RF DEFOCUSING IN SUPER-CONDUCTING STRUCTURE WITH CONSTANT GEOMETRY *

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Abstract

Due to higher accelerating gradient in the super-conducting linear accelerator the RF defocusing factor plays significant role in the beam dynamics. Together with the space charge it is a main reason for the stability loss. Usually the RF defocusing is estimated in frame of the travelling wave formalism with synchronous motion. However, the super-conducting cavity is desirable to have the constant geometry, when synchronous motion is absent. In this case the quasi-synchronous phase velocity is adjusted by RF phasing. In this paper we investigate RF defocusing factor in absent of synchronism between the beam and the accelerating structure.

INTRODUCTION

In order to simplify the construction of SC cavity and increase the accelerating gradient the focusing system is desirable to place outside cavity using the external quadrupoles. However, together with the higher accelerating rate in the super-conducting linear accelerator we get much higher defocusing factor in comparison with the normal conducting linear accelerator. Since it affects on the total transverse frequency similarly the space charge tune shift, the total current and the maximum possible cavity length (focusing period) are strongly determined by this effect. The RF defocusing is the derived effect from accelerating field alteration, and usually it is estimated in frame of the traveling wave formalism, when we take into consideration only one synchronous harmonics. It means in case of perfect synchronism the synchronous particle sees the constant transverse force.

In the super-conducting linear accelerator many cavities owing to one family have an identical geometry, and the phase velocity changes step by step from family to family. Number of gaps in cavity and number of cavities in one family can be varied from one to few tens and it depends on many factors. The particles are sliding down or up relatively of RF wave in dependence on ratio between the particles and the wave velocities. Thus, the particles are never in synchronism with the equivalent traveling wave. The proper law of RF phasing of cavities provides the longitudinal stability in SC linear accelerator [1]. But simultaneously with that we can expect the RF defocusing factor will be modified.

RF DEFOCUSING IN SINGLE CAVITY WITH CONSTANT GEOMETRY

First of all we consider the RF defocusing factor as the single integrated kick in one cavity with the axial symmetrical field and with the n_{cell} absolutely identical accelerating cells, having the same sizes with periodicity $L_{cell} = \frac{\beta_{str}\lambda}{2}$. The structure velocity β_{str}, some analogue of the phase velocity, is therefore constant in the cavity. The transverse motion equations are:

$$\frac{d^2x}{dt^2} = \frac{e}{m_0\gamma^3}E_x, \qquad \frac{d^2y}{dt^2} = \frac{e}{m_0\gamma^3}E_y \tag{1}$$

where $E_{x,y}$ is the transverse component of the RF field. On the cavity axis the radial RF field can be represented as:

$$E_x(x) = E_x(0) + \frac{\partial E_x}{\partial x}\cdot x, \qquad E_y(y) = E_y(0) + \frac{\partial E_y}{\partial y}\cdot y \tag{2}$$

In the case of the field with the axial symmetry we have: $\frac{\partial E_x}{\partial x} = \frac{\partial E_y}{\partial y}$ and $E_x(0) = E_y(0) = 0$.

Substituting the latter in the Maxwell equation $\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$, we can easily get the ratio between the transverse and the longitudinal components:

$$\frac{\partial E_x}{\partial x} = \frac{\partial E_y}{\partial y} = -\frac{1}{2}\frac{\partial E_z}{\partial z} \tag{3}$$

For instance, in the H resonator with the angular frequency $\omega = \frac{2\pi c}{\lambda}$ the accelerating field can be represented by the Fourier series:

$$E_z(z,t) = \sum_n E_n \sin\frac{2\pi n}{\beta_{str}\lambda}z\cdot\sin(\omega t + \varphi_0) \tag{4}$$

Then the motion equations (1) can be rewritten in the longitudinal coordinates system $dz = c\beta dt$:

$$\frac{d^2x}{dz^2} = -\frac{e}{2m_0c^2\gamma^3\beta^2}\cdot\sum_n k_nE_n\cos k_nz\cdot\sin(\omega t+\varphi_0)\cdot x, \qquad \frac{d^2y}{dz^2} = -\frac{e}{2m_0c^2\gamma^3\beta^2}\cdot\sum_n k_nE_n\cos k_nz\cdot\sin(\omega t+\varphi_0)\cdot y \tag{5}$$

where $k_n = \frac{2\pi n}{\beta_{str}\lambda}$ is the wave number of n-th field harmonic.

Retaining in (4) the nearest harmonics to the quasi-synchronous particle $E_1\cos k_1 z$ with the wave number

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$k_1 = \dfrac{2\pi}{\beta_{str}\lambda}$, we can use the traveling wave system. In this system, the phase of the arbitrary particle relative to the RF field is determined in accordance with the common definition:

$$\varphi(z,t) = \omega t + \varphi_0 - k_1 z = \frac{2\pi}{\lambda}\int_0^z \frac{d\xi}{\beta} + \varphi_0 - \frac{2\pi}{\lambda}\int_0^z \frac{d\xi}{\beta_{str}}, \qquad (6)$$

where φ_0 is the initial phase. In order to provide the quasi-synchronism in the longitudinal plane the particle RF phase at the entrance and exit of the cavity has to be in the relation determined in [1]:

$$\begin{aligned} \varphi_{L_{\mathrm{cav}}} &= \overline{\varphi}_{qs} + 0.5\cdot\Delta\varphi_{slide} \\ \varphi_0 &= \overline{\varphi}_{qs} - 0.5\cdot\Delta\varphi_{slide} \end{aligned}, \qquad (7)$$

where $\Delta\varphi_{slide} \approx \pi n_{cell}\left(\dfrac{\beta_{str}}{\beta} - 1\right)$ is the phase sliding factor due to the non-synchronism, $\overline{\varphi}_{qs}$ is the average meaning of quasi-synchronous phase.

Now we can calculate the trajectory-refracting angle of the quasi-synchronous particle due to the RF defocusing effect over the whole cavity with the length of $L_{\mathrm{cav}} = \dfrac{\beta_{str}\lambda}{2}\cdot n_{cell}$:

$$\Delta\begin{pmatrix} \dfrac{dx}{dz} \\ \dfrac{dy}{dz} \end{pmatrix} = -\frac{e\pi\, k_1\, E_1 \sin\overline{\varphi}_{qs}}{4m_0c^2\gamma^3\beta^2}\cdot\frac{\sin\dfrac{\Delta\varphi_{slide}}{2}}{\dfrac{\Delta\varphi_{slide}}{2}}\cdot\frac{n_{cell}\beta_{str}\lambda}{2}\cdot\begin{pmatrix} x \\ y \end{pmatrix} \quad (8)$$

In order to have stable longitudinal motion the average quasi-synchronous phase has to be negative, that is to say the RF accelerating field always defocuses the particle. Let us call $T_{slide} = \dfrac{\sin\dfrac{\Delta\varphi_{slide}}{2}}{\dfrac{\Delta\varphi_{slide}}{2}}$ the time sliding factor. We can see that due to the non-synchronism the RF defocusing factor decreases proportionally to the time sliding factor.

RF DEFOCUSING IN SEQUENCE OF CONSTANT GEOMETRY CAVITIES

The expression (8) allows the RF defocusing integrating kick to be estimated when the particle is flying through the single cavity. In reality, the kick is modulated with the frequency of the cavities' repetition. Now we shall consider the case when the cavities are joined up in families, and all the m cavities of one family have the same structure phase velocity $\beta_{str} = const$ for $i \in 1 \div m$. Because of non-synchronism the particle always slides in the cavity relative to the RF field and it is returned to the required RF phase at the entrance of next cavity by the RF phase shift. The RF phase shift law in each cavity $\varphi_{str}(i) = \sum_i \Delta\varphi_{RFi}$ determines the quasi-synchronous particle motion. Due to a proper choice of the RF phase shift $\Delta\varphi_{RF}$ between cavities we can create a quasi-synchronous motion and, in total, stable motion in the whole accelerator [1]. The quasi-synchronous particle oscillating instantaneously in the single resonator around $\varphi = 0$ (for "sin" wave) is forced by the intersection RF shift to oscillate on average around $\overline{\varphi}_{qs}$. Figure 1 shows how the quasi-synchronous particle oscillates around the average value $\overline{\varphi}_{qs}$ in the linear accelerator with one family. For each section the initial and final phase deviations are not equal to each other.

Instead (6) the corrected phase of the quasi-synchronous particle is $\varphi_{qs}(z) = \dfrac{2\pi}{\lambda}\left(\int_0^z \dfrac{d\xi}{\beta_{qs}} - \int_0^z \dfrac{d\xi}{\beta_{str}}\right) + \varphi_{str}(z)$ and the full equation system of quasi-synchronous motion in the normalized coordinate system $\tau = z/\beta_{str}\lambda$ takes the form:

$$\begin{aligned} \frac{d^2x}{d\tau^2} &= -\frac{e\lambda^2\beta_{str}^2}{4m_0c^2\gamma^3\beta_{qs}^2}\cdot k_1E_1\sin\varphi_{qs}\cdot x \\ \frac{d^2y}{d\tau^2} &= -\frac{e\lambda^2\beta_{str}^2}{4m_0c^2\gamma^3\beta_{qs}^2}\cdot k_1E_1\sin\varphi_{qs}\cdot y \\ \frac{d\varphi_{qs}}{d\tau} &= 2\pi\left(\frac{\beta_{str}}{\beta_{qs}} - 1\right) + \frac{d\varphi_{str}}{d\tau} \\ \frac{d\beta_{qs}}{d\tau} &= \frac{eE_1\lambda\beta_{str}}{2m_0c^2\gamma^3\beta_{qs}}\cos\varphi_{qs} \end{aligned} \qquad (9)$$

The last two equations of the longitudinal motion are solved in [1] together with the determination of the optimum law for the RF phase shift.

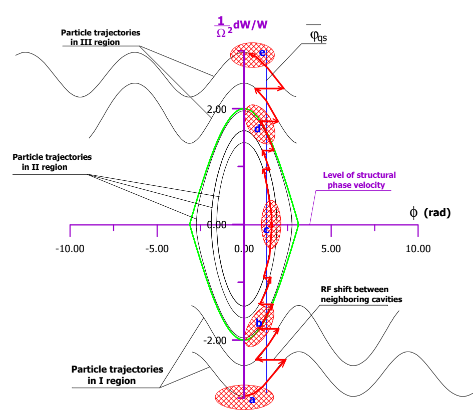

Figure 1: Longitudinal motion in the stepped RF phase structure with one family

In particular, the instantaneous quasi-synchronous phase is:

$$\varphi_{qs}(\tau) = \overline{\varphi_{qs}} + f(\tau)$$

with $$f(\tau) = -2\frac{d\overline{\varphi}_{str}}{d\tau}\cdot\sum_{l=1}^{\infty}\frac{l\nu_{cav}\cdot\sin(2\pi\, l\nu_{cav}\tau)}{\Omega^2\cos\overline{\varphi}_{qs} - l^2\nu_{cav}^2}, \quad (10)$$

where $\nu_{cav} = 1/T_{cav}$ is the frequency of the RF phase shift and Ω is the longitudinal frequency in the sinus wave separatrix with φ_s =0. The period $T_{cav} = n_{cell}/2$ measured in the number of $\beta_{str}\lambda$ has to coincide with the cavity length $L_{cav} = n_{cell}\beta_{str}\lambda/2$.

For the practical case we can take $\nu_{cav} >> \Omega\cos^{1/2}\overline{\varphi_{qs}}$, and the first harmonic retains only $f(\tau) = \varphi_a(\tau)\cdot\sin(2\pi\nu_{cav}\tau)$ with the slowly time dependent amplitude $\varphi_a(\tau) = \frac{2}{\nu_{cav}}\cdot\frac{d\overline{\varphi}_{str}}{d\tau}$. Thus, the quasi-synchronous particle oscillates around the average value with the amplitude dependent on the non-synchronism $\frac{d\overline{\varphi}_{str}}{d\tau} = 2\pi\left(1 - \frac{\beta_{str}}{\beta(\tau)}\right)$. For the higher relative velocity and the same number of cavities in one family the amplitude $\varphi_a(\tau)$ decreases.

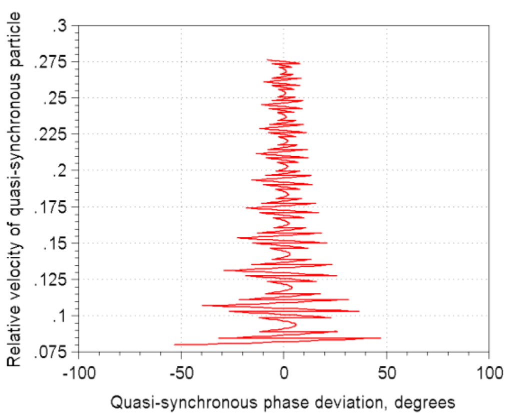

Figure 2: Quasi-synchronous particle oscillation $f(\tau) = \varphi_{qs}(\tau) - \overline{\varphi_{qs}}$ relative to the average value in the linear accelerator with 10 families

Figure 2 shows the quasi-synchronous phase oscillation versus the relative velocity in the case of 10 families with 6 cavities in each family. Substituting $\varphi_{qs}(\tau) = \overline{\varphi_{qs}} + f(\tau)$ in the transverse motion equation (9) and inserting the new coordinate $\xi = \frac{\tau}{T_{cav}}$ normalized to the cavity periodicity $T_{cav} = \frac{n_{cell}}{2}$, we have, for instance, in the horizontal plane:

$$\frac{d^2x}{d\xi^2} + \frac{A_{RF}n_{cell}^2}{4}\sin\left(\overline{\varphi}_{qs} + \varphi_a\sin 2\pi\xi\right)\cdot x = 0, \quad (11)$$

where $A_{RF} = \frac{e\lambda^2\beta_{str}^2}{4m_0c^2\gamma^3\beta_{qs}^2}\cdot k_1E_1$ is the amplitude of the RF defocusing factor.

Comparing the superconducting linear accelerator with the fully synchronized normal conducting option, we can see that the sinus argument has the oscillating term. Thus, we get the alternating phase focusing component in the RF defocusing term. Let us open the sinus with the sinus argument, using the Bessel functions and retaining no terms higher than the first-order term:

$$\frac{d^2x}{d\xi^2} + A_{RF}\left(\frac{n_{cell}}{2}\right)^2\cdot \quad (12)$$
$$\left\{\sin\overline{\varphi}_{qs}\left[J_0(\varphi_a) + ...\right] - \cos\overline{\varphi}_{qs}\cdot\left[2J_1(\varphi_a)\sin 2\pi\xi + ...\right]\right\}\cdot x = 0$$

The Bogolyubov-Metropolsky averaging method [2] allows us to find a common case for equation

$$\frac{d^2x}{d\xi^2} + (a + 2\cdot\sum_k q_k\cos 2\pi k\xi)\cdot x = 0 \quad (13)$$

the solution:

$$x(\xi) = A\cdot\left(1 + \sum_k\frac{q_k}{2\pi^2k^2}\cos 2\pi k\xi\right)\cos\mu\xi \quad (14)$$

with the phase advance per one focusing period:

$$\mu_{x,y}^2 = \frac{1}{2\pi^2}\sum_k\frac{q_k^2}{k^2} + a \quad (15)$$

For equation (12) we have:

$$\mu_{x,y}^2 \approx \frac{1}{2\pi^2}\left(A_{RF}\left(\frac{n_{cell}}{2}\right)^2\cos\overline{\varphi}_{qs}\cdot J_1(\varphi_a)\right)^2 + \quad (16)$$
$$+ A_{RF}\left(\frac{n_{cell}}{2}\right)^2\sin\overline{\varphi}_{qs}J_0(\varphi_a)$$

As usual, since $\sin\overline{\varphi}_{qs} < 0$, the second term defocuses, but it creates the focusing term as well, which is proportional to $J_1(\varphi_a)$. Additionally, due to sliding and phase oscillating, the RF defocusing is smaller by factor $J_0(\varphi_a)$. From (16) it follows that the sliding factor can provide the beam focusing without quadrupoles, when $\mu^2 > 0$, that is:

$$\left|\sin\overline{\varphi}_{qs}\right| < \frac{A_{RF}n_{cell}^2\cos^2\overline{\varphi}_{qs}J_1^2(\varphi_a)}{8\pi^2J_0(\varphi_a)} \quad (17)$$

We can conclude that the sliding factor positively affects the transverse beam stability without any significant loss in accelerating rate.

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[2] A. N. Bogolyubov and Yu. Mitropol`skij, Asymptotic Methods in the Theory of non-linear oscillations, (Hindustan Publ., Delhi, 1961)