# REALISTIC NON-LINEAR MODEL AND FIELD QUALITY ANALYSIS IN RHIC INTERACTION REGIONS\*

J. Beebe-Wang# and A. Jain, Brookhaven National Laboratory, Upton, NY 11973, USA

#### Abstract

The existence of multipole components in the dipole and quadrupole magnets is one of the factors limiting the beam stability in the RHIC operations. So, a realistic nonlinear model is crucial for understanding the beam behavior and to achieve the ultimate performance in RHIC. A procedure is developed to build a non-linear model using the available multipole component data obtained from measurements of RHIC magnets. We first discuss the measurements performed at different stages of manufacturing of the magnets in relation to their current state in RHIC. We then describe the procedure to implement these measurement data into tracking models. including the implementation of the multipole feed down effect due to the beam orbit offset from the magnet center. Finally, the field quality analysis in the RHIC interaction regions (IR) is presented.

#### INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) consists of two rings, Blue and Yellow. The two beams, traveling in opposite directions in the two rings, are maneuvered to collide head on at the interaction regions (IRs). The field quality in the IR magnets influences the ultimate luminosity performance of RHIC. In order to study the performance of the machine, it is desirable to build a nonlinear model using the field quality measured in the individual magnets.

Unfortunately, not all of the IR magnets were measured in the superconducting state at full field. Furthermore, even where cold data are available, the geometric configuration of the leads during these measurements was not the same as the final as-installed magnet assemblies. This makes it difficult to estimate the field quality under normal operating conditions in these magnets.

In this paper, we describe the available measurements in these magnets, and the procedure followed to estimate the field quality under operating conditions by extrapolating from low field measurements at room temperature. We then describe the procedure to implement these data into tracking models, including the multipole feed down effect due to the beam orbit offset from the magnet center. Finally, the field quality analysis in the RHIC interaction regions (IR) is presented.

## MAGNETIC FIELD MEASUREMENTS

The six interaction regions of RHIC consist of a total of 72 quadrupoles of 13 cm aperture, 24 dipoles of 100 mm aperture (D0, 3.6 m long) that initiate the beam cross over, and 12 dipoles of 180 mm aperture (DX, 3.7 m long) through which both beams pass. Three quadrupoles of lengths 1.44 m (Q1), 3.40 m (Q2) and 2.10 m (Q3) are located on either side of all six of the RHIC crossing points. The three quadrupoles are close together as a "triplet" and perform the final strong focusing for the experiments. More details about the design and performance of these magnets can be found in [1, 2].

The field quality of all magnets in RHIC was measured at room temperature as part of the magnet quality control and acceptance. For the magnets in the arc, only about 20% were cold tested. It was planned to cold test 100% of the 108 insertion region magnets (plus spares) and to measure the field quality in the superconducting state. However, due to schedule constraints, only the magnets of Q1 and Q2 series could be fully measured cold. Of the remaining insertion region magnets, only 13 of the Q3 (including one spare), 10 of the D0, and 7 of the DX series magnets were measured cold.

## **ESTIMATING FIELD QUALITY**

In order to create a realistic model of the IR, one needs to estimate the field quality under operating conditions for the magnets for which no cold data are available. Furthermore, the lead configurations had changed after the cold test when the magnets were finally prepared for installation. This affected a few harmonics in the lead region. Since the magnets were also measured at room temperature in their final states just before installation in RHIC, we have used these data to estimate the needed corrections to the cold data. The details for estimating field quality are given below for each magnet type.

## Estimating Field Quality in the IR Quads

The cold data in the IR quads were obtained using a long rotating coil, which gave only the integral harmonics. For modeling, the magnets were divided into three regions – a "lead end" region, a central "body" region, and a "non-lead end" region. An estimate of contribution from different regions was obtained from the warm measurements in the final assemblies, which were carried out using a 0.91 m long measuring coil. It was assumed that the harmonics in the ends did not change with excitation. Although not perfect, this assumption is good for most harmonics. The contribution from the "body" was obtained by subtracting the contributions of the two ends from the measured integral values cold.

05 Beam Dynamics and Electromagnetic Fields

<sup>\*</sup> This work was supported by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy and by a sponsored research grant from Renaissance Technologies. This manuscript has been authored by Brookhaven Science Associates, LLC under contract with the U.S. Department of Energy. The United States Government retains, and the publisher, by accepting the article for publication, acknowledges, a worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for the United States Government purposes. # bbwang@bnl.gov

In the case of the 13 Q3 magnets that were not measured cold, we needed to estimate the field quality from the warm measurements. Each of the IR quads was individually shimmed using magnetic tuning shims after initial warm measurements in order to achieve the best possible field quality. The integral harmonics at lower operating fields, where iron is not saturated, were estimated by applying the average warm to cold shift in harmonics measured in the 11 Q3 magnets. At higher fields, each IR quad saturates differently. This difference between magnets was accounted for by calculating the saturation behavior of each Q3 magnet using a POISSON model, and comparing the computed saturation with the measured values in magnets where cold data are available.

## Estimating Field Quality in the IR dipoles

Estimation of cold field quality in the IR dipoles (D0 and DX) was relatively straightforward since the cold and the warm measurements were both made with a 0.91 m long rotating coil, and the body, ends and integral harmonics were separately available. The average warm to cold shifts obtained from the magnets that were measured cold were applied to the warm data to estimate the cold data in magnets that were not measured cold. In the case of D0 magnets, however, the magnet yoke cross section was changed during production after magnet serial number 6. This affected the saturation behavior of the normal sextupole, decapole and 14-pole terms. Thus, the D0 magnets were divided into two groups for these harmonics, with a different warm to cold shift for each group. For all the other allowed and unallowed terms, all magnets could be treated as a single group.

### **REALISTIC NON-LINEAR MODEL**

Following the above procedures, all the normal and skew coefficients,  $b_n$  and  $a_n$ , of the multipolar expansion of the magnetic field were estimated up to the 22-pole (n = 10) for all IR quadrupole and dipole magnets that had no cold measurements. The measured and estimated data are stored separately in the RHIC optical database. The realistic non-linear RHIC model is built by including complete data of the multipole components of all IR magnets into tracking packages SixTrack[3] and UAL[4]. Generally, these tracking packages treat non-linear multipoles as thin lenses to ensure simplecticity. In view of the large variations of the beta functions within the RHIC interaction regions, the IR magnets are subdivided into 8 slices and the multipoles are placed in 7 thin lenses between the slices. In addition, two thin lenses are placed at the ends of each magnet to account for the multipoles from the end fields.

The reference orbits of beams in both the Blue and the Yellow ring are off centered in the DX magnets. Each order of multipole component thus contributes to all the other multipoles of lower orders due to the feed down effect. The general expression of the multipoles in the DX magnets, including the feed-down effect, can be derived using a binomial expansion as follows:

$$B_{y} + iB_{x} = \frac{C(m)}{10^{4}} \sum_{n=0}^{\infty} (b_{n} + ia_{n}) \left[ \frac{(x + x_{0}) + iy}{R_{ref}} \right]^{n}$$
$$= \frac{C(m)}{10^{4}} \sum_{n=0}^{\infty} (b_{n} + ia_{n}) \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} \left( \frac{x_{0}}{R_{ref}} \right)^{m} \left( \frac{x + iy}{R_{ref}} \right)^{n-m}$$

where C(m) is the strength of the main 2m-pole term,  $x_0$  is the horizontal offset, and  $R_{ref}$  is the reference radius at which the harmonics are specified. One can define new multipole components  $\tilde{a}_n$  and  $\tilde{b}_n$  which include the feed-down effect as:

$$B_{y} + iB_{x} = 10^{-4} \times C(m) \sum_{n=0}^{\infty} \left( \widetilde{b}_{n} + i\widetilde{a}_{n} \left( \frac{x + iy}{R_{ref}} \right)^{n} \right)$$

Comparing the two expressions above, we get:

$$\widetilde{b}_{n} = \sum_{m=0}^{\infty} \frac{(m+n)!}{m!n!} \left(\frac{x_{0}}{R_{ref}}\right)^{m} b_{m+n}$$
$$\widetilde{a}_{n} = \sum_{m=0}^{\infty} \frac{(m+n)!}{m!n!} \left(\frac{x_{0}}{R_{ref}}\right)^{m} a_{m+n}$$

In the model they are implemented in finite form up to the 22-pole (n = 10) as:

$$\widetilde{b}_n = b_n + \sum_{m=1}^{10-n} \left[ \prod_{k=1}^m \frac{k+n}{k} \right] \left( \frac{x_0}{R_{ref}} \right)^m b_{m+n}$$
$$\widetilde{a}_n = a_n + \sum_{m=1}^{10-n} \left[ \prod_{k=1}^m \frac{k+n}{k} \right] \left( \frac{x_0}{R_{ref}} \right)^m a_{m+n}$$

#### FIELD QUALITY ANALYSIS

In order to study the field quality, the multipole components of IR magnets are analyzed using both measured and estimated data. The normal sextupole  $(b_2)$  in the dipoles and the normal 12-pole  $(b_5)$  in the quadrupole magnets were found to give the major contributions to the field errors. Fig. 1 shows the  $b_2$  component in D0 magnets as a function of magnet current in 10 D0 magnets (5 in the Blue ring, 4 in the Yellow ring, and one spare). The values of  $b_2$  are at a radius of 31 mm, and are in "units" of  $10^{-4}$  of the dipole field. There is a significant magnet to magnet variation in the



Figure 1: Normal sextupole (integral value) as a function of magnet current in the D0 dipoles.

D02 Non-linear Dynamics - Resonances, Tracking, Higher Order

1-4244-0917-9/07/\$25.00 ©2007 IEEE

<sup>05</sup> Beam Dynamics and Electromagnetic Fields



Figure 2: Warm-cold offsets in normal sextupole as a function of magnet current in the D0 dipoles.

absolute value of the sextupole. However, the values measured cold are well correlated to the values measured warm. The warm to cold shift in all the D0 dipoles is given in Fig. 2, showing good consistency between magnets. The difference in saturation behavior of magnet serial numbers 1 to 6 due to change in yoke design is also clear. The uncertainty in predicting the sextupole term in magnets with no cold data is well below 1 unit.

Figs. 3 and 4 show similar plots of  $b_2$  at a reference radius of 60 mm in the DX dipoles. These results are similar to those in the D0 dipoles. The mean values of the warm-cold offsets, depicted by solid lines in Figs. 2 and 4, were used to estimate the cold data in the dipoles that were not measured cold. A similar procedure was used for estimating all the harmonics up to the 22-pole.

In the case of the Q3 quadrupoles, the warm-cold correlation in the allowed 12-pole ( $b_5$ ) term is shown in Fig. 5. Large deviations from mean is evident at higher currents in at least one magnet. The high field data can be explained by the saturation behavior computed for each magnet based on the custom tuning shims. A combination of measured warm-cold offsets and calculations based on custom shims was therefore used to estimate the field quality in the Q3 magnets.

#### CONCLUSION

A procedure was developed to estimate the field quality under operating conditions by extrapolating from low



Figure 3: Normal sextupole (integral value) as a function of magnet current in the DX dipoles.



Figure 4: Warm-cold offsets in normal sextupole as a function of magnet current in the DX dipoles.



Figure 5: Warm-cold offsets in normal 12-pole in the Q3 quadrupoles. Larger scatter at high fields is a result of custom iron tuning shims used in each magnet, and is consistent with calculations.

field measurements at room temperature. A realistic nonlinear RHIC model was built by including the complete set of multipole components in the body and ends of all IR magnets from either measured or estimated data. The simulation studies using this non-linear model have been performed and are also presented at this conference [5,6].

#### REFERENCES

- R. Gupta et al., "RHIC IR quadrupoles and field quality state of the art in superconducting accelerator magnets," Proc. of PAC1999, New York, pp. 185-7.
- [2] M. Anerella et al., "The RHIC Magnet System," Nucl. Instrum. Meth. in Physics Research, A499, pp. 280-315 (2003).
- [3] F. Schmidt, "SIXTRACK: Single Particle Tracking Code Treating Transverse Motion with Synchrotron Oscillations in a Symplectic Manner", CERN SL/94– 56 (AP).
- [4] N. Malitsky and R. Talman, "Unified Accelerator Libraries", AIP 391, 1996.
- [5] Y. Luo, et al., "Dynamic aperture evaluation at the current working point for RHIC polarized proton run", these proceedings, paper FRPMS111.
- [6] C. Montag, et al., "A near-integer working point for polarized protons in RHIC", these proceedings, paper TUPAS099.
- D02 Non-linear Dynamics Resonances, Tracking, Higher Order

05 Beam Dynamics and Electromagnetic Fields 1-4244-0917-9/07/\$25.00 (c)2007 IEEE