NUMERICAL STUDIES OF THE FRICTION FORCE FOR THE RHIC ELECTRON COOLER*

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Abstract

Accurate calculation of electron cooling times requires an accurate description of the dynamical friction force. The proposed RHIC cooler will require ~55 MeV electrons, which must be obtained from an RF linac, leading to very high transverse electron temperatures. A strong solenoid will be used to magnetize the electrons and suppress the transverse temperature, but the achievable magnetized cooling logarithm will not be large. In this paper, we explore the magnetized friction force for parameters of the RHIC cooler, using the VORPAL code [1]. VORPAL can simulate dynamical friction and diffusion coefficients directly from first principles [2]. Various aspects of the friction force are addressed for the problem of high-energy electron cooling in the RHIC regime.

SIMULATIONS

The first step towards accurate calculation of cooling times is to use an accurate description of the cooling force. The achievable Coulomb logarithm in the analytic expression for the magnetized cooling force is not very large [3]. In addition, in some regimes there is a significant discrepancy between available formulas. For this reason the ParSEC project at Tech-X Corp. is being used to develop a parallel code capability based on the VORPAL code [1] to simulate from first principles the friction force and diffusion coefficients for the RHIC parameters. This project is using molecular dynamics techniques (i.e. simulating every particle in the problem) to explicitly resolve close binary collisions and thus capture the friction force and the diffusion tensor with a bare minimum of physical assumptions. Careful testing indicates that the algorithm is working well [2] for magnetized and unmagnetized cases that are relevant to the proposed electron cooling section for RHIC [4].

The primary goals of VORPAL simulation for the RHIC cooling project can be summarized as follows: 1) resolve differences in analytic calculations (approximations of uniform electron density, no space charge, infinite magnetic field, etc.); 2) determine validity of Z^2 scaling for friction force (non-linear plasma effects in magnetized plasma Debye shielding); 3) understand the effects of the space charge and diffusion dynamics; 4) understand the effects of magnetization (from strong to weak magnetization, effect of magnetic field errors); 5) accurate calculations of the friction force in the regime of small Coulomb logarithm due to magnetized collisions; 6) if the friction force for RHIC regime significantly deviates from description based on simple formulas, provide a numerical table of friction coefficients for use in other codes.

Preliminary studies with VORPAL found good agreement with available formulas in some parameters regimes and deviate in others. Some of these studies, which are directly relevant for the parameters of the magnetized cooling in RHIC are presented in this paper.

MAGNETIZED FRICTION FORCE THEORY

The process of magnetized cooling is based on the energy loss of an ion due to Coulomb interaction with the superimposed electron beam, which is guided by the longitudinal magnetic field. For a strong magnetic field, the electron dynamics in the transverse direction is effectively frozen, which together with the flattened velocity distribution of the electrons results in very fast cooling. There exist two standard approaches for the theoretical description of magnetized cooling. The first one is based on the dielectric response of the plasma to the perturbation caused by an ion. The treatment is valid for weakly coupled plasmas, and needs a carefully chosen cutoff parameter for close encounters, because linear theory cannot treat close encounters between the ion and plasma electrons. The alternative approach is based on the calculation of energy transfer due to successive binary collisions, where an approximation is needed for the shielding of the Coulomb potential by the plasma. Apart from the fact that the binary collisions approach cannot describe collective plasma excitation correctly (which may be important for some parameters), both treatments may be regarded as complementary. In fact, similar results can be achieved with the appropriate choice of the cutoff parameter [5, 6].

Closed form analytic expressions can be obtained only in the limiting cases of zero and infinite magnetic field, with numerical simulations required for finite magnetic field values. A simplified kinetic model can be described by considering two types of collisions: 1) fast collisions, where effective interaction time is small compared to the Larmor period of electrons; 2) adiabatic (magnetized) collisions, where interaction time is much longer than the Larmor period. As a result, the friction force can be approximated as a non-magnetized part (fast collision) and magnetized part (adiabatic collision) as described by Derbenev and Skrinsky (D-S) [7]. The relative role of fast and adiabatic collisions depends on the relative velocity of ions. The general treatment results in three regions of the impact parameters and relative ion velocities (with an additional repetitive type of collisions also being included). In such a form, suitable for cooling dynamics simulations, the friction force formulas were presented by Meshkov [8], and are sometimes referred to as Derbenev-

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Skrinsky-Meshkov (D-S-M) formulas. In contrast, Parkhomchuk presents a parametric formula – a generalization of the unmagnetized result, which is predicted [9] to agree reasonably well with experiments for a wide range of parameters. The Derbenev-Skrinsky-Meshkov (D-S, D-S-M) [7, 8] and Parkhomchuk (VP) [9] formulas disagree strongly in some parameter regimes, while showing approximate (roughly factor of two) agreement in other regimes. For completeness and comparison with simulations these formulas are listed below. The magnetized part of the Derbenev-Skrinsky (D-S) formulas [7] is given by the following expressions:

$$\mathrm{F}_{\perp}^{DS} = -\frac{1}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\Lambda^A(V_{ion})\frac{\left(V_\perp^2 - 2V_\parallel^2\right)}{V_{ion}^2}\frac{V_\perp}{V_{ion}^3} \quad (1a)$$

$$\mathrm{F}_{\parallel}^{DS} = -\frac{3}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\left[\Lambda^A(V_{ion})\left(\frac{V_\perp}{V_{ion}}\right)^2 + \frac{2}{3}\right]\frac{V_\parallel}{V_{ion}^3} \quad (1b)$$

for $V_{ion} >> \Delta_{e\parallel}$ (longitudinal rms velocity of electrons) and:

$$\mathrm{F}_{\perp}^{DS} = -\frac{\omega_{pe}^2}{\sqrt{2\pi}}\frac{(Ze)^2}{4\pi\varepsilon_0}\Lambda^A(\Delta_{e\parallel})\ln\left(\frac{\Delta_{e\parallel}}{V_\perp}\right)\frac{V_\perp}{\Delta^3_{e\parallel}} \quad (1c)$$

$$\mathrm{F}_{\parallel}^{DS} = -\frac{1}{\sqrt{2\pi}}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\Lambda^A(V_\perp)\frac{V_\parallel}{\Delta^3_{e\parallel}} \quad (1d)$$

for $V_{ion} < \Delta_{e\parallel}$, where

$$\Lambda^A = \ln\left(\rho^{DS}{}_{\max} / \rho^{DS}{}_{\min}\right) \qquad V_{ion}^2 = V_\parallel^2 + V_\perp^2 \quad (1e)$$

$$\rho_{\min}^{DS} = \max(r_L, \rho_{\min}) \quad \rho_{\max}^{DS} = V_{rel}^{DS}/\max(\omega_{pe}, 1/\tau) \quad (1f)$$

$$r_L = V_{rms,e,\perp}/\Omega_L \qquad V_{rel}^{DS} = \max(V_{ion}, \Delta_{e\parallel}) \quad (1g)$$

The Parkhomchuk's (VP) empiric formula is [9]:

$$\mathbf{F}^{VP} = -\frac{1}{\pi}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\Lambda^M \frac{\mathbf{V}_{ion}}{\left(V_{ion}^2 + V_{eff}^2\right)^{3/2}} \quad (2a)$$

$$\Lambda^M = \ln\left(\frac{\rho_{\max}^P + \rho_{\min}^P + r_L}{\rho_{\min}^P + r_L}\right) \quad \rho_{\max}^P = V_{ion}/\max(\omega_{pe}, 1/\tau) \quad (2b)$$

$$\rho_{\min}^P = \left(Ze^2/4\pi\varepsilon_0\right)/m_e V_{ion}^2 \qquad V_{eff}^2 = \Delta_{e\parallel}^2 + \Delta V_{\perp e}^2 \quad (2c)$$

Parkhomchuk's formula includes an effective velocity which may help to relate formulas to realistic situations for experiments. Magnetized cooling is governed by the longitudinal rms velocity spread of electrons $\Delta_{e\parallel}$ (since transverse motion is effectively suppressed), which can be a very small quantity, ensuring "fast cooling". However, in real life, the spread of electrons can be increased due to various effects such as imperfection of the solenoidal field or the space-charge of the electron beam in the cooling section, for example. Their effect can be parametrically described with an additional contribution to an rms velocity spread (ΔV_e), which results in the definition of V_{eff} in Eq. (2c) [9].

Order-of-magnitude agreement between various magnetized friction force formulas has been acceptable for existing electron cooling facilities, because they operate in a regime where the friction force is strong enough by a large margin. In contrast, the proposed RHIC cooler will operate in a completely new regime, where the estimated friction force is expected to be adequate but with little margin for error [10]. As a result, we are trying to understand the accuracy of available formulas for various degrees of magnetization, based on numerical simulations with the VORPAL code.

NUMERICAL STUDIES

Recent advances in code development [2] allowed us to begin systematic study of the magnetized friction force for parameters of the RHIC cooler. As a result of aggressive beam dynamics studies (both of magnetized beam transport and cooling dynamics), the design parameters are also evolving. Various design solutions are being studied to address technical issues and possible effects of cooling on ion beam dynamics. For example, it is technically difficult to produce one long superconducting solenoid of length L=30 or 40 meters. To compensate for coupling (introduced by a solenoid) it may be beneficial to have half-length solenoids with reversed magnetic fields. However, this would result in a reduction of magnetized Coulomb logarithm. As a result, the optimum length of solenoid section (as well as total number of such sections) is presently under study [3]. Similar discussion is true for field strength in the solenoid with present consideration in the range of B=2-5 T (depending on the achievable emittance for magnetized transport of electron bunches with very high charge), density of electrons in the range n_e=0.5-2x10^{15} m^{-3} (depending on several effects).

As a result, validity of available formulas should be established with VORPAL for a wide range of parameters, to resolve many outstanding issues. Detailed study for RHIC parameters will be reported later elsewhere. Here, we choose the upper end of RHIC parameters to show capabilities of the code. The parameters chosen for simulations are the following (in the beam moving frame): Au ions with Z=79, time of flight through the solenoid τ=0.4 ns (length of solenoid section L=13 m), magnetic field B=5 T; n_e=2x10^{15} m^{-3}, transverse rms spread of electrons $\Delta_{e,\perp}$=8x10^6 m/s (which corresponds to typical parameters of the 20 nC magnetized electron beam with transverse normalized rms emittance of 30 µm), longitudinal rms spread of electrons $\Delta_{e,\parallel}$=1x10^5 m/s (corresponds to an rms momentum spread of 3*10^{-4}), and $V_{eff}=\Delta_{e\parallel}$ (no solenoid imperfections).

Figure 1 shows dependence of the longitudinal component of the friction force on ion velocity for ion motion along the magnetic field lines (zero component of the transverse ion velocity): 1) Eq. (1b,d) (blue curve in two regions of applicability) 2) Eq. (2) (green dotted curve) 3) VORPAL results (pink dots with error bars). For these parameters, one can see tendency of formula in Eq. (2) to underestimate the friction force compared to numerical results. Also, VORPAL results indicate that all possible type of collisions should be taken into account in formulas (see Ref. [8], for example), especially at low relative velocities.

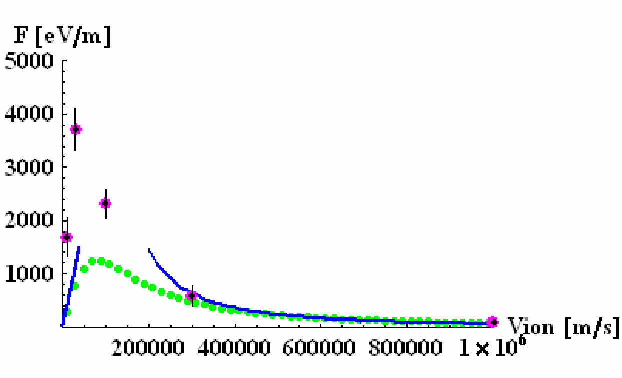

Figure 1: Dependence of the longitudinal component of the friction force on ion velocity: 1) Eq. (1b,d) (blue curve in two regions) 2) Eq. (2) (green dotted curve) 3) VORPAL results (pink dots with error bars).

For realistic simulations of magnetized cooling dynamics [10], it is very important to accurately capture the friction force for ion trajectories at different angles with respect to the magnetic field lines. Equations (1) and (2) show dramatic differences in this respect. Equations (1) show anisotropy introduced by a strong magnetic field, while Eq.'s (2) neglect such behavior. In fact, the main argument in Ref. [9] in favor of Eq. (2a) was that Eq. (1b) gives zero results for the longitudinal component of the friction force when the transverse component of ion velocity goes to zero. This happens only if the logarithmic term is kept in Eq. (1b), while the constant term is neglected. However, the magnetized Coulomb logarithm typically has a small value of around 2-3 (even smaller for small relative velocities), so that for small transverse angles ($\sin(\theta)=V_{ion,tr}/V_{ion}$), the constant and logarithmic terms in Eq. (1b) become comparable.

Figure 2 shows the dependence of the longitudinal component of the friction force on the transverse angle θ for simulation parameters close to those described above, and fixed ion velocity of $3*10^5$ m/s. The conditions of applicability for Eq. (1b) are satisfied: 1) red curve - Eq (1b) with a constant term being neglected (as used in [9]); 2) blue curve – Eq. (1b) with both the logarithmic and constant terms; 3) green curve – Eq. (2); 4) pinks dots with error bars - VORPAL results. One may argue about accuracy and validity of the formulas derived in logarithmic approximation, but we see a remarkable agreement with the direct numerical simulation obtained with VORPAL, which are not based on these logarithms or any analytic approximations.

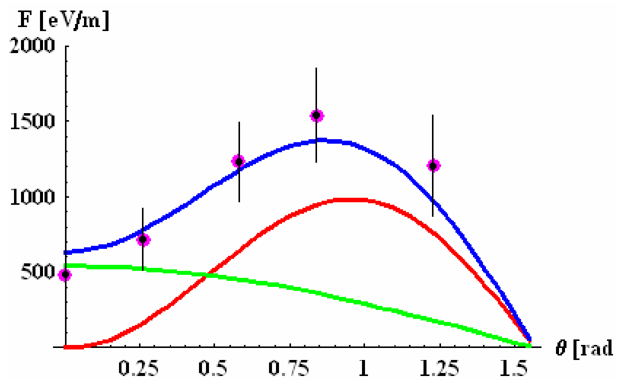

Figure 2: Dependence of the longitudinal component of the friction force on the transverse angle of ion velocities – comparison of formulas vs. direct numeric simulations.

The on-going simulations with the VORPAL code show great promise for resolving ambiguities in the theoretical understanding of the magnetized friction force under idealized conditions, and also for determining quantitatively the effect of complicating factors, including the impact of errors in the magnetic field [11]. The benchmarking of the code vs. recent precision measurements of the friction force at CELSIUS cooler [12] is also in progress and will be reported later.

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