# STUDY OF THE BEAM-BEAM EFFECT FOR CRAB CROSSING IN KEKB AND SUPER KEKB

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#### Abstract

Crab crossing has been proposed to boost up the luminosity performance at KEKB. The luminosity is boosted up by removing x-z coupling due to the crossing angle. For a high beam-beam parameter ( $\xi > 0.05$ ), linear couplings between degree of freedom induces symplectic diffusion, with the result that emittance growth and luminosity degradation arises. We have to care not only x - z coupling (crossing angle) but also x - y and y - z couplings. Chromaticity may be next order contribution of the luminosity degradation, External diffusion due to fast noises, for example kicker noise and phase jitter of cavities in crab crossing, could affect the luminosity performance. We present the tolerance for linear couplings, chromaticity and the fast noise in the crab crossing at KEKB.

#### **INTRODUCTION**

We try to achieve a high beam-beam parameter  $\xi \equiv 2r_e\beta_y L/N\gamma \geq 0.1$  using crab crossing at KEKB. The highest beam-beam parameter is  $\xi \approx 0.16$  for a tune operating point of  $(\nu_x, \nu_y) = (0.503, 0.550)$  in B factories with a damping time of several thousand turns [1]. The beambeam limit, which is considered as fundamental limit, is determined by the diffusion due to quantum excitation of synchrotron radiation [2]. Essential to achieve the beambeam parameter is to reduce other diffusion mechanisms compare with the radiation excitation.

We discuss two types of diffusion which degrade the fundamental beam-beam limit. One is the nonlinear diffusion, which is caused by linear coupling and other errors. Crossing angle is equivalent to linear x - z coupling in arc transfer matrix. The crab cavity is expected to boost up the beam-beam parameter due to removing the x - z coupling. As a matter of course, other linear couplings (x - y, y - z) should be reduced so as not to disturb the improvement due to the crab cavity. Nonlinearity of lattice may contribute the nonlinear diffusion. An effect of chromaticity is discussed as lowest nonlinearity in this paper.

Second is an external diffusion due to artificial noise. Since the electron-positron beam has radiation damping time of several  $\times 10$  ms, noises slower than the damping time does not affect luminosity performance: that is, emittance is considered as an adiabatic invariant for the slow noise. We focus fast noise with the time scale between the revolution time to the damping time. In recent accelerator, bunch by bunch feedback systems are popularly used to suppress coupled bunch instabilities. The feedback system can be a source of a small transverse offset noise at the

collision point. Perhaps the noise is fast, has no correlation turn by burn.

RF cavity can be another source. Phase jitter of RF wave induces fluctuation of longitudinal position of bunches. The jitter of longitudinal position of a bunch is not serious for the beam-beam effect [3]. However the situation changes worse for crab crossing. The phase jitter of crab cavity and accelerating cavity induces a fast transverse offset noise. These noises affect beam-beam performance due to the chaotic nature of the beam-beam system with strong nonlinearity.

We discuss the degradation of the fundamental beambeam limit due to these diffusion processes using a strongstrong beam-beam smiulation [4].

### LINEAR COUPLING

We examine tolerance for linear coupling. The linear coupling enhances nonlinear diffusion [5], with the result that luminosity degradation arises.

The simulation is performed by putting off-diagonal elements for the  $6 \times 6$  transfer matrix (M) in arc section,

$$M(s^*) = V^{-1}(s^*)UV(s^*) \qquad V(s) = B(s)R(s)H(s)$$
(1)

where U, B, R and H are matrices for phase space (betatron) oscillation, tilt of the phase space ( $\beta$  and  $\alpha$ ), x-y coupling and xy-z coupling at the collision point, respectively. Detailed expression is seen in Ref. [5]. The crossing angle is essentially equivalent to x-z coupling, which is represented by H [4].

Figures 1-2 shows the luminosity degradation due to xy coupling  $(r_4)$  and vertical dispersion  $(\eta_y)$ , respectively, at the collision point. The tolerances are  $r_4 = 0.02$  and  $\eta_y = 0.1mm$  for 5% degradation of the luminosity. Other parameters  $r_1 - r_3$ , shift of the waist position of  $\beta$  function and  $\eta'_y$  was examined. The tolerances were acceptable for the present knob tuning in KEKB.



Figure 1: Luminosity and vertical beam size as a function of coupling error,  $r_4$  at the collision point.



Figure 2: Luminosity and vertical beam size as a function of vertical dispersion at the collision point.

Figure 3 shows the sensitivity for the crossing angle. The tolerance is 0.3 mrad for 5% degradation of the luminosity.



Figure 3: Luminosity as a function of the half crossing angle.

#### **CHROMATICITY**

One turn map including the chromaticity is expressed by a Hamiltonian as follows,

$$H_c(y, p_y) = \frac{c_1 y^2 + c_2 y p_y + c_3 p_y^2}{2} \delta.$$
 (2)

where  $\delta = \Delta E/E$ .  $c_i$  is connected with the three types of chromaticities: i.e.,  $d\nu_y/d\delta$ ,  $d\beta_y/d\delta$  and  $d\alpha_y/d\delta$ . The canonical transformation for the Hamiltonian in Eq.(2) is expressed by

$$p_y = \bar{p}_y + \frac{\partial H_c(y, \bar{p}_y)}{\partial y} \qquad \bar{y} = y + \frac{\partial H_c(y, \bar{p}_y)}{\partial \bar{p}_y} \quad (3)$$

New variables after transformation  $(\bar{y}, \bar{p_y})$  is expressed by those before  $(y, p_y)$ . Since the relation is linear, it is easy to get an explicit relation for new variables. The transformation is directly connect with the three types of chromaticities.

Simulation has been done for the chromaticity  $\xi_x \leq 3$  and  $x_y \leq 7$ . The chromaticity is the same level as is used in KEKB operation. There was no clear luminosity degradation in the chromaticity range.

#### NOISE OF COLLISION OFFSET

#### Noise of Vertical offset

The vertical offset noise could be sensitive for the luminosity performance because the beam size is very small for  $e^+e^-$  colliders,  $10000\varepsilon_y \approx 100\varepsilon_x \approx \varepsilon_z$ . To study the noise effect, vertical offset is applied turn by turn with Gaussian distribution with a deviation,  $\Delta y$ , in the simulation

$$\langle y(t)y(t')\rangle = \Delta y^2 \delta(t-t') \qquad y(t) = \Delta y \hat{r},$$
 (4)

where  $\hat{r}$  is a Gaussian random number with unit deviation.

Figure 4 shows the luminosity and vertical beam size as a function of amplitude of vertical offset noise. The luminosity is sensitive for the offset noise: 5% degradation for offset noise of  $0.01\sigma_y$  is seen in the figure. In the beam-beam limit, radiation excitation played a important role. The diffusion rate of the radiation excitation is  $\Delta y \approx \sigma_y \sqrt{2/\tau_y}$ , where  $\tau_y$  is damping time in unit of turn. In our case, since  $\tau_y$  is 4000-6000 turns, the diffusion rate, which is  $0.02\sigma_y$ , is comparable with the sensitivity.



Figure 4: Luminosity and vertical beam size as a function of amplitude of vertical offset noise at the collision point.

We compare the degradation due to the fast noise with that for a static offset. Figure 5 shows the luminosity and beam size as a function of the static vertical offset. The luminosity is less sensitive (1/20) for the static offset. The tolerance for the static offset is somewhat severer than the geometrical degradation, but is acceptable. The luminosity performance is very sensitive for a fast noise, therefore it should be treated carefully.



Figure 5: Luminosity and vertical beam size as a function of amplitude of static vertical offset at the collision point.

## Phase errors of crab and accelerating cavities: noise of horizontal offset

The crab cavity can be source of diffusion: i.e., since the crab cavity is operated by a transverse mode, the deviation and jitter of RF phase give a dipole kick to the beam, with the result that transverse offset at the collision point is generated. Both of phases of main RF and crab cavity can be source of the transverse offset.

Jitters of RF phase of main cavity causes a deviation of timing of beam arrival at the crab cavity. The transverse offset, which arise from the jitter of main RF system, is expressed by

$$\delta x = \frac{c \tan \phi}{\omega_{RF}} \delta \psi_{RF}.$$
 (5)

where  $\delta \psi_{RF}$  is the phase error of the main RF system.

The crab cavity gives a transverse kick due to its jitters of RF phase, with the result that the offset given by the kick is expressed as follows,

$$\delta x = \frac{c \tan \phi}{\omega_{RF}} \frac{\cos(\pi \nu_x - \Delta \Psi(s^*, s_c))}{2 \sin \pi \nu_x} \delta \psi_{crab}, \quad (6)$$

where  $\Delta \Psi(s^*, s_c)$  and  $\delta \psi_{RF}$  are the betatron phase difference between the collision point and the crab cavity and the deviation of the RF phase of the crab cavity, respectively. In the both cases, the jitter of transverse offset is given by  $\delta x \approx c \tan \phi \delta \psi / \omega_{RF}$ .

The noise of RF phase has a fast variation, but the correlation time is longer than the revolution time. Since the quality factor of the cavity is Q = 200,000, noise may have a correlation time of  $Q/\omega_{RF}$  which correspond to several 10 turns. The offset noise is applied so as to satisfy  $\langle y(t)y(t')\rangle = \Delta y^2 e^{-|t-t'|/\tau}$  in the simulation, where  $\tau$  is correlation time in unit of turn.

Figure 6 shows the luminosity and horizontal beam size as a function of amplitude of horizontal offset noise. Two results, which are obtained for  $\tau = 1$  and 10, are depicted in the figure. The horizontal axis is displayed with  $\Delta x/\sigma_x$ . The tolerance is  $0.02\sigma_x$  for 5% degradation at  $\tau = 10$ , which correspond to the phase error of 0.1 degree. The tolerance is acceptable, though it is severe. The beam phase change along bunch train due to beam loading. Since the change is static for each collision, it is not serious. We have to care for fast noise which occurs collision by collision.



Figure 6: Luminosity and horizontal beam size as a function of amplitude of horizontal offset noise at the collision point.

## OTHER POSSIBILITIES FOR LUMINOSITY DEGRADATION

There are many sources to degrade the luminosity performance. We did not discuss coherent motion in this paper. Beam life time is also important issue [6].

We here discuss unbalance between two beams briefly as a last part. Figure 7 shows the evolution of beam size for unequal damping time; 4000 and 6000 turns for HER and LER, respectively. The beam size variation occurs in the time scale of 1 ms. Such an unbalance could occur also due to the breaking the transparency condition. If the enlarged beam has an enough life time, equalization procedure, for example control the vertical dispersion of HER as is done in KEKB, may help to keep the unbalance. We need fine feedback system for keeping the balance.



Figure 7: Beam size asymmetry due to different damping time, 4000 and 6000 turns for HER beam, respectively.

#### **SUMMARY**

We have various hurdles to achieve the fundamental beam-beam limit. Success of super B factories depends on how we get over the hurdles. Study for crab crossing and a challenge toward the high beam-beam parameter ( $\xi > 0.1$ ) starts from 2006 at KEKB.

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