ENHANCED OPTICAL COOLING OF PARTICLE BEAMS IN STORAGE RINGS*

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Abstract

An enhanced optical cooling method (EOC) based on external selectivity of interaction between particles and theirs amplified undulator radiation wavelets (URW) is discussed. The selectivity arranged by a moving screen located on the image plane of optical system projecting URW there. A non-exponential damping time in this scheme of cooling is not limited by any restriction like Robinson's damping criterion.

INTRODUCTION

For any particle in a storage ring, the change of the square of its amplitude of betatron oscillations caused by sudden energy change δE in smooth approximation is determined by the equation

$$\delta A_x^2 = -2x_{\beta,0}\delta x_\eta + (\delta x_\eta)^2 , \qquad (1)$$

where $x_{\beta,0}$ is the initial particle deviation from it's closed orbit; $\delta x_\eta = \eta_x \beta^{-2}(\delta E/E)$ is the change of it's closed orbit position; η_x is the dispersion function in the storage ring; β is the normalized velocity. In the approximation $|\delta x_\eta| < 2|x_{\beta 0}| < 2A_x$ the amplitude will be decreased, if the product $x_{\beta,0}\delta x_\eta > 0$. Based on this observation, two schemes of EOC of particle beams based on external selectivity were suggested in [1] for unbunched beam (RF turned off). Pick-up and kicker undulators, laser and optical systems in these schemes are similar to ones in the method of OSC [2]-[4]. The possibility to enhance OSC by screening the radiation from some part of the beam was mentioned in [5].

In the first scheme of EOC two or more identical undulators are installed in different straight sections of a storage ring at a distance determined by a betatron phase advance for the lattice segment $(2p+1)\pi$ between pick-up and first kicker undulator and $2p'\pi$ between next kicker undulators; where p, p' = 1,2,3... are integer numbers. Undulator Radiation Wavelets (URW) are emitted by a particle in the pick-up undulator passed through an optical system with movable screens located on the image plane of the particle's beam. Then this radiation is amplified and passes through the following kicker undulators together with the particle.

First, the screens in the optical system open the way for URW emitted in the pick-up undulator by particles with higher energies and higher positive deviations $x_\beta > 0$ from their closed orbits. The beam of amplified URW in the kicker undulators, in this case is similar to the moving prototype of the target T_2 considered in [1], [6] if definite phase conditions are fulfilled in the optical system to inject particles in the kicker undulators at decelerating phases. If the betatron phase advance for the lattice segment between pick-up and kicker undulators is $(2p+1)\pi$ and the deviation of the particle in the pick-up undulator $x_\beta > 0$, then the deviation of the particle in the kicker undulators comes to be $x_\beta < 0$. In this case the energy loosed by particles is accompanied by a decrease in both energy spread and amplitudes of betatron oscillations of the beam. So the EOC is going both in the longitudinal and transverse degrees of freedom. After the screen will open images of all particles of the beam the optical system must be switched off. Then the cooling process can be repeated.

Second modification of EOC method uses the pick-up undulator followed by even number of kicker undulators installed in straight sections of a storage ring. The distances between neighboring undulators are determined by the phase advance equal to $(2p+1)\pi$. In this scheme, the deviations of particles in undulators "*i*" and "*i*+1" are $x_{\beta_i} = -x_{\beta_{i+1}}$ and that is why the decrease of energy of particles in undulators does not lead to change of the particle's betatron amplitude at the exit of the last undulator. So it leads the cooling of the particle beam in the longitudinal coordinate only.

A third modification of EOC method can be suggested. It uses the pick-up undulator and even number of kicker undulators installed in straight sections of a storage ring at distances determined by a phase advance $(2p+1)\pi$ between the pick-up and next neighboring kicker undulators as well. If the particle decreases its energy in odd undulators and increases it in even ones, then the change of the energy of the particle in undulators leads to decrease of their betatron amplitudes and do not lead to change of their energy at the exit of the last undulator. In this case cooling of the particle beam is going in the transverse coordinate only.

The wavelets of UR emitted by a particle in the pick-up undulator after amplification in the optical amplifier interact efficiently with the particle in the kicker undulators. Radiation from one particle does not disturb trajectories of other particles if an average distance between particles in a longitudinal direction is more, than the length of the URW, $M\lambda_{UR}$, where M is the number of the undulator periods; λ_{UR} is the wavelength of the emitted undulator radiation (UR). This case is named "single particle in the sample". It corresponds to the beam current

$$i < i_c = \frac{Zec}{M\lambda_{UR}} = \frac{4.8\cdot 10^{-9} Z}{M\lambda_{UR}}[A] \qquad (2)$$

If overlapping of other particles with URW occurs (more than one particle in the sample), then amplified

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URW does not disturb the energy and amplitudes of betatron oscillations of other particles of the beam in the first approximation. It leads to an increase of their amplitudes in the second approximation because of the stochasticity of the initial phase of the URW for other particles.

In the second method the URW emitted by a particle in the pick-up undulator and amplified in the optical amplifier interact efficiently with the same particle in the kicker undulators and do not disturb amplitudes of betatron oscillations of other particles independently of the average distance between particles.

In the third method the amplified URW do not disturb the energy and amplitudes of betatron oscillations of other particles of the beam in the first approximation. It leads to a week increase in theirs betatron amplitudes in the second approximation because of the stochasticity of the initial phase of the URW for other particles.

In these schemes of cooling at first approximation, the degree of cooling of high current beams is higher, if the transverse dimensions of the URW in the kicker undulators are less then the transverse total (dispersion + betatron) dimensions of the being cooled particle beam as in this case the particles outside the URW do not interact with the URW and the characteristic current (2) is increased in the ratio of the areas of the particle beam and URW. It means that high dispersion- and beta- functions in the straight section of the storage ring have to be used at the location of the pick-up and kicker undulators.

The considered schemes are of great interest for cooling of fully stripped ion, proton and muon beams. Laser cooling, based on nuclear transitions has problems with low-lying levels [7]. Enhanced optical cooling of heavy ions, on the level with optical stochastic cooling, is the most efficient. In this case the emitted power $\cong Z^2$, where Z is the atomic number [8].

THE RATE OF COOLING

The total energy radiated by a relativistic particle traversing a given undulator magnetic field B of finite length is given by

$$E_{tot} = \tfrac{2}{3} r_p^2 \overline{B^2} \gamma^2 M\lambda_u , \qquad (3)$$

where $\overline{B^2}$ is an average square of magnetic field along the undulator length $M\lambda_u$; γ is the relativistic factor; $r_p = Z^2e^2/M_pc^2$ is the classical radius of the particle; $Z=1$ for electrons, protons and muons. All other symbols have theirs standard meanings. For a plane harmonic undulator, $\overline{B^2} = B_0^2/2$, where B_0 is the peak of the undulator field. For helical undulator $\overline{B^2} = B_0^2$.

The expression for the wavelength of the undulator radiation is

$$\lambda_{UR,k} = \lambda_u (1 + K^2 + \vartheta^2)/2k\gamma^2 , \qquad (4)$$

where $\lambda_{UR,k}$ is the wavelength of the k^{th} harmonic of UR; $\vartheta = \gamma\theta$; θ, the azimuth circle angle and K is the deflection parameter given by

$$\sqrt{\overline{K^2}} = \frac{Ze\sqrt{\overline{B^2}}\lambda_u}{2\pi M_p c^2} . \qquad (5)$$

The number of the equivalent photons in the URW, according to (3) – (5) becomes

$$N_{ph} = \frac{E_{tot}}{\hbar\omega_{1,\min}} = \tfrac{2}{3}\pi\alpha M Z^2 \overline{K^2} , \qquad (6)$$

where $\omega_{1,\max} = 2\pi c/\lambda_{UR,1}|_{\theta=0}$.

In the regime of small deflection parameter $K<1$, the spectrum of radiation emitted in the undulator having harmonically varying transverse magnetic field, is given by

$$\frac{dE}{d\xi} = E_{tot} f(\xi) , \qquad (7)$$

where $f(\xi) = 3\xi(1-2\xi+2\xi^2)$, $\xi = \frac{\lambda_{UR,1,\min}}{\lambda}$, $(0 \le \xi \le 1)$,

$$\int f(\xi)d\xi = 1 , \quad \lambda_{UR,\min} = \lambda_u(1+K^2)/2\gamma^2 .$$

The bandwidth of the UR emitted at a given angle θ is

$$\frac{\Delta\omega}{\omega} = \frac{1}{kM} . \qquad (8)$$

Below we accept a Gaussian distribution for the URW, its Rayleigh length $Z_R = 4\pi\sigma_w^2/\lambda_{UR,1.\min} = M\lambda_u$, the rms waist size $\sigma_w = \sqrt{Z_R\lambda_{UR,1,\min}/4\pi}$.

The rms electric field strength E_w of the wavelet in the kicker undulator

$$E_w = \sqrt{\frac{2E_{tot}}{\sigma_w^2 M\lambda_{UR,1,\min}}} = \frac{2\sqrt{2\overline{B^2}}\gamma^2 r_p}{\sqrt{3}\sigma_w} . \qquad (9)$$

The rate of the energy loss for particles in the amplified URW is

$$P_{loss} = eE_w M\lambda_u \beta_{\perp m} f \cdot N_{kick}\sqrt{\alpha_{ampl}} , \qquad (10)$$

where $\beta_\perp = K/\gamma$ is the maximum deflection angle of a particle from the direction of its closed orbit; f is the revolution frequency; N_{kick} is the number of kicker magnets; α_{ampl} is the gain in optical amplifier.

The damping time for the particle beam in the longitudinal degree of freedom is

$$\tau = \Delta E_b / P_{loss} , \qquad (11)$$

where ΔE_b is the energy spread of the particle beam.

According to (11), the damping time of the particle beam in the longitudinal plane is shorter, proportionally to its energy spread (not to the initial energy of particles). Moreover, because of non-exponential decay of its both energy and angular spreads the degree of cooling is much higher than 1/e reduction of the beam emittance [1].

If in the pick-up method of cooling the screen of the optical system will open images of all particles of the beam and at this moment it will be stopped, the open particles will continue to loose their energies up to the moment when they will be displayed inward to the distances corresponding to overlapping their URW by the screen. After this time all particles will stay at a threshold energy with the energy spread determined by the jump of the particle energy $\Delta E_{loss} = P_{loss}\sqrt{n_c}/f$, where $n_c = (i/i_c)(\sigma_{URW}/\sigma_b)$ is the number of particles in a sample;

σ_{URW} is the transverse area occupied by the URW and σ_b is the area occupied by the particle beam.

The minimum rms transverse dimension of the beam σ_x is determined by jumps δx_η of the closed orbit of particles. According to (1),

$$\overline{A_x^2} = (\delta x_\eta)^2 N_c \ , \qquad (12)$$

where $N_c = (\Delta E_b / \Delta E_{loss}) n_c$ is the number of interactions of particles with URWs. In this case the value

$$\sigma_x = \sqrt{\overline{A_x^2}} = \delta x_\eta \sqrt{\frac{\Delta E_b}{\Delta E_{loss}} n_c} = \frac{\partial x_\eta}{\partial E} \Delta E_{loss} \sqrt{\frac{\Delta E_b}{\Delta E_{loss}} n_c} =$$

$$= \Delta x_{\eta,0} \sqrt{\frac{\Delta E_{loss}}{\Delta E_b} n_c} \ , \qquad (13)$$

where $\Delta x_{\eta,0}$ is the initial spread of closed orbits of the beam.

In the smooth approximation the relative phase shifts of particles in their URWs radiated in the pick-up undulator and displaced to the entrance of kick undulators depend on theirs energy and amplitude of betatron oscillations. If we assume that the longitudinal shifts of URWs $\Delta l < \lambda_{UR}/2$, then the amplitudes of betatron oscillations, transverse horizontal emittance of the beam and the energy spread of the beam, in the smooth approximation, must not exceed the values

$$a << \frac{\sqrt{\lambda_{UR}\lambda_{bet}}}{\pi}, \quad \varepsilon_x < 2\lambda_{UR}, \quad \frac{\Delta\gamma}{\gamma} < \frac{\beta^2}{\eta_c}\frac{\lambda_{UR}}{\lambda_{bet}}, \qquad (14)$$

where $\eta_c = \alpha_c - \gamma^{-2}$ and α_c are local slip and momentum compaction factors between undulators. Strong limitations (14) to the energy spread can be overcame if, according to the decrease of the high energy edge of the being cooled beam, a change in time of optical paths of URWs is produced. Special elements in storage ring lattices (short inverted dipoles, quadrupole lenses et al.) to decrease the slip [9] can be used as well. With cooling of fraction of the beam at a time only, the lengthening problem diminishes also as the $\Delta E/E$ now stands for the energy spread in the part of the beam which is under cooling at the moment.

The power of the amplifier is equal to the power of the amplified URWs

$$P_{ampl} = \varepsilon_{sample} \cdot f \cdot N_p / N_{kick} \ , \qquad (15)$$

where $\varepsilon_{sample} = \hbar\omega_{1,\max} N_{ph} \alpha_{ampl}$ is the energy in a sample; N_p, the number of particles in the ring.

The transverse selectivity of radiation (movable screen) can be arranged with help of electro-optical elements. These elements contain crystals, which change its refraction index while external voltage applied. This technique is well known in optics [10]. In simplest case the sequence of electro-optical deflector and a diaphragm followed by optical lenses, allow controllable selection of radiation generated by different parts of the beam.

EXAMPLE

Now let us consider an example of enhanced cooling of fully stripped $^{82}_{207}Pb$ ion beam in the CERN LHC. The relevant parameters of the LHC: circumference C=27 km, $f = 1.1 \cdot 10^4$, $\alpha_c = 3 \cdot 10^{-4}$, $\gamma = 10^3$, $M_p c^2 \gamma = 192$ TeV, $\Delta\gamma/\gamma = 10^{-4}$, $N_p = 3 \cdot 10^9$. One pick-up and 10 kick undulators with parameters $\sqrt{\overline{B^2}} = 10^5$ Gs, $\lambda_u = 100$ cm, M=30 are used. The amplifier gain goes to be $\alpha_{ampl} = 10^6$.

In this case: $N_{ph} = 422$, $i_c = 0.26$ mA ($N_c = i_c / ef = 1.7 \cdot 10^9$), $\lambda_{UR,1} = 5 \cdot 10^{-5}$ cm, K=0.37, $\sigma_w = 7.7 \cdot 10^{-2}$ cm, $E_w \cong 0.356$ V/cm, $P_{loss} = 4.35 \cdot 10^8$ eV/sec, $\Delta E_{loss} = 3.95 \cdot 10^4$ eV/rev, $\tau_u = 44.1$ sec, $P_{ampl} = 450$ W, the bandwidth of the URW $\Delta\omega/\omega \cong 1/K = 1/30 << \Delta\gamma/\gamma$ (particles overlapped with the URWs interacting with them), $a < 5$ mm.

CONCLUSION

We considered EOC of particle beams in storage rings for unbunched beam. The rate of EOC defined by the ratio of the energy spread to the rate energy loss is more than the rate of ordinary optical stochastic cooling (OSC) and of the order of the rate of laser cooling based on internal selectivity. EOC for bunched beam, peculiarities of cooling with $i > i_c$, influence of a noise of optical amplifier, other examples of ion cooling in RHIC, HERA et al. and EOC of other particles will be considered in separate publications.

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