# OPTIMIZED SHAPE OF CAVITY CELLS FOR APERTURES SMALLER THAN IN TESLA GEOMETRY\*

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#### INTRODUCTION

With superconductivity as the chosen technology for the future International Linear Collider, the responsible choice of all technical solutions is an on-going process.

In our previous papers [1, 2] we have shown that the accelerating rate of superconducting cavities for ILC can be increased for the same iris aperture if

- 1. Some increase of  $E_{pk}/E_{acc}$  is permitted so that the value of  $H_{pk}/E_{acc}$  can be lowered in comparison with the original ILC regular cell shape. ( $E_{pk}$  and  $H_{pk}$  are maximal electric and magnetic fields on the surface,  $E_{acc}$  is the acceleration rate in the given cell).
- 2. Shape of the cells is described by two elliptic arcs instead of "circular arc straight segment elliptic arc" contour as in the original TESLA shape.
- 3. The reentrant cavities obtained as a result of consecutive optimization with this two-elliptic-arcs approach are treated as a possible version of the accelerating cells in spite of some technological complications by fabrication.

On November 16, 2004, an accelerating gradient of 46 MV/m (CW) and 47 MV/m (pulsed) were achieved in a superconducting niobium cavity [3]. This represents a world record gradient in a niobium RF resonator. This 1.3 GHz cavity has a reduced (by 10 % in comparison to TESLA cavity) ratio of  $H_{pk}/E_{acc}$  obtained by sacrificing the value of  $E_{pk}/E_{acc}$  (by 20 %), and its geometry is close to the optimized reentrant regular cell geometry.

Not only the values of  $H_{pk}/E_{acc}$  can be improved but also values of cell-to-cell coupling k, values of R/Q, and of the geometry constant G grow with the transition to the reentrant shapes.

However, not all benefits of this shape are employed. First of all, increased cell-to-cell coupling prompts that the aperture of the original cell is big enough to be decreased without loss of field flatness in comparison with the original design. This decrease will lead to further increase of the  $E_{acc}$  for the same  $H_{pk}$ , also as to improvement of others important parameters. Here, a broader range of calculations for the same as the original and for smaller apertures is presented, and proposals for a better choice of ILC cavity cells are derived.

### **OPTIMIZATION CURVES**

For the TESLA accelerating cavity as reported in [4] the defining field ratios are

$$E_{pk}/E_{acc} = 2$$
,  $H_{pk}/E_{acc} = 42 \text{ Oe/(MV/m)}$ .

More recent data for the same values are 2.0 and 42.6 [5], but we will use for convenience the old "round" numbers.

We will compare values of calculated fields with values for TESLA and introduce for this purpose the normalized peak electric and magnetic fields:

$$e = E_{pk} / 2E_{acc}$$
,  $h = H_{pk} / 42E_{acc}$ , (1)

so that for the regular TESLA cells e = 1, h = 1.

The process of optimization consists in searching a cell shape with a minimal value of the normalized peak magnetic surface field in a cell for each value of the normalized peak electric field. The result of optimization is a function h(e). We presented this function earlier for the iris (beam-pipe) aperture  $R_{bp} = 35$  mm. Now in Fig. 1 are shown the same dependences for  $R_{bp} = 32.5$  and 30 mm as well. This picture reflects how the magnetic field can be decreased if we sacrifice by the electric field but keep the same value of acceleration per cell.

However, it would be more physically demonstrative if we keep the same value of peak magnetic field that was achieved in the TESLA structure (whatever it is), and calculate the gain in acceleration in dependence on the increased value of peak surface electric field. If we designate the maximal obtained magnetic field in TESLA by  $H_{pk}^T$ , the corresponding electric field by  $E_{pk}^T$ , and take into account equations (1), we can write:

$$h = H_{pk}^{T} / 42E_{acc}, \ 1 = H_{pk}^{T} / 42E_{acc}^{T},$$
  
$$e = E_{pk} / 2E_{acc}, \ 1 = E_{pk}^{T} / 2E_{acc}^{T}.$$

It follows herefrom:

$$E_{acc}/E_{acc}^{T} = 1/h$$
,  $E_{pk}/E_{pk}^{T} = e/h$ .

Now, we can reconstruct the curves of the Fig. 1 to new coordinates 1/h and e/h which will show us how much we need to increase the peak surface field if we want to increase the acceleration by a certain value keeping the same  $H_{pk} = H_{pk}^T$ , Fig. 2. For example, the previous statement: to decrease the magnetic field by 10 % with the same acceleration will need to increase the electric field by 20 %, now will sound for the same point as follows: to keep the same magnetic field and to have the acceleration 11.1 % higher we need to increase the surface electric field by 33.3 % in comparison with the TESLA cavity. This point is shown on both graphs, Fig. 1 and Fig. 2, as the point A.

Supported by NSF

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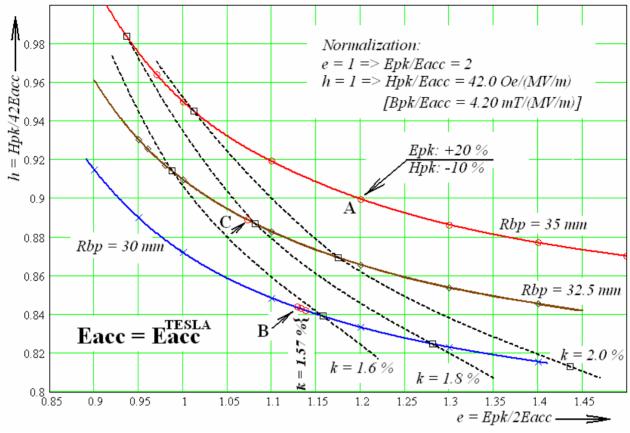


Fig. 1. Optimization curves for inner cells: normalized electric and magnetic fields for different apertures of the iris; *k* is cell-to-cell coupling; cells have the reentrant shape.

## **COUPLING**

Curves for coupling coefficient k for optimized reentrant cells are also shown in Fig. 1 and 2. One can see that coupling rapidly increases when we go to higher e or h. TESLA regular cells have k about 1.87 %.

One of principal limitation factors of cells number for superconducting cavity is field nonuniformity [6] which is proportional to  $\Delta f / f \cdot N^{3/2} \cdot k^{-1}$ , where  $\Delta f / f$  is average relative error of cells frequency, N is number of cells in the cavity. When we go to smaller radii of the aperture, coupling rapidly decreases. However, one can choose higher accelerating rate, and then take smaller number of cells. Nonuniformity of electric field at the point B (k = 1.57 %), see Fig. 1 or 2, is the same as at the point A, if number of cells is decreased from 9 to 8 because  $1.87 \cdot (8/9)^{1.5} = 1.57$ . Normalized acceleration at the point B  $(1.187 \cdot 8 = 9.50)$  will stay higher than acceleration in the original TESLA cavity with 9 cells  $(1 \cdot 9 = 9)$  with the same peak magnetic field. Acceleration rate at the point B will be 49 MV/m if we believe that we have 46 MV/m in the experiment for the point A.

As an intermediate choice, we can take the point C with strong coupling and moderate overvoltage with 8 cells having the same acceleration as 9 TESLA cells have:  $1.125 \cdot 8 = 9$ , see Fig. 2.

The function h(e) for any given  $R_{bp}$ , and for any given e, is a minimal value of h over an array of parameters A, B, and a, where A and B are horizontal and vertical half-axes of the bigger and a is horizontal half-axis of the smaller ellipse defining the cell shape. This is unconditional minimum. However, we can make an attempt to find a minimum under condition k=1.57 %, for the same  $R_{bp}=30$  mm. This curve is shown as a chain of points from the point B. It goes very close to the main curve for  $R_{bp}=30$  mm. Unfortunately, this curve is a limited one, it cannot be continued below  $1/h \approx 1.185$ . Nevertheless, a possibility exists to slightly decrease  $E_{pk}$  for this geometry and make it the same as at the point A.

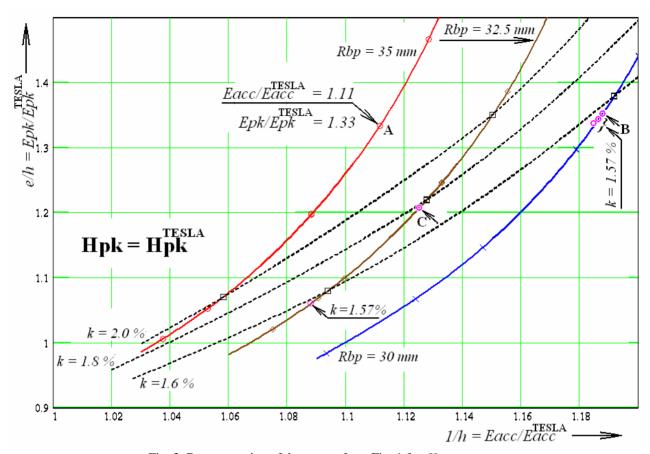


Fig. 2. Reconstruction of the curves from Fig. 1 for  $H_{nk} = \text{const}$ .

#### CONCLUSIONS

The cavities for International Linear Collider upgrade can be made substantially shorter and with higher acceleration rate if the reentrant shape of cells is adopted. This shape can be used with smaller iris radii without loss of field uniformity in comparison with original TESLA cavities. Issues of HOM, wakefields, and Lorenz force detuning should be taken into account. Hopefully, the trade-off between HOM extraction and the cavity length also exists. If we reach the same surface fields as in our record experiment, with smaller aperture we can come right up to 50 MV/m accelerating rate.

The author is thankful to Hasan Padamsee for very helpful remarks.

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