BEAM-BEAM SIMULATIONS FOR DOUBLE-GAUSSIAN BEAMS *

C. Montag, N. Malitsky, I. Ben-Zvi, V. Litvinenko,
Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

Electron cooling together with intra-beam scattering results in a transverse distribution that can best be described by a sum of two gaussians, one for the high-density core and one for the tails of the distribution [1]. Simulation studies are being performed to understand the beam-beam interaction of these double-gaussian beams. Here we report the effect of low-frequency random tune modulations on diffusion in double-gaussian beams and compare the effects to those in beam-beam interactions with regular gaussian beams and identical tune shift parameters.

BEAM DYNAMICS CONSIDERATIONS

The beam-beam tuneshift parameter ξ for round gaussian beams is defined as

$$\xi = \frac{N_p r_p \beta^*}{4\pi\gamma\sigma^2}, \quad (1)$$

with N_p being the number of protons in the oncoming bunch, $r_p = 1.54 \cdot 10^{-18}\,\mathrm{m}$ the classical proton radius, β^* the β-function at the interaction point (IP), $\gamma = 267$ the Lorentz factor of the beam, and σ the rms beam size of the oncoming beam.
In the case of a bi-gaussian beam with beam sizes σ_1 and σ_2, and corresponding populations N_1 and $N_2 = N_p - N_1$, the resulting beam-beam tuneshift $\xi_{1,2}$ is just the sum of the two tuneshifts resulting from these two contributions,

$$\begin{aligned} \xi_{1,2} &= \xi_1 + \xi_2 \\ &= \frac{N_1 r_p \beta^*}{4\pi\gamma\sigma_1^2} + \frac{N_2 r_p \beta^*}{4\pi\gamma\sigma_2^2}. \end{aligned} \quad (2)$$

The luminosity $\mathcal{L}$ of two identical round gaussian beams is

$$\mathcal{L} = \frac{f}{\pi}\frac{N_p^2}{\sigma^2}, \quad (3)$$

In the case of two identical bi-gaussian beams, the resulting luminosity $\mathcal{L}_{1,2}$ can be written as

$$\mathcal{L}_{1,2} = \sum_{i=1,2}\sum_{j=1,2}\frac{f}{\pi}\frac{N_i N_j}{\sigma_i^2 + \sigma_j^2}. \quad (4)$$

Combining Equations (3) and (4), we can therefore compute the beam size σ of a regular gaussian beam with identical intensity $N = N_1 + N_2$ which provides the same luminosity $\mathcal{L} = \mathcal{L}_{1,2}$ as

$$\begin{aligned} \sigma^2 &= \frac{f}{\pi}\frac{N_p^2}{\mathcal{L}} \\ &= \frac{f}{\pi}\frac{N_p^2}{\mathcal{L}_{\infty,\in}} \\ &= \frac{N_p^2}{\sum_{i=1,2}\sum_{j=1,2}\frac{N_i N_j}{\sigma_i^2+\sigma_j^2}}. \end{aligned} \quad (5)$$

Inserting this result into Equation (1) yields the beam-beam tuneshift for this equivalent beam,

$$\begin{aligned} \xi &= \frac{N_p r_p \beta^*}{4\pi\gamma\sigma^2} \\ &= \frac{r_p\beta^*}{4\pi N_p\gamma}\sum_{i=1,2}\sum_{j=1,2}\frac{f}{\pi}\frac{N_i N_j}{\sigma_i^2+\sigma_j^2}. \end{aligned} \quad (6)$$

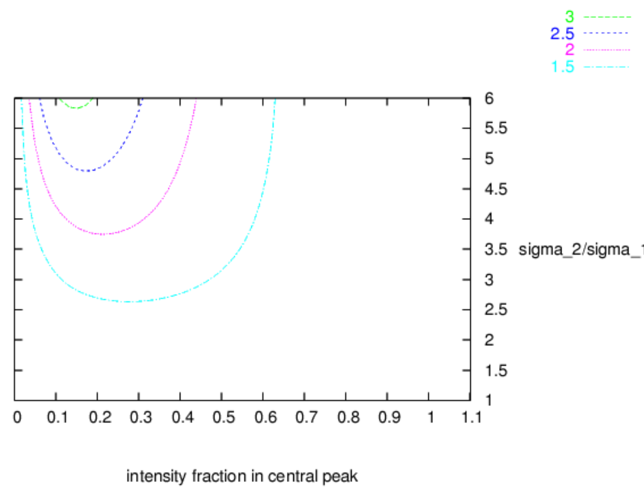

Figure 1: Beam-beam contour plot

Figure 1 shows a contour plot of the resulting normalized beam-beam tuneshift $\xi_{1,2}/\xi$ as a function of the intensity fraction N_1/N_p in the central core and the ratio of the rms widths of the two gaussian components, σ_2/σ_1.

SIMULATIONS

To study the beam-beam interaction effect of a double-gaussian distribution, a weak-strong simulation code was

* Work performed under the auspices of the U.S. Department of Energy

Q_x	.21
Q_y	.23
N_p	$2.5 \cdot 10^{11}$
N_1	$1.4 \cdot 10^{11}$
N_2	$0.6 \cdot 10^{11}$
σ	$66\,\mu$m
σ_1	$50\,\mu$m
σ_2	$150\,\mu$m
ξ	-0.007
γ	266
γ_t	23
U_{RF}	2 MV
harmonic number	2520
number of IPs	1

Table 1: Parameter table.

used. The accelerator lattice is described by a linear matrix, with a chromaticity of $\chi_x = \chi_y = +2.0$. Longitudinal motion was included with parameters close to the RHIC case, as listed in Table 1.

In the transverse planes, a random tune fluctuation with rms variation of $\sigma_Q = 3 \cdot 10^{-5}$ and a coherence length $n_c = 10^4$ turns was added to the model. This random drift is modeled as

$$d_{n+1} = \frac{n_c}{1+n_c} d_n + \frac{1}{1+n_c} k_n, \tag{7}$$

where k_n is a random white noise signal with unit standard deviation and zero mean, and n is the turn number. n_c determines the correlation time of the random drift in terms of revolutions. Using the random drift signal d_n, the drift of the tunes is simulated as

$$Q_n = Q + \frac{\delta Q \cdot d_n}{\sqrt{\langle d_0^2 \rangle}}, \tag{8}$$

where

$$d_0 = \frac{k_0}{\sqrt{1+n_c}} \tag{9}$$

is the initial condition, and

$$\langle d_0^2 \rangle = \frac{1}{1+n_c}. \tag{10}$$

This random tune fluctuation is assumed to be caused by power supply ripple, vibrations of non-linear magnetic elements, etc.

To study the effect of beam-beam interactions of beams with a double-gaussian distribution, 1000 particles were launched with specific initial phase space distributions and tracked over $3 \cdot 10^6$ turns. The total transverse emittance was averaged over 997 turns and recorded. For comparison, the interaction of two regular gaussian beams with the same beam-beam tune shift parameter $\xi = -0.007$ as in the double-gaussian case was also studied. Three different working points have been investigated, indicated by their respective tune footprints shown in Figure 2.

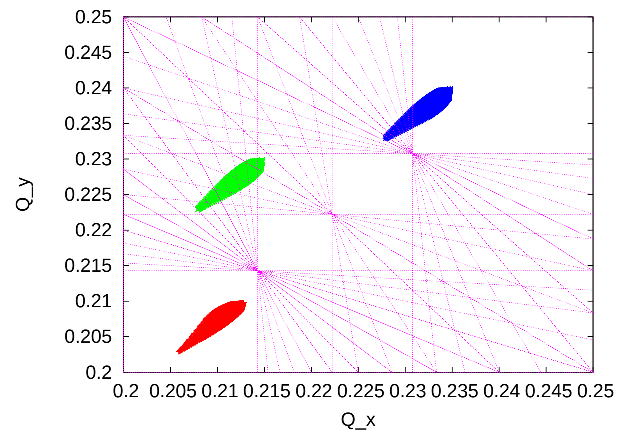

Figure 2: Sum resonances up to 14th order and tune footprints of the three working points studied in this paper. The colors a reconsistent with those in the following pictures.

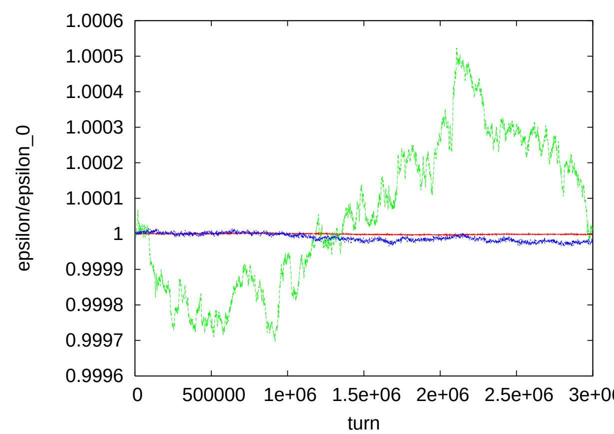

Figure 3: Normalized beam emittance in the regular case of two gaussian beams. The three colors indicate the three working points, as shown in Figure 2.

In the regular case, the particles were launched with a gaussian phase space distribution in both transverse planes, as well as in the longitudinal direction. No significant emittance growth was observed, as Figure 3 shows. In the bi-gaussian case, the particles were launched according to the rms width σ_2 of the tails of the distribution. In the longitudinal plane, initial phase space coordinates were chosen such as to resemble a beam with 20 cm rms bunch length. As in the regular case, they were tracked for 3 million turns, and the sum emittance was averaged over 997 turns and written to file every 997 turns. In this case, a significant emittance blow-up occurs (Figure 4). The observed emittance growth becomes even more pronounced when the longitudinal phase space coordinates resemble the tails of the distribution in this plane, with an rms bunch length of

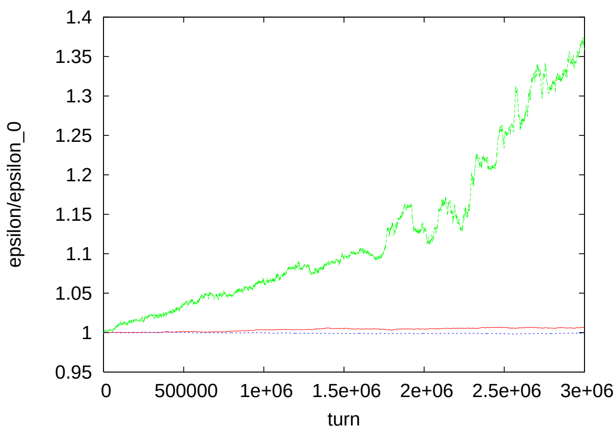

Figure 4: Normalized emittance of the 3σ transverse tails, for particles in the longitudinal core. The colors indicate the three working points, shown in Figure 2.

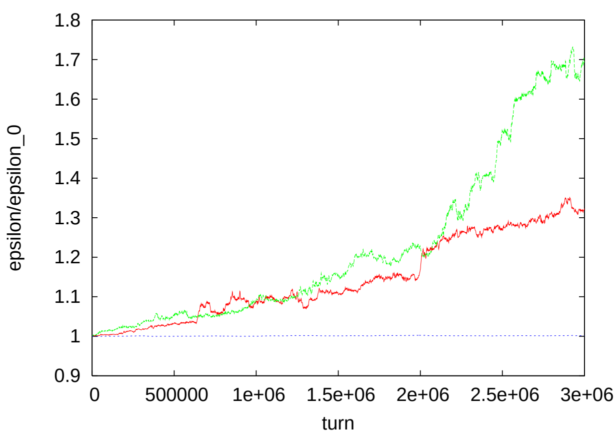

Figure 5: Normalized emittance of the 3σ transverse tails, for particles in the longitudinal tail.

$60\,\mathrm{cm}$. This is shown in Figure 5. Particles launched according to the rms width σ_1 of the beam core do not show emittance blow-up, as depicted in Figure 6.

When the double-gaussian strong beam is replaced by a regular gaussian beam providing the same beam-beam tuneshift, no significant difference between this case and the double-gaussian case was observed for the transverse 3σ tails of the weak beam. This indicates that the observed effect is mostly due to the mismatched beam sizes and not due to the fact that the strong beam has a double-gaussian distribution.

For a distribution with $\sigma_2 = 10 \cdot \sigma_1$, $N_2 = 100 \cdot N_1$, and a total beam-beam tuneshift parameter of $\xi = 0.007$, nonlinear terms are completely dominated by the core and differ by a factor of two from the regular gaussian case with the same linear tuneshift. In this case, the strong regular gaussian beam results in a significantly faster emittance blow-up of the weak beam tails, see Figure 7.

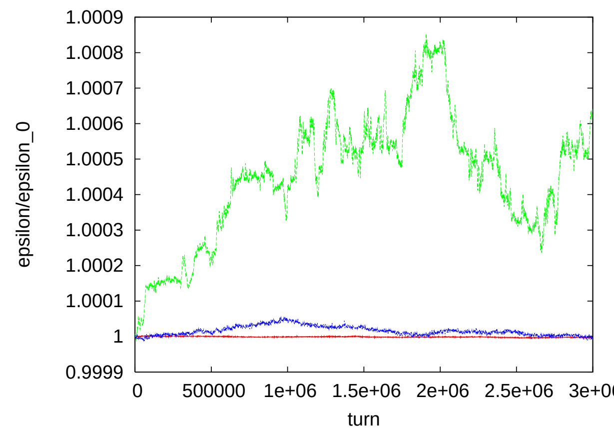

Figure 6: Normalized emittance of the core of the double-gaussian beam, for particles in the longitudinal core. Colors are consistent with Figure 2.

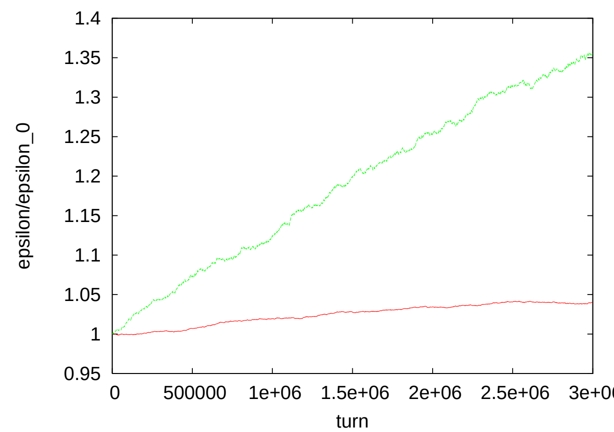

Figure 7: Emittance of the 10σ tail when colliding with a regular gaussian strong beam (green line), and when colliding with a double-gaussian strong beam (red curve), for equal linear beam-beam tuneshifts. The emittance growth is much faster when collding with the regular gaussian strong beam due to the twice larger nonlinear terms in the beam-beam kick.

REFERENCES

[1] ZDR Electron Cooling for RHIC, http://www.cadops.bnl.gov/eCool/ZDRDraft_edit4.pdf

