A NEW TUNING METHOD FOR RESONANT COUPLING STRUCTURES

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Abstract

In order to have efficient particle acceleration it is fundamental that the particles experience, in the accelerating gap, field amplitudes as uniform and as high as possible from gap to gap. Because of the unavoidable fabrication errors, an accelerating structure, when assembled, exhibits field values lower than the nominal ones and/or not uniform. All the usual procedures developed in order to adjust the parameter deviations responsible of the malfunctioning of these structures, are based on field amplitude measurements, by using the bead pull technique, which is a very invasive technique. In this paper the philosophy is reversed: it is assumed that all the information can be got by Sounding the Modes of the whole System (SMS) and correct the deviation of each frequency mode from its nominal value by means of an appropriate tuning of the cavities: resorting to a perturbative technique applied to a circuit model representing this kind of structures, it is possible to calculate the amount of tuning to give to the cavities. It will be shown that a very good equalization and maximization of the fields in the cavities can be achieved by using this technique.

INTRODUCTION

In this paper it will be analyzed the accelerating field optimization procedure of a Side Coupled Linac (SCL) which are very suitable devices for medical applications as cancer radiotherapy by using protons (protontherapy).

A SCL is a structure formed by a chain of on-axis Accelerating Cavities (AC) and off-axis Coupling Cavities (CC) each other coupled by means of a slot [1]. Each cavity resonates at a frequency depending on its geometrical parameters. The cavities, when coupled, exhibit a peculiar behaviour: they loose their individuality and resonate on eigenfrequencies of the whole system, each characterized by a phase advance between adjacent cavities.

The working frequencies of such a structures is pushed as high as possible, in such a way to increase the breakdown electric field [2]. This will reduce the longitudinal dimensions of the linac.

Because of the extremely high frequency of the RF feeders, the fabrication tolerances play a crucial role in the performances of these kinds of devices. The assembling and brazing procedures give additional random errors to the whole system. This behaviour leads to serious problems: because the structure does not resonate to the nominal frequency, there is a reactive power which flows back to the feeder (klystron) and this is extremely dangerous for the klystrons, unless one resorts to special and expensive devices; the field is not uniform from cell to cell; and, even if uniform, the field is smaller than the nominal value; therefore, one needs more power in order to have acceptable field amplitude values. In any case, without a proper tuning of the system, one has, in the best case (if there are not extreme reactive power feedbacks towards the klystron and acceptable uniform fields), to spend much more to have quite good field level in the cells.

It is worth noting that, after assembling the system, it is impossible to make direct measurements on the single cavity since it cannot be isolated: the only measurement allowed being on the whole system, the behaviour of which depends on cooperative dynamics of all cavities. Nevertheless one would like to change the parameters of the resonators responsible of the malfunctioning.

It will be described a new conceived tuning method named System Mode Sounding (SMS) by which only two probes are necessary, in the first and in the last cavity in order to span the whole frequency bandwidth. No mechanical device is introduced on the beam-line, no bead pull measurements are required: the sounding is purely electromagnetic. This behaviour makes it very attractive for many cases. It will be also shown that the application of SMS produces an equalization and maximization of the field in the cavities.

THE MODEL

Lumped circuit representation very well suits the behaviours of resonant coupling structures, even if one is dealing with devices working in the Gigahertz range, because the system modes stay in a narrow bandwidth proportional to the nominal coupling coefficient; this is irrespective of the number of cavities. Remembering that an SCL structure is formed by a certain number of AC cavities and a certain number of CC off-axis cavities, it is possible to represent the total structure as a biperiodic chain of resonant circuits. The first chain is formed by the AC cavities, coupled with the nearest neighbors (CC) and with the second nearest neighbors (AC); the second chain is formed by the CC cavities coupled with the nearest neighbors (AC) and with the second nearest neighbors (CC) as shown in Fig. 1. In the following, the odd circuits (cells) will represent the AC's, while the even ones the CC's; therefore it is possible to de-

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fine two resonant frequencies, one for the AC's and another for the CC's, respectively, as: $f_a = (2\pi)^{-1} (L_a C_a)^{1/2}$ and $f_c = (2\pi)^{-1} (L_c C_c)^{1/2}$.



Figure 1: Circuit representation for a biperiodic chain of cavities with first (k_1) and second nearest neighbor $(k_{a,c})$.

Let it consider a system half cell ended with $R \rightarrow 0$ and define the quantities

$$\mathbf{D} = \begin{cases} \epsilon_{ii} \equiv \epsilon_i = \frac{1}{\sqrt{2}} & \text{for } i \text{ first and last} \\ \epsilon_i = 1 & \text{for } i = 2, \dots, N \\ \epsilon_{ij} = 0 & \text{else,} \end{cases}$$
(1)

$$\mathbf{f} = \begin{cases} f_{ii} \equiv f_i = f_a & \text{for } i \text{ odd} \\ f_{ii} \equiv f_i = f_c & \text{for } i \text{ even} \\ f_{ij} = 0 & \text{else}, \end{cases}$$
(2)

and

$$\mathbf{K} = \begin{cases} k_{ii\pm1} = k_1 \\ k_{ii\pm2} = k_a & \text{for } i \text{ odd} \\ k_{ii\pm2} = k_c & \text{for } i \text{ even} \\ k_{22} = k_{NN} = k_c \\ k_{ij} = 0 & \text{else}, \end{cases}$$
(3)

The circuit equation system

$$\mathbf{f}^{-1}(\mathbf{I} + \frac{1}{2}\mathbf{D}^{-1}\mathbf{K}\mathbf{D}^{-1})\mathbf{f}^{-1}\mathbf{E} = F^{-2}\mathbf{E}, \qquad (4)$$

has only N+1 non trivial solution of eigenvectors (\mathbf{E}_m) and eigenfrequencies (F_m). It has to be noted that the equation system representing the circuit which can be found in the literature, e.g. eq.(25) in [3], is incomplete: indeed the coupling coefficient k_{22} and k_{NN} are missing in the second and in the last but one equations, respectively. Without these terms the dispersion relation, eq.(25) in [3], is incorrect. A discussion on the reasons of the presence of this terms will be given in [4].

The eigenvectors are:

$$\mathbf{E}_{m} = \mathbf{D} \begin{pmatrix} A_{m} \\ B_{m} \cos \varphi_{m} \\ A_{m} \cos 2\varphi_{m} \\ B_{m} \cos 3\varphi_{m} \\ \cdots \\ A_{m} \end{pmatrix},$$
(5)

with $\varphi_m = \frac{(m-1)\pi}{N}$ (m = 1, ..., N+1).

The dispersion relation can be found in the relevant literature [3, 5]. In the case of compensated structures [6], it

is interesting to give the eigenfrequency formula

$$\frac{f_0^2}{F_m^2} = \frac{1 - (k_a + k_c) \sin^2 \varphi_m - k_c k_a \cos 2\varphi_m}{(1 - k_a)(1 - k_c)} + \frac{k_1 \cos \varphi_m \left[(1 - k_a)(1 - k_c) + \cos^2 \varphi_m \frac{(k_a - k_c)^2}{k_1^2} \right]^{\frac{1}{2}}}{(1 - k_a)(1 - k_c)},$$
(6)

where f_0 is the $\frac{\pi}{2}$ mode frequency. The explicit expression of the A_m and B_m coefficients can be found by using the eq.(27) in [3] and then by normalizing the eigenvector.

THE PERTURBATION ANALYSIS

Let us consider an equation similar to eq. (4) where the cell resonant frequencies have small random deviations, δf_i , from their nominal values. This implies a deviation, δF_m , of the eigenmode frequencies from their original values. We disregard any deviation of the coupling coefficients. By adopting the standard first order perturbation technique we get the deviations of the eigenfrequencies:

$$\delta(F_m^{-2}) = \mathbf{E}_m^T \delta(\mathbf{f}^{-2}) \mathbf{E}_m \tag{7}$$

where the eigenvector set is taken normal. Equation (7) can be considered as a system of N+1 equations with N+1 unknowns δf_i . However, since the eigenvector components satisfy the following equation

$$E_{mi}^2 = E_{mi'}^2 \quad \text{if } i + i' = N + 2, \tag{8}$$

the eq. (7) cannot be inverted as a function of δf_i . Neverthless, as it can be written in the following way:

$$\delta(F_m^{-2}) = \epsilon_m^2 \sum \epsilon_i^2 \left[\delta(f_i^{-2}) + \delta(f_{i'}^{-2}) \right] E_{im}^2; \quad (9)$$

from the above equation we may infer that by making zero the sum of the detuning in the symmetric cavities we may tune all the eigenfrequencies. The key statement of this paper is the following: this tuning gives an acceptable optimization of the field level in the cavities.

THE TUNING PROCEDURES

After some algebra, since $f_i = f_{i'}$, the eq. system (9) can be rewritten in the following way:

$$\frac{\delta F_m}{F_m^3} + \frac{\delta F_{m'}}{F_{m'}^3} = \epsilon_m^2 \frac{4}{N} \sum_{i=1}^{N/2+1} \epsilon_i^2 \frac{\delta f_i + \delta f_{i'}}{f_i^3} \cos^2 \frac{(i-1)(m-1)\pi}{N},$$
(10)

with m+m' = N+2 and i+i' = N+2. The r.h.s. term can be regarded as a measurable quantity, while the deviations of the single cavity frequencies are the unknown. In the system of the eq. (10) the number of the equations is equal to the unknowns and the determinant is never singular. Not only, but it can be even analytically inverted.

The proposed tuning procedure, named System Mode Sounding (SMS)[7], consists in:

1) the detection of the mode frequencies by sounding the system in a suitably large frequency band $f_0(1 \pm k_1)$;

2) the calculation of the correction by means the inversion of eq.(10);

3) the introduction (extraction) of tuners producing the desired tuning refinement;

4) the iteration of the procedure, if the case.

Because of eq.(10) the iteration stops when the sum of the frequency deviations of symmetrical resonators is zero. The tuning converges very fast: in general two iterations are sufficient. The detection of the mode frequencies is made by feeding a variable frequency signal in the first cell and by picking up the response in the last one.

In a similar way to what was done in [7], we optimized by tuning the field distribution in the cavities of an SCL. We made a "virtual measurement". We solve the most general circuit equation where each cell is slightly detuned by a random error generated with a uniform distribution in the interval $(-\delta f, \delta f)$. The solution gives the eigenfrequency deviations δF_m . These values are considered as "measured quantities". They are introduced in eq.(10) which gives by inversion the symmetric cell detuning $(\delta f_i + \delta f_{i'})$. One half of this output is the correction to the initial detuning of the relevant cavities (i and i' indices). If the case, the procedure restarts. At the end of the tuning procedure we "measure", by means of the same circuit solver, the field distribution in the cavities at the $\frac{\pi}{2}$ mode. This distribution is compared to the nominal one and to the one affected by the starting random detuning.

As an example, the correction procedure has been applied to two linacs of 9 and 13 cavities. We allow for second order couplings. The relevant parameters of interest are: Q = 10000, $f_0 = 3GHz$, $k_1 = 3.6\%$, $k_a = 0.7\%$ and $k_c = -0.4\%$. The adopted procedure foresaw two tunings and a statistical evaluation on 40 samples affected by random errors. The results are given in Fig.2 and 3.

CONCLUSION

In order to give a quantitative evaluation of the results of the SMS tuning, we define three quality indices:

1) Field Amplitude Spread (FAS) which is the mean square deviation of the field in the AC's referred to the field mean values;

2) Power Efficiency (PE) which is the ratio between the total amount of the power dissipated in the AC's and the total amount delivered by the feeder;

3) Field Amplitude Distance (FAD) which is the mean square value of the distance from the nominal value in percentage.

One can see in Fig.2 and 3 that, in both cases, we get very good values of the quality indices after tuning. FAS and PE, even if the starting situation seems acceptable, indicate a remarkable improvement. More impressive is the improvement of the field level in the AC's: starting from a FAD almost 90% we may reach less than 2%. Remark that FAD=0 is the optimum.

The SMS tuning with 6 iteration was applied with the same structures: the differences are irrelevant.



Figure 2: Field equalization for $\pi/2$ mode $(\delta f = \pm 4MHz)$ of 9 cavities.



Figure 3: Field equalization for $\pi/2$ mode $(\delta f = \pm 4MHz)$ of 13 cavities.

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