ROBUST AUTORESONANT EXCITATION IN THE PLASMA BEAT-WAVE ACCELERATOR: A THEORETICAL STUDY*

A.E. Charman†, R.R. Lindberg, J.S. Wurtele, U.C. Berkeley, LBNL, Berkeley, CA 94720
L. Friedland, Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

Abstract

A modified version of the Plasma Beat-Wave Accelerator scheme is presented, which is based on autoresonant phase-locking of the nonlinear Langmuir wave to the slowly chirped beat frequency of the driving lasers via adiabatic passage through resonance. Compared to traditional approaches, the autoresonant scheme achieves larger accelerating electric fields for given laser intensity; the plasma wave excitation is more robust to variations in plasma density; it is largely insensitive to the details of the slow chirp rate; and the quality and uniformity of the resulting plasma wave for accelerator applications may be superior.

INTRODUCTION

The Plasma Beat-Wave Accelerator (PBWA) was first proposed by Tajima and Dawson [1] as an alternative to the short-pulse Laser Wake-Field Accelerator. In its original incarnation, two lasers co-propagating in an under-dense plasma are detuned from each other by a frequency shift close to the electron plasma frequency. The modulated envelope resulting from the laser beat resonantly excites a large-amplitude, high phase velocity plasma wave suitable for particle acceleration. Because plasma is not subject to material breakdown, accelerating gradients can be much higher than those of existing structures, and the requires acceleration length can be dramatically reduced.

Because the PBWA relies on resonant excitation, it is sensitive to the natural oscillation frequency of the plasma electrons, which is a function of both the plasma density (via the linear response at $\omega_p^2 = 4\pi e^2 n_0/m_e$ for a plasma with density n_0) and the amplitude (through relativistic effects for large amplitude waves). The latter of these effects was first studied by Rosenbluth and Liu [2], who showed that the longitudinal field E_z is limited by relativistic detuning to an amplitude given by the Rosenbluth-Liu limit:

$$E_{RL} \equiv E_0 \left[\frac{16\omega_p^2}{3\omega_1\omega_2} \frac{|E_1||E_2|}{E_0^2} \right]^{1/3} < E_0 \equiv \frac{mc\omega_p}{e}. \quad (1)$$

Here, $E_{1,2}$ and $\omega_{1,2}$ are the electric fields and frequencies of the two driving lasers, and E_0 is the cold, non-relativistic wave-breaking limit.

To overcome this detuning, Deutsch, Meerson, and Golub (DMG) [3] proposed a chirped beat-wave excitation scheme, where the laser frequencies are chirped downward starting from the linear resonance so as to compensate for the change in the nonlinear frequency of the growing plasma wave. However, the DMG scheme sometimes fails to produce appreciable phase-locking. Informed by more recent results, we elaborate on a novel variant of the chirped plasma PBWA concept which exploits autoresonant phase-locking via Adiabatic Passage Through Resonance (APTR) [4, 5]. Rather than chirping downward from the linear resonance, we start with a frequency shift well above the resonance, and then slowly sweep the beat frequency through and below resonance. With this approach, the final state of the plasma wave is insensitive to the exact chirp history, and the excitation is more robust to imprecise characterization of the plasma density.

* Work supported Depr. of High Energy Physics, U.S. DOE
† acharman@berkeley.edu

AUTORESONANT EXCITATION

Due to the limited room, we only sketch our derivations, highlighting relevant results; interested readers are directed to [6, 7]. Our study uses an approximate, but widely-used model of laser-plasma interactions. The plasma is treated as a fully relativistic Maxwell-fluid electron system in a stationary ion background. We restrict our analysis to one-dimension and make the quasi-static approximation, wherein we assume that the laser envelope does not appreciably evolve on the time scale of the interaction with the plasma electrons. In this approximation, the dynamics depend solely on the coordinate ξ moving at the laser group velocity v_g: $\xi \equiv t - z/v_g$. For a highly under-dense plasma, $\omega^2 \gg \omega_p$, the dimensionless electrostatic potential $\phi \equiv \frac{e}{mc^2}\Phi$ satisfies the quasi-static equation,

$$\frac{d^2\phi}{d\xi^2} = \frac{\omega_p^2}{2}\left[\frac{1+\vec{a}^2}{(1+\phi)^2} - 1\right], \quad (2)$$

with the normalized vector potential $\vec{a} \equiv \frac{e}{mc^2}\vec{A}_\perp$ given by

$$\vec{a} = \tfrac{1}{2}\left[a_1 e^{i\psi_1}\hat{e}_+ + c.c.\right] + \tfrac{1}{2}\left[a_2 e^{i\psi_2}\hat{e}_+ + c.c.\right], \quad (3)$$

Defining the beat amplitude $\epsilon \equiv a_1 a_2$, the average intensity $\bar{a}^2 \equiv (|a_1|^2 + |a_2|^2)/2$, and the beat phase $\psi \equiv \psi_1 - \psi_2$, (2) can be obtained from the Hamiltonian

$$\mathcal{H}(\phi, p; \xi) = \frac{p^2}{2} + \frac{\phi^2}{2(1+\phi)} + \frac{\bar{a}^2 + \epsilon\cos\psi}{2(1+\phi)}. \quad (4)$$

To transform (4) to action-angle coordinates useful for autoresonant analysis, we define the action as the phase space area in an unperturbed orbit $\mathcal{I} \equiv \frac{1}{2\pi}\oint d\phi\, p$. Furthermore,

0-7803-8859-3/05/$20.00 ©2005 IEEE

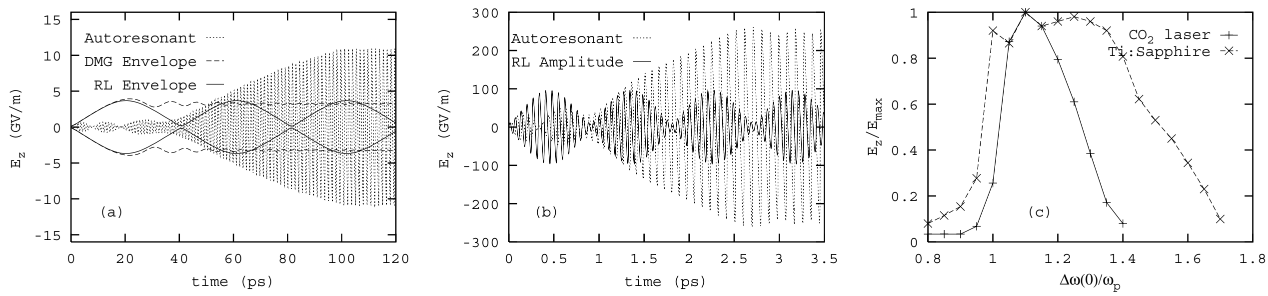

Figure 1: Plasma wave excitation for (a) 10μm CO_2 laser with intensity 2.7×10^{14} W/cm^2 ($\epsilon = 0.02$) and (b) 800 nm Ti:Sa laser with intensity 2×10^{17} W/cm^2 ($\epsilon = 0.09$). Both excite accelerating gradients greater than the nonrelativistic limit E_0 and than that without a chirp. (c) demonstrates robust field excitations of 5-10GeV/m for density errors of order $\pm 20\%$ for the CO_2 laser, and fields ~ 250 GeV/m for density errors of order $\pm 40\%$ for the CPA system.

since the potential is a periodic function of the canonical angle θ, we expand the last term in (4) as a Fourier series

$$\frac{\bar{a}^2 + \epsilon \cos\psi}{2(1+\phi)} = \left(\bar{a}^2 + \epsilon \cos\psi\right) \sum b_n(\mathcal{I}) e^{in\theta}. \quad (5)$$

We use a linear chirp such that $\partial_t \psi = \omega_p(1 - \alpha t)$ and define the phase difference $\Psi \equiv \theta - \psi$. Assuming that only the slowly varying terms are relevant ($n = 0, \pm 1$) we find

$$\mathcal{H} = \mathcal{H}_0(\mathcal{I}) - \omega_p(1 - \alpha\xi)\mathcal{I} + \epsilon \frac{\partial b_1}{\partial \mathcal{I}} \cos\Psi + \bar{a}^2 \frac{\partial b_0}{\partial \mathcal{I}}. \quad (6)$$

From this Hamiltonian, one can derive the conditions under which the phase Ψ is trapped about a stable equilibrium and autoresonance persists. In the linear regime, one can show that the chirp α must be smaller than a critical value α_c:

$$\alpha \leq \alpha_c \approx 0.169\epsilon^{4/3}. \quad (7)$$

As the plasma wave grows to large amplitude, nonlinear effects make autoresonance more difficult to maintain, and de-phasing occurs; the condition can be found in [6].

EXPERIMENTAL CONSIDERATIONS

Unfortunately, the PBWA does not have unlimited time to be excited, as instabilities will eventually destroy wave coherence. For cold plasmas and moderately intense lasers, the oscillating two-stream (or modulational) instability destroys plasma wave coherence for times approximately equal to $5/\omega_{pi}$, where ω_{pi} is the ion plasma frequency [8]. Thus, for a total drive frequncy shift equal to $\delta\omega$, the chirp rate is limited to $\alpha \geq 0.2\,\frac{\omega_p}{\omega_i}\delta\omega = 2.3 \times 10^{-3}\,\delta\omega$ for singly-ionized Helium.

Below, we choose two experimentally relevant parameter sets, one corresponding to a $10\,\mu$m CO_2 laser; the other, to a 800 nm Chirped Pulse Amplification (CPA) Ti:Sapphire laser system. We demonstrate how, beginning with the laser frequency above the linear resonance and then slowly decreasing it, one can robustly excite plasma waves to amplitudes larger than the cold, linear wave-breaking limit in times commensurate with onset of the oscillating two-stream instability.

CO_2 Laser at $10\,\mu$m

For parameters roughly corresponding to the CO_2 laser at UCLA [9]. We envision two pulses of duration 100 ps which enter the plasma at $t = 0$, central wavelengths near $\lambda = 10\,\mu$m, and normalized intensities $a_1 = a_2 = 0.14$, i.e., $\epsilon = 0.02$. We choose a linear chirp, so that the beat frequency is given by $\Delta\omega(t) = \omega_p(1.15 + \alpha t)$, with $\alpha = 0.00065$ chosen below the critical value $\alpha_c = 0.0009$ from (7). The results from simulations integrating the quasi-static equation 2 are in Fig. 1. 1(a) demonstrates the excitation of a uniform accelerating field E_z of 10 GV/m, which is above the wave-breaking limit of $E_0 \approx 8.8$ GV/m. The total chirp is modest, only about 1.5% of the laser carrier frequency. For comparison, we include the simulated envelopes of the longitudinal field for the resonant case $\Delta\omega(t) = \omega_p$, and for the chirped DMG scheme starting on linear resonance, $\Delta\omega(t) = \omega_p\,(1 - \alpha t)$. The resonant case demonstrates the characteristic RL limit of $E_z \leq E_{RL} \approx 4.2$ GV/m, whereas the DMG scheme fails to achieve appreciable phase-locking, and the final plasma wave amplitude is about the same as in the unchirped case.

An additional advantage is that autoresonant excitation is very robust with respect to experimental mismatches between the beat frequency and the plasma frequency. Because one sweeps over a reasonably broad frequency range and only need pass through the resonance, no precise matching is required. In Fig. 1(c), we plot the final accelerating gradient achieved via APTR when we vary the value of ω_p over a range of $\pm 10\%$, from its "design" value above, while keeping the laser parameters fixed. We see large levels of excitation for a wide range in plasma variation, corresponding to density mismatch/errors up to 20%.

Ti:Sapphire Laser at 800 nm

For a Ti:Sapphire CPA laser in a singly-ionized He plasma, with $n_0 = 1.4 \times 10^{18}$ cm^{-3}, so that $\omega_p/\bar{\omega} = 1/25$, and a laser duration $T = 3.2$ ps (chosen to correspond to the modulational instability limit). We imagine two 1 J pulses focused to a waist of $w_0 = 6\,\mu$m for intensities of $I_0 = 2.0 \times 10^{17}$ W/cm^2, so that, with $\lambda \approx 800$ nm, we

 0-7803-8859-3/05/$20.00

have $a_1 = a_2 = 0.3$, and $\epsilon = 0.09$. We choose the laser detuning $\Delta\omega(t) = \omega_p(1.2 - \alpha t)$, with $\alpha = 0.0025$. The resulting plasma wave excitation is shown in Fig. 1(b). We see maximum longitudinal electric fields $E_z \approx 260\,\text{GV/m}$, corresponding to $\sim 1.6\,E_0 \approx 0.25E_0$. For comparison, we also plot the resonant case, for which detuning results in maximum fields corresponding to the RL limit $(16\epsilon/3)^{1/3}E_0 \approx 125\,\text{GV/m}$. Furthermore, Fig. 1(c) shows the robustness of autoresonant excitation, for which density imperfections of $\pm 35\%$ have little effect on the accelerating gradients achieved.

ELECTRON ACCELERATION

For the purpose of matched particle injection, some phase-locking mechanism is desired. While the nonlinear frequency of the Langmuir wave may be closely entrained to the known drive frequency, this does not necessarily imply that the absolute phase of the Langmuir wave is known.

Autoresonant excitation may provide good frequency-locking either with comparably good or disappointingly inferior phase-locking. Nevertheless, the sample parameters used in the CO_2 example demonstrate the plasma wave phase can tightly lock with the drive phase, and remain so until adiabaticity is lost and the excitation saturates. We show these results in Fig. 2. In this case, phase-locked injection scheme could reliably place electron bunches near the maximum acceleration gradient. It appears that the extent of this phase error is determined by how deeply the oscillator phase is trapped in its effective potential, which improves with greater drive strength, slower chirp rate, and initial detuning further above resonance.

A consideration is how extending our one-dimensional analysis to multiple dimensions might affect results. One point that needs to be specifically addressed is the additional Gouy phase shift ϕ_g incurred by a Gaussian mode of finite Rayleigh range z_R. Because this phase shift is a function of z, $\tan\phi_g = z/z_R$ for the lowest order, vacuum Gaussian mode, one may worry that this phase shift will destroy the ξ-only dependence essential for autoresonance, possibly destroying phase locking near the laser focus. Fortunately, in the case of two lasers we will find that the respective Guoy phases cancel out of the ponderomotive beat $\sim a_1 a_2^*$. Thus, if the two lasers have similar focal properties, we expect autoresonance to drive the plasma wave to high amplitudes.

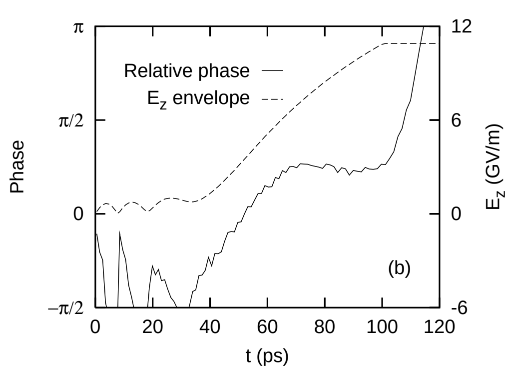

Figure 2: Evolving phase difference and E_z for the CO_2 laser parameters of Fig. 1. For t between 60 and 100 ps there is a constant phase difference between the lasers and accelerating field, permitting control of electron injection.

CONCLUSIONS

We have introduced a straightforward, seemingly minor, modification of the DMG scheme for the chirped-pulse PBWA, based on the nonlinear phenomenon of autoresonance, which nevertheless enjoys certain advantages over previous approaches. By beginning the excitation above resonance and sweeping the beat frequency downward sufficiently slowly, the plasma wave frequency automatically self-locks to the drive frequency, so that the plasma wave amplitude adjusts itself consistent with this nonlinear frequency. This autoresonant excitation achieves higher plasma wave amplitudes, is much more robust to inevitable variations in plasma and laser parameters, and appears to provide an avenue toward timed bunch injection. The most obvious drawback to the APTR scheme, in comparison to the standard approach, is the added complication of a chirped drive. But the total chirp required is modest, on the order of only ω_p, typically corresponding to only a few percent of the laser carrier frequency. These results warrant extending investigations to higher-dimensional geometries, including self-consistent laser evolution, and using fluid and particle-in-cell codes.

REFERENCES

[1] J. Tajima and J.M. Dawson, “Laser electron accelerator”, Phy. Rev. Lett. 43 (1979) 267.

[2] M.N. Rosenbluth and C.S. Liu, “Excitation of plasma waves by two laser beams”, Phys. Rev. Lett. 29 (1972) 581.

[3] M. Deutsch, B. Meerson, and J. Golub, “Strong plasma wave excitation by a chirped laaser beat wave”, Phys. Fluids B 3 (1991) 1773.

[4] L. Friedland, “Spatial autoresonance: Enhancement of mode conversion due to nonlinear phase locking”, Phys. Fluids B 4 (1992) 3199.

[5] E. Grosfeld and L. Friedland, “Spatial control of a classical electron state in a Rydberg atom by adiabatic synchronization”, Phys. Rev. E 65 (2002) 046230.

[6] R.R. Lindberg, A.E. Charman, J.S. Wurtele, L.Friedland, “Robust autoresonant excitation in the plasma beat-wave accelerator”, Phys. Rev. Lett. 93 (2004) 055001.

[7] R.R. Lindberg, A.E. Charman, J.S. Wurtele, L.Friedland, arXiv:physics/0403081.

[8] P. Mora et. al., “Modulational instability and its consequences for the plasma beat-wave accelerator”, Phys. Rev. Lett. 61 (1988) 1611.

[9] C.E. Clayton et. al., “Second generation beat-wave experiments at UCLA”, Nucl. Instr. Meth. A 410 (1998) 378.