SIMULATIONS OF ERROR-INDUCED BEAM DEGRADATION IN FERMILAB'S BOOSTER SYNCHROTRON*

P. S. Yoon [1†], W. Chou[2], C. L. Bohn[2,3]

[1]University of Rochester, Rochester, NY 14627, USA
[2]FNAL, MS 220, Fermilab, P.O. Box 500, Batavia, IL 60510, USA
[3]Northern Illinois University, DeKalb, IL 60115, USA

Abstract:
Individual particle orbits in a beam will respond to both external focusing and accelerating forces as well as internal space-charge forces. The external forces will reflect unavoidable systematic and random machine errors, or imperfections, such as jitter in magnet and radio-frequency power supplies, as well as translational and rotational magnet alignment errors. The beam responds in a self-consistent fashion to these errors; they continually do work on the beam and thereby act as a constant source of energy input. Consequently, halo formation and emittance growth can be induced, resulting in beam degradation and loss. We have upgraded the ORBIT-FNAL package and used it to compute effects of machine errors on emittance dilution and halo formation in the existing FNAL-Booster synchrotron. This package can be applied to study other synchrotrons and storage rings as well.

INTRODUCTION

Beam loss during the injection is a major concern regarding high-intensity circular accelerators. We investigate this phenomenon in Fermilab's Booster synchrotron [1]. To do so, we use an upgraded ORBIT-FNAL 3D object-oriented simulation package [2] to compute beam dynamics calculations in the presence of time-dependent *colored* noise that is unavoidable in real machines. We find that this noise, coupled self-consistently to the space-charge force, contributes to beam degradation, halo formation, and beam loss in the Booster synchrotron. This happens despite the fact that the Booster beam is relativistic; even though the space-charge force is weak, both it and the noise act on the beam over a long time scale.

Our motivations for performing realistic simulations involving colored noise and space-charge effects are:

- To explore phenomena induced by unavoidable systematic and random machine errors/imperfections in the Booster that may generate continuous emittance growth and halo formation.
- To search for other general sources of halo formation in synchrotrons or storage rings.
- To augment analytic predictions and inferences arising from simple models [3, 4].

*Work supported by the Universities Research Association, Inc. under U.S. Department of Energy (DOE) contract No. DE-AC02-76-CH03000, by DOE Grant DE-FG02-04ER41323 to NIU, and by DOE Grant No. DE-FG02-91ER40685 to University of Rochester.
†E-mail: syoon@fnal.gov

THE FERMILAB BOOSTER SYNCHROTRON

The Fermilab Booster is an alternating-gradient, rapid-cycling synchrotron containing 96 combined-function magnets. These magnets are excited with a 15-Hz biased sine wave. Capacitors are used to form a resonant network with the magnets. The proton-beam energy in the Booster ranges from 0.4 GeV at injection to 8.0 GeV at extraction. The parameters we use in simulating the Booster appear in Table 1.

Table 1: Booster Parameters

Ring Circumference [m]	474.2
Injection Kinetic Energy [MeV]	400.0
β at Injection*	0.7131
γ at Injection*	1.4263
Revolution Time at Injection [μsec]	2.2
γ at Transition*	5.4696
$\lvert\eta\rvert$ at Injection	0.458
Transverse Distribution	Bi-Gaussian
Longitudinal Distribution	Uniform
RF Peak Voltage ($\hat{V}$) [KV/Turn]	205.0
Maximum # of Macroparticles	110 K
Injection Turns	11
Total Proton Intensity [E+10 ppb]	6.00
Maximum Intensity / batch [E+12]	5.0
Harmonic Number	84
ν_x/ν_y (nominal betatron tune)	6.7 / 6.8
$\Delta P/P_0$	± 0.15 %
Dimensionality of Space Charge	2.5D

∗Relativistic Lorentz Factor

MACHINE IMPERFECTIONS

Magnet Power Supplies, Current Regulators

The Booster has 4 programmed thyristor power supplies connected in series and inserted at symmetric points in the resonant system. Each power supply drives 12 resonant cells in series; there are 2 loops in the magnet regulator. Fluctuations in the maximum current I_{max} from cycle to cycle must be held below $\Delta I/I = 1.0\times10^{-4}$. The tolerance on the minimum current I_{min} is $\Delta I/I = 2.0\times10^{-4}$. From the relationship between magnetic-field gradient and current, the following relations can be derived:

$$\frac{\Delta I}{I} = \frac{\Delta K_n}{K_n} \qquad (1)$$

$$\theta = \Delta K_n \cdot \Delta x \cdot L \sim 1(\mu rad)$$

where K_n, Δx, and L denote the normalized magnetic coefficient, transverse length, and longitudinal length of a quadrupole, respectively. These fluctuations constitute dynamic colored noise. To simulate this noise while the beam is tracked during a simulation run, we randomly impart an angular-dipole kick to each macro-particle on 809 nodes, while it is propagated through the ring lattice.

Magnet Misalignments

Based on recent magnet-survey data, we incorporate into the simulation all types of magnet misalignments, which are averaged over the 96 magnets in the Booster. These misalignments are static, and as for colored noise and space charge, we find that they likewise contribute to beam degradation and halo formation.

NOISE MODULE

To reiterate, noise is inherent to all types of accelerators because of their inherent imperfections: power-supply jitter, magnet misalignments, lattice transitions, etc. We model dynamic noise (such as jitter) as exponential Gaussian-driven colored noise sampling an Ornstein-Uhlenbeck stochastic process [5]. The major features of the new noise module are as follows:

- It is configured to generate different types of random noise per user input (i.e., colored and white noise).
- It enables statistical properties of the random noise to be controlled via input script files.
- It is parallelized with Message-Passing Interface (MPI).
- It calculates properties inherent to the collection of particle orbits: halo amplitudes, root-mean-squared (rms) transverse emittances, and beam loss.

Algorithm

For improved accuracy and speed, we generate the noise using an integration method [5] rather than the more conventional differential method. The random deviates are generated by the Box-Muller method [6]. We are careful to select sensible parameters (i.e., autocorrelation time and noise strength) for the Booster; how the noise quantitatively affects halo formation depends on these parameters. As previously mentioned, we also add realism by including various inherent machine errors: magnet alignment errors, power-supply ripples and jitter.

Upon integrating a stochastic differential equation, we can obtain a Gaussian-driven term $H(t,\Delta t)$:

$$\frac{d\eta}{dt} = -\lambda\eta + \lambda G_W$$

$$\eta(t) = e^{-\lambda t}\cdot\eta(o) + \lambda\int_0^t ds\cdot e^{-\lambda(t-s)}\cdot G_W(s) \qquad (2)$$

$$\eta(t+\Delta t) = e^{-\lambda\Delta t}\cdot\eta(t) + \lambda\int_t^{t+\Delta t} ds\cdot e^{-\lambda(t+\Delta t-s)}\cdot G_W(s)$$

$$= e^{-\lambda\Delta t}\cdot\eta(t) + H(t,\Delta t)$$

$$H(t,\Delta t) \equiv \lambda\int_t^{t+\Delta t} ds\cdot e^{-\lambda(t+\Delta t-s)}\cdot G_W(s) \qquad (3)$$

where η, λ, and G_W denote random noise, the reciprocal of the autocorrelation time, and Gaussian white noise, respectively.

Noise Parameters

Parameters for the colored noise are: autocorrelation time, time step, and noise strength. The first and second moments of $H(t,\Delta t)$ determine the statistical properties of the noise. Because $H(t,\Delta t)$ is Gaussian and $\langle H(t,\Delta t)\rangle=0$, all of its statistical properties are determined by its second moment:

$$\left\langle H^2(t,\Delta t)\right\rangle = S_{noise}\cdot\lambda\cdot(1-e^{-2\lambda t}). \qquad (4)$$

Here, S_{noise} denotes the noise strength, which is computed as the amplitude of the random noise based on the angular-dipole-kick calculation from the power-supply ripple. The transit time of a particle around the Booster is taken to be the autocorrelation time.

SIMULATION RESULTS

With the procedure described above, we include the self-consistent influences of space charge, static errors (magnet misalignments) and dynamic colored noise (power-supply jitter) in our simulation of beam dynamics in the Booster. As the macroparticles orbit through the Booster, we compute at each time step the rms emittances for the two transverse phase-space planes, the halo profile, the fractional beam loss, and statistical properties of the noise. To add clarity, we have smoothed the data in all of the plots presented here using a spline function.

Fig.1 depicts that as the magnitude of the colored-noise strength increases, so does rms transverse emittance growth rate accordingly. In case of the Booster, a noise-strength of 1.0×10^{-11} is matched to the machine imperfections based on the equation (1) and (4). Upon including dynamic noise, this beam degradation worsens, and a more prominent halo forms. This halo is quantified in Fig. 2, which shows the fraction of particles that must be excluded to produce a given rms emittance after just the first 1,000 turns through the Booster. For example, we see that 93% of the macroparticles combine to produce an 80 (π-mm-mrad) rms emittance, and 96% combine to produce 100 (π-mm-mrad). This is a signature of a substantial halo; particles that are far from the beam

 0-7803-8859-3/05/$20.00

centroid are weighted heavily when computing the emittance. Another measure of halo is beam loss, which of course arises from particles having a transverse displacement that exceeds the hardware aperture. The cumulative beam loss after every turn is plotted in Fig. 3; after 1,000 turns, nearly 2 % of the beam has impinged somewhere along the machine. What is more, the emittance continually grows, as implied in Fig. 1; the rms emittance growth vs. time due to colored noise in the presence of space charge only (no magnet misalignments) appears in Fig. 4. The synergistic mechanism between colored noise *alone* arising from machine imperfections and the space-charge effects causes sizable transverse emittance growth of about 50 %.

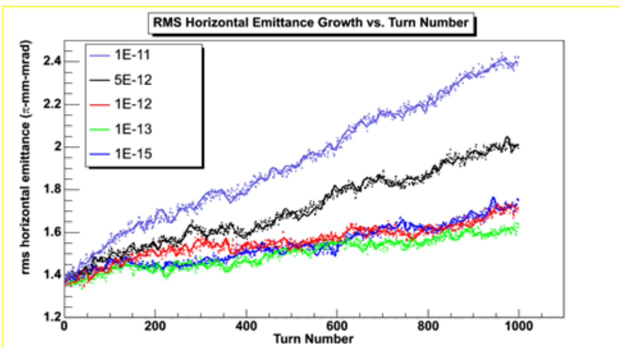

Figure 1. The RMS horizontal emittance vs. time with various noise strengths matched to the Booster lattice.

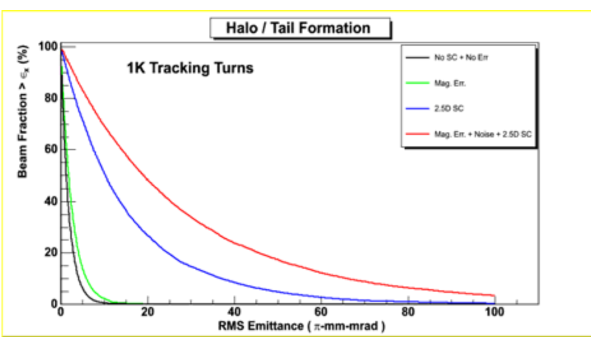

Figure 2. Fraction of macro-particles that lie outside of a given rms emittance at each rms transverse emittance in the absence of space charge, errors, and noise (black); with just magnet misalignments (green); with just space charge (blue); and with all effects combined (red).

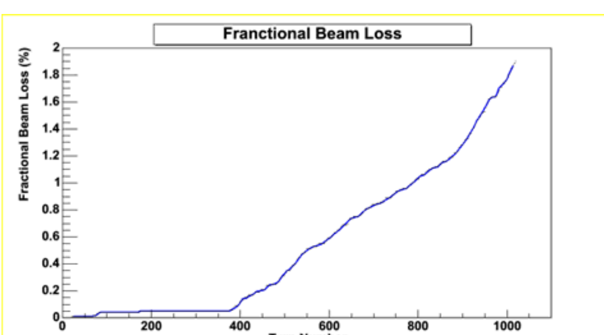

Figure 3. Cumulative fractional beam loss in the Booster synchrotron vs. time (turn number).

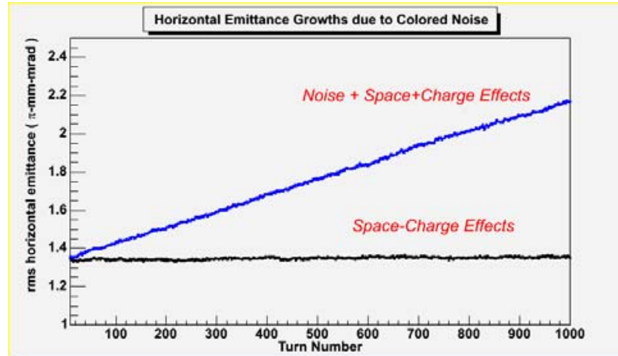

Figure 4. RMS horizontal emittance growth vs. time (turn number). The Booster noise strength is 1.0×10^{-11}.

CONCLUDING REMARKS

If colored noise in the presence of even very weak space charge acts on a beam over a sufficiently long time, it will significantly degrade the beam quality. This phenomenon could account for large beam loss for the first few ms in the Booster operation. This has been seen in simulations involving a simplistic model [3], and it is seen clearly in the realistic simulations described herein. Motivated by the results of the Booster simulations, the authors plan to investigate these phenomena (space charge, hardware irregularities, and dynamic noise, all three of which have proven to be important to the beam dynamics) in other synchrotrons and storage rings as well.

ACKNOWLEDGEMENTS

We are grateful for advice from several members of the Fermilab Booster Group. We also thank J. Holmes and A. Shishlo of SNS/ORNL, and W. Brown and J.-F. Ostiguy of FNAL, for their support in upgrading and commissioning the ORBIT-FNAL package.

REFERENCES

[1] E.L. Hubbard, FERMILAB-TM-405 (1973).
[2] J.D. Galambos et al., ORBIT User Manual, Version 1.10, Rev. 1 (SNS/ORNL/AP Technical Note, 1999).
[3] C.L. Bohn, AIP Conf. Proc. **737**, 456 (2004).
[4] C.L. Bohn, I.V. Sideris, Phys. Rev. Lett. **91**, 264801 (2003).
[5] N.G. van Kampen, Stochastic Processes in Physics and Chemistry (North Holland, Amsterdam, 1981), see especially p. 252.
[6] W.H. Press, S.A. Teukolsky, W.T. Vertterling, B.P. Flannery, Numerical Recipes in C (Cambridge University Press, Cambridge, 1995).