MEASUREMENT OF LINEAR LATTICE FUNCTIONS IN THE ESRF STORAGE RING USING TURN-BY-TURN DATA

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Abstract

Model-independent methods to measure linear optics functions have been tested in turn-by-turn data from the ESRF storage ring. The methods do not necessitate neither the knowledge of the model nor magnetic element manipulation. They use only the positions measured in consecutive BPM of betatron oscillations issued by small transverse kicks. The phase advances and tunes necessary to construct the transfer matrices are issued by refined Fourier analysis. Measurements of off-momentum optics parameters using longitudinal beam excitation are also presented.

OUTLINE AND EXPERIMENTAL SETUP

Several techniques for measuring linear beam optics functions in rings have been proposed in the last decade [1]. Two methods among them, and their variants have been the most popular: one uses the phase advances between three BPM measured by harmonic analysis of turn-by-turn (TBT) data and the machine model in order to reconstruct the beta variation around the ring [2]. The second is based on a response matrix construction issued by the variation of the orbits or tunes when machine elements like steerers or quadrupoles are varied [3]. The linear optics can be reconstructed by a machine model fitted to the beam response through a Singular Value Decomposition (SVD) algorithm. The main drawback of the first approach is the need of an accurate machine model which most of the times does not exist. The second method necessitates manipulation of machine elements which, apart from distorting the machine itself by introducing higher order effects, can be quite time consuming, e.g. a complete steerer response matrix for the ESRF storage ring takes around 30 minutes.

Recently, a Model Independent Analysis method has been developed, based on a spatio-temporal mode decomposition which is obtained by an SVD of the matrix formed by beam orbit histories measured on a large number of BPM [4]. Through this technique, it was possible to measure phase advances and beta functions at all BPM of the APS storage ring [5]. In this paper, an alternative fast method based on refined Fourier analysis of betatron oscillations issued by small transverse kicks and an iterative TBT linear transfer matrix reconstruction is proposed. This method along with novel techniques enables the estimation of off-momentum beam parameters when applied to TBT data issued by longitudinal beam excitations.

Three different equipments producing the necessary beam excitation for the TBT measurements are available in the ESRF storage ring, enabling the experimental exploration of the full 6D phase space: an horizontal injection kicker, a dedicated vertical kicker and an RF phase shifter. Note that both horizontal and vertical maximum kicker amplitudes of 10 and 7 mm are limited due to the dynamic and physical aperture, respectively. The same is true for the maximum longitudinal excitation.

Table 1: Kicker parameters.

Type	Hor. Kicker	Vert. Kicker	Phase Shifter
Pulse length [μs]	1	1	Output of the
Rep. rate [Hz]	10	1-100	master source
Defl. angle [mrad]	2	0.6	with maximum
β_x,β_y [m]	5	35	kick of
Max. ampl. [mm]	10	7	$dp/p \approx 1.5\,\%$

A bunch train filling 1/3 of the machine with a current of 10 mA is kicked and beam positions are recorded on the standard "1000-turn" system [6] comprising 224 BPM. This system provides a "pseudo"-TBT acquisition, as the position is reconstructed from the multi-plexed signal on each electrode in four different kicks [6]. The linearity is guaranteed due to a signal pre-processing algorithm based on the BPM electro-magnetic model. The data is averaged over many groups of kicks (16 to 256) in order to provide a resolution of 1 μm. Apart from the high resolution the averaging smoothes out any kicker jitter and BPM calibration error.

TRANSFER MATRIX RECONSTRUCTION

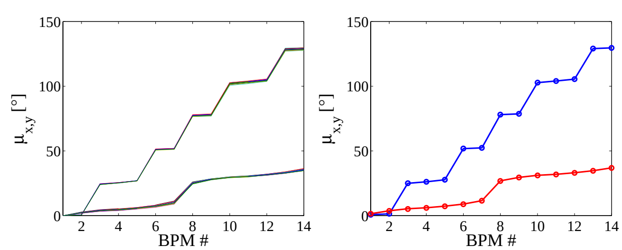

Figure 1: Horizontal and vertical phase advances measured on all BPM of the 16 cells (left) and theoretical horizontal (blue) and vertical (red) values from the perfect machine.

Neglecting coupling, damping and non-linear effects, the transfer matrix between BPM 1 and 2 provides two equations involving positions, angles and the linear optics' functions on these two locations. There are seven unknowns, namely the two slopes $x'_{1,2}$, beta $\beta_{1,2}$ and alpha $\alpha_{1,2}$ functions and the phase advance $\mu_{1\mapsto 2}$. Alternative ways have to be found for estimating some of these parameters in order to reconstruct the transfer matrix. The phase advance can be measured by an alternative of the harmonic analysis applied at LEP [2], using the frequency analysis of Laskar [7]. The method approximates the TBT positions by a quasi-periodic series: $x(s) = A_1 e^{i\phi_1} e^{i\nu_1 t} +$

$\sum_{k=2}^{N} A_k e^{i\phi_k} e^{i\nu_k t}$ where the first term provides the tune ν_1, with a very high precision [7]. The phase advances between consecutive BPM are estimated by the difference of the Fourier phases ϕ_1. In Fig. 1, the measured horizontal and vertical phases are plotted and superimposed for the 14 symmetric BPM of all 16 cells around the ring. The measured phase advances are very close to the ones provided by the perfect model of a completely symmetric lattice, showing an rms difference of only 5%. Another indication of the precision in the phase advance determination is given by the difference of the fractional part of the phase advance in one turn with respect to the tune, which is of the order of 10^{-3}, in both planes.

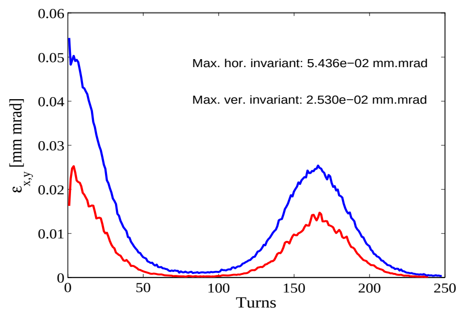

Figure 2: Mean values of the measured horizontal (blue) and vertical (red) transverse invariants for the recorded number of turns.

Although the Fourier amplitudes A_1 can be used for estimating the beta function on each BPM, a different approach will be followed. The next step is to estimate the angles. This can be done by using a "virtual" BPM, i.e. shifting the Fourier approximation of the position by $\pi/2$ which is equivalent to a differentiation with respect to the independent variable [2]. An alternative way can be used in machines as sychnotron light storage rings or colliders where a few pick-ups are located on straight sections on either sides of the insertion devices or interaction points. In the case of the ESRF, 64 out of 224 BPM are on straight sections. In that case, the angles are known by the position and the distance between the BPM. Using the 1-turn matrix, the beta and alpha functions on these BPM can be estimated. Now the number of unknowns is limited to three, i.e. the beta and alpha functions and the angles in the consecutive BPM. The third equation needed to estimate these parameters can be found by building a turn-by-turn "invariant" $\epsilon = \gamma x^2 + 2\alpha x x' + \beta x'2$ on each BPM located in the straight sections. This parameter can be considered constant for one turn and indeed its rms variation taken on all straight section BPM is of the order of 10^{-2}. Its oscillation can be observed in Fig. 2, where the typical decoherence and re-coherence due to chromaticity can be observed in both planes.

Using the "invariant" and the transfer matrix equations, the beta and alpha functions of the BPM upstream and downstream of the ones located in the straight section can be measured. The solutions are propagated in order to estimate all betas and alphas up to the next straight section. This process can be repeated for each turn up to the point

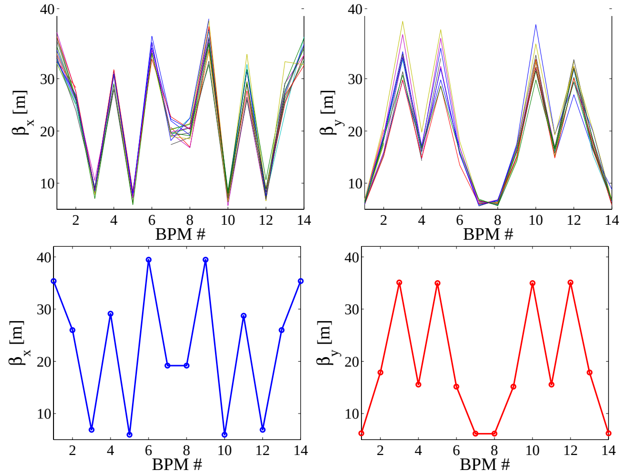

Figure 3: Horizontal (left) and vertical (right) beta functions measured on all BPM of the 16 cells (top) and theoretical values computed from the perfect model (bottom).

that the requested accuracy is obtained. Actually, the convergence of the method is quite fast, giving a precision of the order of less than 1% in around 50 turns. The measured horizontal and vertical beta functions are shown in Fig. 3 along with the theoretical values estimated by the symmetric model. The agreement between the model and measurements is good considering the fact that the model is an idealisation of the actual machine. A better evaluation can be achieved when comparing measurements done by two different methods: in Fig. 4, the horizontal and vertical normalised beta variation on each symmetric BPM is plotted and compared to the beta estimation using a method based on steerer response matrix analysis. Both methods show an equivalent beta variation, i.e. the TBT method predicts an rms horizontal beating of 11% and vertical beating of 10% whereas the response matrix method gives 7% on horizontal and 11% on vertical.

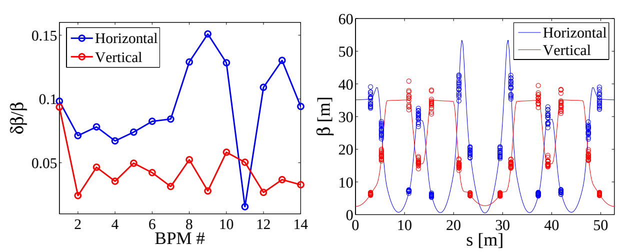

Figure 4: Horizontal (blue) and vertical (red) beta variation measured on all BPM of the 16 symmetric cells (left) and measured beta values (right) from a response matrix analysis (points) as compared to the perfect machine (lines).

OFF-MOMENTUM OPTICS

Off-momentum optics measurements are possible through the TBT oscillations issued by longitudinal beam excitations coupled with transverse kicks. The longitudinal kicks are produced by pulsing a phase shifter in the output of the storage ring RF, in a step like form, for a time longer than the longitudinal damping time. The TBT measurement system is synchronised with the fall part of the pulse. In that way, clean transverse oscillations with

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the synchrotron frequency are produced. An example of the positions recorded on a dispersive BPM, are shown in Fig. 5. The phase shifter enables the application of very large momentum kicks of up to 1.5%. Through the frequency analysis of the TBT data, the sychnotron tune can be measured with a normalised precision of better than 10^{-3}. This measurement can be used for calibrating the RF voltage read-out as in Fig. 5 or other non-linear longitudinal phase space measurements.

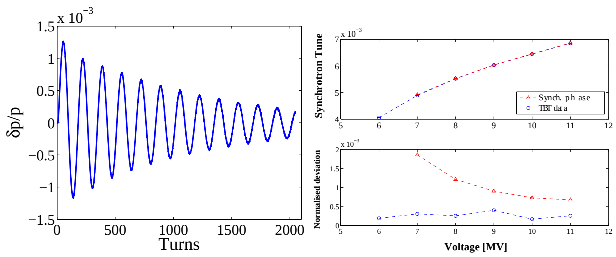

Figure 5: Momentum oscillations issued by a longitudinal kick, recorded on a dispersive BPM (left) and sychnotron tune with respect to the RF voltage (right top) with the associated precision (right bottom), using TBT (blue) and synchrotron phase measurements (red).

Horizontal and vertical dispersion along the storage ring can be measured by the linear fit between the TBT displacements and the momentum spread (Fig. 6). Due to the large number of turns available, the precision is excellent (it represents the size of the points). There is also a very good agreement between this measurement and the one provided by a classical RF scan with a mean rms difference of 1 % in the horizontal and 5 % in the vertical plane.

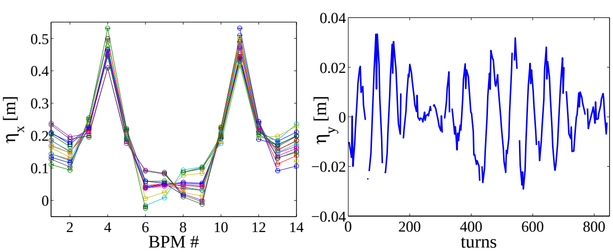

Figure 6: Horizontal (left) and vertical dispersion measured by the TBT data issued by longitudinal kicks.

Using the transfer matrix reconstruction method, horizontal off-momentum β can be measured (Fig. 7). Comparing with the on-momentum optics, an off-momentum β variation is estimated and plotted around the ring, as in the bottom part of Fig. 7. The β variation presents a characteristic beta modulation pattern, with an rms off-momentum β-beating of 8%, for a momentum spread of 1%.

Higher order dispersion can be measured by estimating the fitted coefficients of a polynomial between the displacements and the momentum spread. An alternative method can be followed [9], using the fact that the Fourier amplitudes of nQ_s is associated with dispersion η_n of order n. In particular, the Fourier amplitude of $2Q_s$ is $A_{2s} = \frac{1}{4}\eta_2(s)\sigma_\delta^2 l^2$ where σ_δ^2, the size of the energy spread and l the strength of the longitudinal kick. The average measured Fourier amplitude over symmetric cells around the

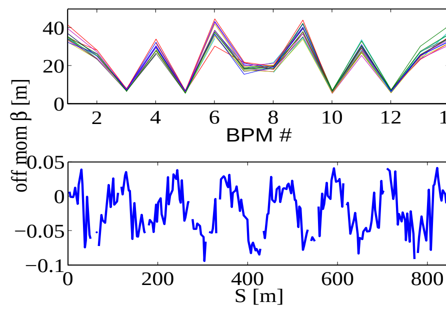

Figure 7: Horizontal off-momentum beta function (top) and corresponding beta variation with respect to on-momentum measurements (bottom).

ESRF ring is shown on the left side of Fig. 8. The error bars correspond to one standard deviation and are associated to second order dispersion beating.

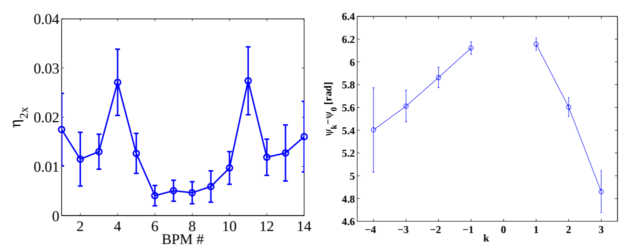

Figure 8: Mean amplitude of the spectral line $2Q_s$ associated to second order dispersion (left) and phase difference of sychno-betatron side-bands versus their order (right). The slope is associated to chromaticity.

Chromaticity of any order can be estimated by the coefficients of a fitted polynomial between the off-momentum tune-shift and the momentum spread [8]. Recently, an interesting measurement was proposed [9] relying on the Fourier phases ψ_k of the synchrotron side-bands. Their difference with respect to the phase of the main Fourier term ψ_0, should be $\psi_k - \psi_0 = -|k| \arctan(\frac{lQ_s}{Q'\sigma_\delta})$, i.e. proportional to the order k [9]. The proportionality factor depends on the chromaticity Q'. Fig. 8 demonstrates that indeed the dependence of the phase difference is linear with k. Nevertheless, the slope is not symmetric with respect to zero as the above formula should be corrected, taking into account the off-momentum optics beating, as in the case of the Fourier amplitudes of the side-bands [9].

REFERENCES

[1] M.G. Minty, F. Zimmermann, Measurement and control of charged particle beams, Springer, 2003.

[2] J. Borer et al, EPAC'92, Berlin, Germany, p.1082; P. Castro, PAC'99, New York NY, USA, p. 456.

[3] J. Corbett et al., PAC'93, Washington DC USA, p.108; D. Robin et al., EPAC'96, Sitges, Spain, p.971; J. Safranek, NIMA 388, 27 (1997).

[4] J. Irwin et al., PRL 82, 1684 (1999); C. Wang, PhD thesis, Stanford Un., 1999.

[5] C. Wang et al., PRSTAB 6, 104001 (2003).

[6] L. Farvaque et al., DIPAC 2001, Grenoble, 2001.

[7] J. Laskar, PAC'03, Portland OR, USA, 2003, p. 378.

[8] O. Bruning, EPAC'02, Paris 2002, p.1852.

[9] G. Rumolo, R.Tomás, NIMA, 528 (2004).