phase advance of the ion motion in this potential is

$$\Delta\phi = k_i\Delta\xi \cong \sqrt{\frac{2\pi Zr_a N_b \sigma_z \sqrt{r_e n_0 \gamma}}{A\varepsilon_{n,x}}}. \qquad (8)$$

If we insert in the numbers for the drive beam from Table 1, we obtain $\Delta\phi = 15.7$, where we have approximated the beam as having line-charge $\lambda_b = dN/d\xi = N_b/\sqrt{2\pi}\sigma_z$ uniform over the length $\Delta\xi \cong k_i\sqrt{2\pi}\sigma_z$. Total collapse is at $\Delta\phi = \pi/2$, where a sharp ion density spike occurs; we are an order of magnitude past this disastrous increase in ion density.
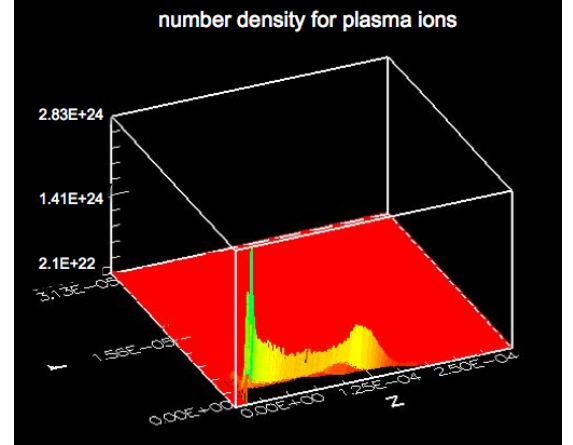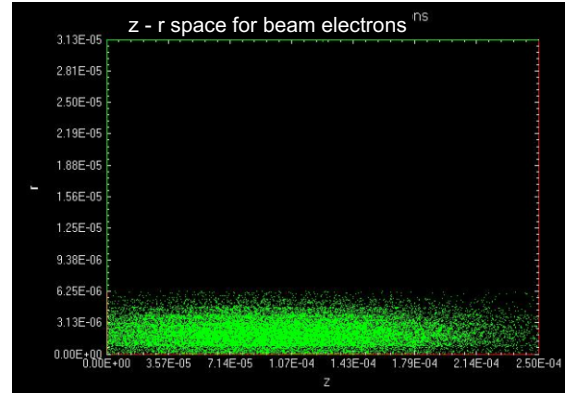

Figure 1: OOPIC simulation of ion collapse.

In order to illustrate the severity of ion collapse, we show the results of OOPIC self-consistent simulations in Fig. 1. The example parameters give $n_b/n_0 = 5 \times 10^4$, which causes instability issues in the simulations. The beam parameters used in the simulations lowered the beam density to $n_b/n_0 = 5 \times 10^3$, raising σ_r and N_b while keeping n_0, σ_z and γ constant. It is seen that, as expected the ions collapse even before the midpoint of the drive beam, with n_0 rising by over a factor of 100 on the axis.

Note that the numbers in Table 1 are based ultimately on Ref. 5, where the beam is assumed to be round, with $\sigma_x = 25$ μm, where in our example $\sigma_x \approx 175$ nm(!), determined self-consistently. The value of the ion focusing wave-number k_i is thus nearly 150 times larger than that deduced from the inconsistent case of Ref. 5. Even with $\Delta\phi \cong 0.1$ deduced for the Ref. 5 case, the motion of the ions is not negligible. This ion motion is also relevant to the accelerating beam. As the ions have time to move after drive beam passage, the ion perturbation is stronger inside the trailing beam.

FLAT BEAM ANALYSIS

The situation is more constrained for the accelerating beams, which have emittance and charge requirements set by the luminosity of the collider. For beams inside of a preformed (by the drive beam) cylindrically symmetric

ion channel, one may assume that the equilibrium β_x and β_y are equal and given by Eq. 1. Thus for the case in Ref. 6, the beam sizes $\sigma_{x,y}$ are a factor of $R=10$ different. Assuming the beam has elliptical symmetry, the transverse electric fields (E_x and E_y) are equal at the beam edges [7] ($y = \sigma_y$, $x = \sigma_x \equiv R\sigma_y$). Thus the ion motion components contributing to the perturbation in the ion density is predominantly vertical.

The vertical field inside of the beam core, in the linear approximation is

$$E_y = -\frac{4\pi e n_{b,0}}{(1+R)} y = -\frac{2eN_b \sqrt{r_e n_0 \gamma}}{\varepsilon_{n,y}\sigma_z(1+R)} y$$

$$\approx -\frac{2eN_b}{\sigma_z}\sqrt{\frac{r_e n_0 \gamma}{\varepsilon_{n,y}\varepsilon_{n,x}}} y, \quad R = \sqrt{\frac{\varepsilon_{n,x}}{\varepsilon_{n,y}}} \gg 1. \quad (9)$$

The linearized vertical equation (for ions initially inside $y < \sigma_y$, $x < \sigma_x$) of motion is

$$y'' = -\frac{2Zr_a N_b}{A\sigma_z}\sqrt{\frac{r_e n_0 \gamma}{\varepsilon_{n,y}\varepsilon_{n,x}}} y = -k_{i,y}^2 y. \quad (10)$$

The frequency of the ion motion is a factor of 2 larger than in the round beam case, if we assume for comparison that for the flat beam $\sqrt{\varepsilon_{n,y}\varepsilon_{n,x}}$ is equivalent to $\varepsilon_{n,x}$ in the round beam — this is the same as assuming the $n_{b,0}$ are identical. Thus in the flat beam scenario the ion problem is worse, and the phase advance in the Ref. 6 case is given by

$$\Delta\phi_y \cong \sqrt{\frac{4\pi Z r_a N_b \sigma_z}{A}\sqrt{\frac{r_e n_0 \gamma}{\varepsilon_{n,y}\varepsilon_{n,x}}}} \approx 9.1. \quad (11)$$

This is again unacceptably large, and should be mitigated by over an order of magnitude in order to preserve the accelerating beam quality.

CONCLUSIONS

One may ask if it is possible to choose parameters that ameliorate the ion motion problem. The most direct method would be to use smaller σ_z and N_b, and larger ε_n, as all these effects reduce $\Delta\phi$, if only as a square-root (note that other parameters have only fourth-root dependence). One may not give up N_b in the drive beam, without losing acceleration gradient, however. Further, we note that the bunch length σ_z chosen is already near the state-of-the-art, and one may not be able to make it

much shorter. On the other hand, the drive beam emittance ε_n may be made significantly larger, at the expense of ease in manipulating the beam; for example, if the emittance is too large, one may not easily compress the beam to shorter lengths. One must also then solve the problem of creating the large emittance driver in the presence of a low emittance trailing beam. The constraints of using the beam in the collider interaction point are much more serious for the accelerating beam. As one may not arbitrarily choose N_b, $\varepsilon_{n,x}$, or R in the trailing beam, it is not likely that the afterburner case discussed in Ref. 6 can be made feasible..

Thus there seem to be two options that may be pursued. The first is a complete redesign of the linear collider beam format to accommodate the ion motion problem, in which case the afterburner concept changes from and add-on to an inherent design constraint. This option is unlikely to succeed, however, given the severity of the ion collapse. A more radical solution would be to eliminate the ions altogether, using a hollow plasma capillary. This has already been proposed in the context of accelerating positrons [5], where the transverse wake is defocusing.

Unfortunately, the loss of ions in the beam path precludes ion focusing of the electrons. It thus also presents an obstacle to implementing another compelling aspect of the afterburner proposal — the use of plasma lens final focusing [8]. Preliminary studies indicate that such plasma lenses are also afflicted by ion motion problems. This issue is under current study. In addition, more detailed simulation studies of ion motion in both cylindrical and in flat (slab) geometries are being pursued.

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