APPLICATIONS OF CHERENKOV RADIATION IN DISPERSIVE AND ANISOTROPIC METAMATERIALS TO BEAM DIAGNOSTICS

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Abstract

We present theoretical and numerical analyses of Cherenkov radiation in bulk anisotropic and dispersive metamaterials and in waveguides loaded with these materials. Anisotropy and dispersion of both permittivity and permeability are taken into account. It is shown that the properties exhibited by these materials allow the design of detectors with unusual and previously unavailable characteristics.

INTRODUCTION

Cherenkov radiation (CR) is extensively used for detection of charged particles moving at relativistic speeds [1]. However, low signal levels and small angles of radiation with respect to the particle trajectory present limitations on the use of traditional detector media. Using modern artificial metamaterials as Cherenkov radiators can provide essential advantages over conventional media. Metamaterials are artificial periodic structures made of small resonating elements that are designed to achieve specific electromagnetic properties. As long as the periodicity and the size of the elements are much smaller than the wavelengths of interest, an artificial structure can be described by a bulk permittivity and permeability just as natural materials. The typical permittivity and permeability tensors have the following form [2,3]:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \quad \hat{\mu} = \begin{pmatrix} \mu_\perp & 0 & 0 \\ 0 & \mu_\perp & 0 \\ 0 & 0 & \mu_\parallel \end{pmatrix}, \qquad (1)$$

$$\varepsilon_\perp = 1 + \frac{\omega_{pe}^2}{\omega_{re}^2 - 2i\omega_{de}\omega - \omega^2}, \qquad \varepsilon_\parallel = 1, \qquad (2)$$

$$\mu_\perp = 1 + \frac{F\,\omega^2}{\omega_{rm}^2 - 2i\omega_{dm}\omega - \omega^2}, \qquad \mu_\parallel = 1, \qquad (3)$$

where ω_{re}, ω_{rm} are resonance frequencies, ω_{pe} is a plasma frequency, and ω_{de}, ω_{dm} are attenuation parameters.

CHERENKOV RADIATION IN UNBOUNDED MEDIA

In this section we present theoretical results for CR in bulk media. A point charge is assumed to be traversing the medium along the z-axis with velocity $V = c\beta$ (or equivalently, in a thin channel through the medium; as long as the channel is much smaller than the wavelength of interest it does not influence the radiation [4]).

A new method of analysis of the fields of moving charges in media with frequency dispersion has been developed in the papers [5,6] (for the case of isotropic media). The technique is based on the theory of functions of a complex variable. This method provides new opportunities for both analytic and numerical treatments of the fields of moving charges. It can be easily extended to the case of an anisotropic dispersive medium as well. However, we will describe here some energetic characteristics only.

We will use a typical characteristic to describe Cherenkov radiation, namely, the spectral density of radiated energy per unit area (as used, for example, for the case of isotropic left-handed media [7]). In the case of anisotropic dispersive media with tensor permittivity and permeability (1), the following approximate expressions for the spectral density components are obtained:

$$W_{\omega\rho} \approx \frac{q^2}{2\pi\rho} \mathrm{Re}\left(\frac{|s|s}{\omega\varepsilon_\parallel} \exp\left(-2\,\mathrm{Im}(s)\rho\right) \right), \qquad (4)$$

$$W_{\omega z} \approx \frac{q^2}{2\pi c\beta\rho} \mathrm{Re}\left(\frac{|s|}{\varepsilon_\perp} \exp\left(-2\,\mathrm{Im}(s)\rho\right) \right), \qquad (5)$$

where q is the charge magnitude, ρ is the radial distance of a measurement point from the particle trajectory, $s^2 = \frac{\omega^2}{V^2}\frac{\varepsilon_\parallel}{\varepsilon_\perp}\left(\varepsilon_\perp\mu_\perp\beta^2 - 1\right)$, and the function $s(\omega)$ is defined so that $\mathrm{Im}\, s(\omega) > 0$. Expressions (4) were derived assuming $|s|\rho >> 1$. We have also derived the exact formulae for arbitrary $s\rho$. The angle between the particle velocity and the vector $\vec{W}_\omega = W_{\omega z}\vec{e}_z + W_{\omega\rho}\vec{e}_\rho$ (directed along the group velocity) is given by the expression $\tan\theta_W \approx W_{\omega\rho}/W_{\omega z} \approx \left|\varepsilon_\perp V s/(\varepsilon_\parallel\omega)\right|$.

Some results for the dependence of the spectral energy density $W_\omega = \sqrt{W_{\omega z}^2 + W_{\omega\rho}^2}$ on the frequency ω and radiation direction θ_W are given in Fig.1. The computations were carried out using the exact formulae.

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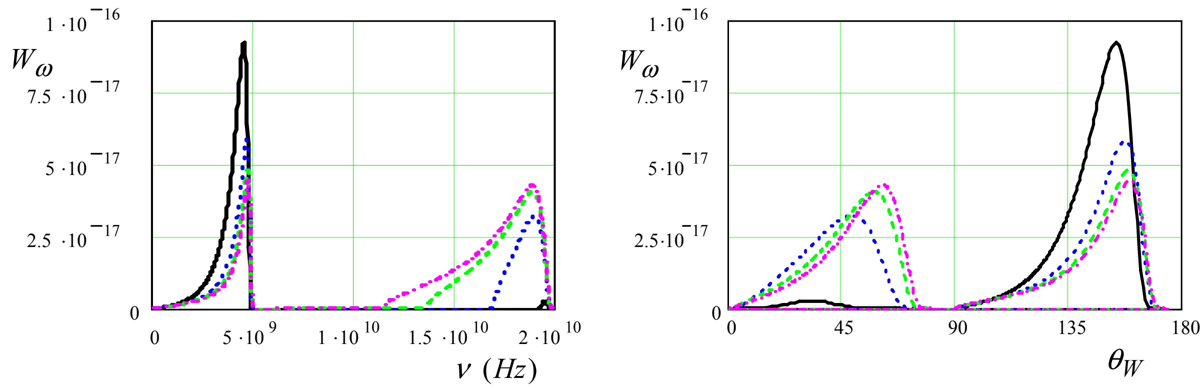

Figure 1: Spectral density of radiated energy per unit area (in SI-units) depending on frequency and angle. Magnitudes of parameters are the following: $\nu_{re}=0$, $\nu_{pe}=5\,\mathrm{GHz}$, $\nu_{rm}=20\,\mathrm{GHz}$, $F=0.5$, $\nu_{de}=\nu_{dm}=10^{-2}$ GHz; $\rho=10\,\mathrm{cm}$; $q=10^{-12}$ pC; $\beta=0.4$ (solid), $\beta=0.7$ (dotted), $\beta=0.9$ (dashed), $\beta=1$ (dash-dotted).

In addition to the case of bi-anisotropic medium, we considered separately the cases of electric and magnetic anisotropies. In the case of dispersive anisotropic permittivity only (when $\mu=1$), the charge radiates backward and the angle of radiation is obtuse (the analogous effect for another functional form of $\varepsilon_\perp(\omega)$ was considered in [1]). It is important to note that in this case there is no threshold for radiation generation. With such a structure it is possible to obtain good velocity selectivity without a velocity threshold. In the case of dispersive anisotropic permeability only (when $\varepsilon=1$) radiation is typical: the particle radiates forward at acute angles, and for a fixed frequency there is a radiation velocity threshold ($\beta^2\,\mathrm{Re}(\mu_\perp(\omega))>1$). In the case of dispersion and anisotropy in both permittivity and permeability (Fig.1) one can observe both forward (for $\omega>\omega_{pe}$) and backward radiation (for $\omega<\omega_{pe}$).

The presence of two distinct peaks in the spectral and angular distributions can be used for construction of a detector with two velocity thresholds. The reverse radiation without threshold allows registration of almost all moving particles, and simultaneously the forward radiation with threshold allows selection of particles with the velocity exceeding a set value.

The frequency and angular distributions in principle allow measurement of the charge velocity for very wide range of β. However this method may not provide enough precision because of errors of measurement of angles or frequencies of the radiation maxima. This disadvantage can be partially eliminated by use of a waveguide loaded with a metamaterial.

CHERENKOV RADIATION IN A WAVEGUIDE

The theory of Cherenkov radiation in waveguides with different dispersive materials has been discussed in a number of papers (see, for example, [1, 4, 7–11]). For a demonstration of principal effects in the case of the medium model given in (1)–(3), it is sufficient to consider a circular waveguide with radius *a*. We assume that the charge moves with a velocity $V=c\beta>0$ along the z axis of the waveguide that coincides with the optical axis of the metamaterial. We will omit the analytical results and describe some numerical examples concerning the behaviour of the fields at the wall of the waveguide (Fig. 2-3).

Examples of the spectra of the fields and the energy flow density at the waveguide wall are shown in Fig. 3 ($H^{(0)}_{\phi jm}$ are amplitudes of the magnetic field for harmonics with mode numbers $j=1,2$ and $m=1,2,3,\dots$; $z<0$ is the region behind the charge at a specified time). One can see that the wave field consists of two series of modes. The low-frequency modes ($j=1$) form the "backward" part of radiation, and the high-frequency modes ($j=2$) form the "forward" part. For the parameters indicated in Fig. 2 the radiation on the wall is backward ($S_z<0$) almost everywhere if $\beta\le0.5$. In the case of $\beta\ge0.7$, we have forward radiation ($S_z>0$) almost everywhere.

Fig. 2 shows frequencies of three modes of each series as a function of velocity. Low-frequency modes exhibit a stronger β dependence at low velocities than at ultra-relativistic velocities, and high-frequency modes have stronger β dependence at ultra-relativistic velocities.

Measuring the bunch velocity can be carried out with the help of measurements of mode frequencies. For example, we can use the first modes of each series: $\nu_{11}(\beta)$ is convenient for measuring relatively low velocities, and the dependence of $\nu_{21}(\beta)$ for measuring higher velocities. Thus, we can design a

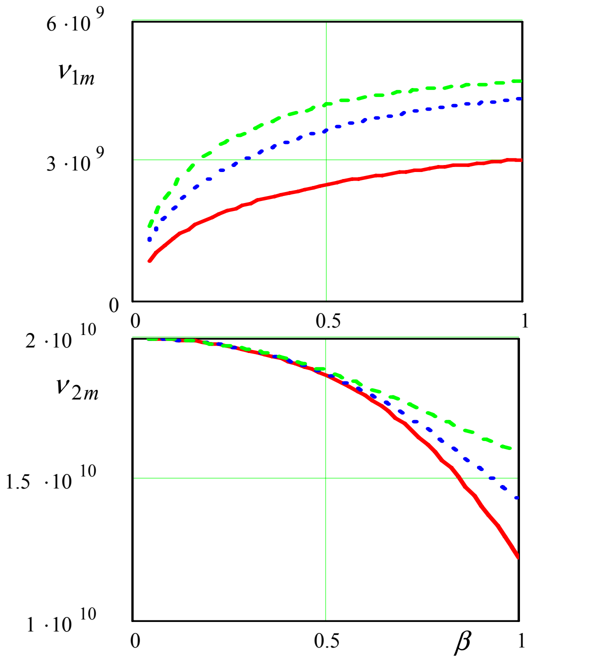

Figure 2: Frequencies of three modes ($m = 1,2,3$ correspond to the solid, dotted, and dashed curves accordingly) of the 1st and 2nd series for a waveguide with bi-anisotropic medium: $\nu_{re} = 0$, $\nu_{pe} = 5\,\mathrm{GHz}$, $F = 0.5$, $\nu_{rm} = 20\,\mathrm{GHz}$, $\nu_{de} = \nu_{dm} = 0$, $a = 3\,\mathrm{cm}$.

velocity detector with approximately identical precision for almost the entire range of velocities. It is also possible to design a detector with two velocity thresholds, as in the case of an unbounded medium. Such a detector will detect almost all moving charges owing to the low-frequency backward radiation, and simultaneously charges with velocities larger than a specified threshold owing to the high-frequency forward radiation.

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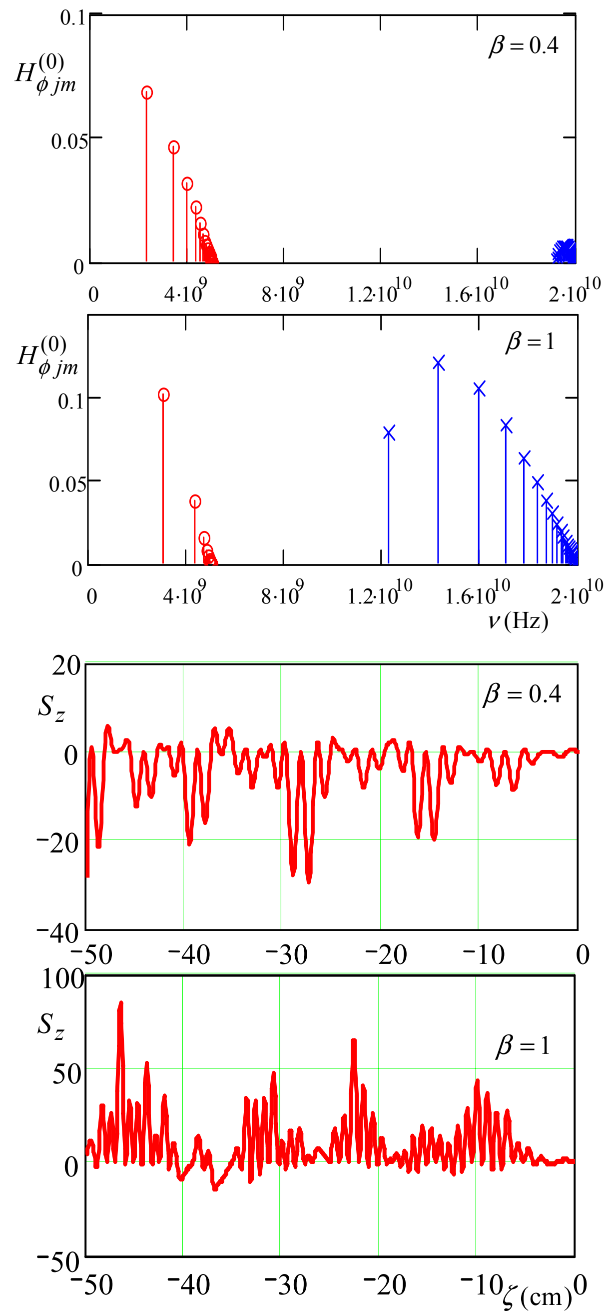

Figure 3: Amplitudes of magnetic components of harmonics and z-projection of energy flow density on the waveguide boundary for the same parameters as in Fig.2 and $q = 10^{-12}\,C$ (SI-units).

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