IMPROVING THE SIS18 PERFORMANCE BY USE OF THE ORBIT RESPONSE METHOD

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Abstract

The GSI heavy ion synchrotron SIS18 will be used as a booster for the new FAIR facility SIS100. A well-controlled linear optics of the SIS18 is necessary for further optimization studies of nonlinear dynamics, resonance induced beam loss, dynamic aperture and nonlinear error measurements. The analysis of the orbit response matrix (ORM) [1] is a powerful tool to calibrate the linear lattice models. We present results of orbit response analysis of several measurement campaigns on the SIS18 and discuss the achieved improvement of the SIS18 performance.

INTRODUCTION

The SIS18 has 12 periods and a nominal working point at injection energy of $Q = (4.29; 3.29)$. There are 24 beam position monitors (BPMs) with 12 in each plane used for the CO diagnostics [2]. For CO control and correction 6 horizontal and 12 vertical steerers are available. 24 dipole magnets are used for the beam bending. For focusing there are 24 horizontal focusing quadrupoles and 12 vertical focusing quadrupoles (triplet structure).

The present SIS18 CO correction is based on the three-steerer local bump method [3, 4]. The local bump is provided by a certain ratio between the steering angles of three neighboring steerers [4]. To hold precisely this ratio in the measurement 'well' calibrated steerers are required [3], i.e. all the steerers have the same calibration factors, which are equal to 1.0. Therefore, the calibration of the steerers and BPMs in SIS18 has to be carried out and can be done by the ORM method [1, 3].

The linear optics errors (model parameters) considered here are: BPM and steerer calibration factors, and quadrupole gradient errors. The linear coupling was not considered and contributes to systematic errors of the considered model parameters. The measurements were performed in conditions to reduce the impact of linear coupling. The condition of linear coupling $Q_x - Q_y = 1$ for the measured tunes is not satisfied. In all three measurements it has been also checked that a kick in vertical (horizontal) direction has no horizontal (vertical) orbit response.

MODEL

The orbit response matrix gives a change in closed orbit $\mathbf{x}_{co}$, $\mathbf{y}_{co}$ by change in steerer angles θ_x, θ_y [1]

$$\begin{pmatrix} \mathbf{x}_{co} \\ \mathbf{y}_{co} \end{pmatrix} = \mathbf{M}_{\text{orm}} \begin{pmatrix} \theta_x \\ \theta_y \end{pmatrix} , \qquad (1)$$

where $\mathbf{M}_{\text{orm}}$ is either the measured or model response matrix, $\mathbf{x}_{co} = (x_{co,1}\, x_{co,2} \ldots)$, $\mathbf{y}_{co} = (y_{co,1}\, y_{co,2} \ldots)$, $\theta_x = (\theta_{x,1}\, \theta_{x,2} \ldots)$, $\theta_y = (\theta_{y,1}\, \theta_{y,2} \ldots)$ and the indices $1, 2, 3...$ are the BPMs and steerer sequence as found in the ring. Now we define the following model parameters: 1) BPM calibration factors $\mathbf{g} = \{g_i\}$, $i = 1...I$, I is the total number of BPMs; 2) steerer calibration factors $\mathbf{f} = \{f_j\}$, $j = 1...J$, J is the total number of steerers; 3) quadrupole gradients $\mathbf{q} = \{q_l\}$, $l = 1...L$, L is the total number of quadrupoles; which form the model parameter vector $\mathbf{x} = (\mathbf{g}, \mathbf{f}, \mathbf{q}) = \{x_m\}$, $m = 1...M$, $M = I + J + L$ is the total number of model parameters. To minimize the difference between the model $\mathbf{M}^{\text{mod}}$ and measured $\mathbf{M}^{\text{exp}}$ orbit response matrices, the vector $V_n = (M^{\text{mod}}_{ij} - M^{\text{exp}}_{ij})/\sigma_{ij}$ is defined, where we have ordered the component of the matrix ij into a vector of component n. The index n runs over all the data points which formed by multiplication of the number of steerers varied j and the number of BPMs used i. To each n corresponds a presice value of ij, that is $n \rightarrow (\bar{i}, \bar{j})$. The total number of fitting data points is $N = I \times J$. The weighting factor σ_{ij} gives more weight to those measured ORM elements with lower noise. The measured matrix always includes the BPM and steerer calibration factors and can be derived via $M^{\text{exp}}_{ij} = M^{\text{data}}_{ij}/(g_i\, f_j)$. The following χ^2-function [5] is introduced as a measure of difference between $\mathbf{M}^{\text{mod}}$ and $\mathbf{M}^{\text{exp}}$ and has to be minimized:

$$\chi^2 = \sum_n {V_n}^2 = \sum_n \frac{(M^{\text{mod}}_{\bar{i}\bar{j}} - M^{\text{data}}_{\bar{i}\bar{j}}/(g_{\bar{i}} f_{\bar{j}}))^2}{{\sigma_{\bar{i}\bar{j}}}^2} . \qquad (2)$$

The multi-dimensional non-linear least-square problem (2), i.e. the minimization of χ^2, can be solved by varying the model parameter vector $\mathbf{x}$, which are quadrupole gradients $\mathbf{q}$, steerer calibration factors $\mathbf{f}$ and BPM calibration factors $\mathbf{g}$. The variation of the Vmth component due to the variation of the m-th component of $\mathbf{x}$ is found [5]

$$V_n(\mathbf{x} + \Delta\mathbf{x}) \approx V_n(\mathbf{x}) + \sum_m \frac{\partial V_n}{\partial x_m} \cdot \Delta x_m . \qquad (3)$$

Imposing Eq. (3) to zero we find an approximate solution by using the multi-dimensional Newton method and build an iterative process $\mathbf{V} + \mathbf{J}\Delta\mathbf{x} = 0$, where the Jacobian $\mathbf{J}$ of the linear system Eqs. (3) consists of partial derivatives of the parameters $J_{nm} = \partial V_n / \partial x_m$. For the case of the overdetermined ($N > M$) least-square problem (2) SVD gives a solution with the minimum norm that is the best approximation in the least-square sense [5] $\mathbf{J} = \mathbf{U}\mathbf{\Lambda}\mathbf{Q}^T$, where $\mathbf{U}$, $\mathbf{Q}$ are orthogonal matrices, $\mathbf{\Lambda}$ is a diagonal matrix consisting of eigenvalues of the matrix

$\mathbf{J}^T\mathbf{J}$. The change in model parameter vector $\mathbf{x}$ in one iteration is performed via pseudoinverse of the decomposed Jacobian $\mathbf{J}$: $\Delta\mathbf{x} = -\mathbf{J}^{-1}\mathbf{V} = -\mathbf{Q}\mathbf{\Lambda}^{-1}\mathbf{U}^T\mathbf{V}$. The model parameter vector $\mathbf{x}$ on the $k+1$ iteration is defined as $\mathbf{x}^{k+1} = \mathbf{x}^k - \mathbf{J}^{-1}(\mathbf{x}^k)\mathbf{V}(\mathbf{x}^k)$. The new value $\mathbf{x}^{k+1}$ has to satisfy the χ^2-convergence condition: $\chi^2_{k+1} < \chi^2_k$. Practically the iterations can be stopped when an extra iteration makes only a small change to χ^2, so that the fitted model can be obtained. Complementary to the SVD method a calculation with the Levenberg-Marquardt method was performed $(\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I})\Delta\mathbf{x} = -\mathbf{J}^T\mathbf{V}$, where $\mathbf{I}$ is the identity matrix, λ is an adjustable parameter, which belongs to the interval $[0, 1]$. At every iteration the parameter λ is adjusted to satisfy the χ^2-convergence condition. The above fitting scheme was applied to the simulated 'noise-free' horizontal and vertical ORMs, generated by MAD [6] for the SIS18 lattice by introducing random quadrupole gradient errors within maximum 5% deviation of the nominal values. 'Noise-free' means that all σ_{ij} are equal to 1.0. To simulate 'wrong' BPM and steerer calibration, the modelled matrices were multiplied by a certain random factor out of the interval [0.9; 1.2]. The model ORMs (as well as their χ^2s) were functions of random BPM and steerer calibration factors, and quadrupole gradient errors. The simultaneous fit of all the parameters was performed by the SVD method and by the Levenberg-Marquardt method: the two methods converge to the same result, χ^2 monotonously converges to 0 with any desirable accuracy and the solution given by a random set of errors for the model parameters is reconstructed.

EXPERIMENT AND ANALYSIS

To fit the ORM several measurements in the SIS18 were performed in February 28, April 8 and April 20 2006, where the ORM was measured three times [1].

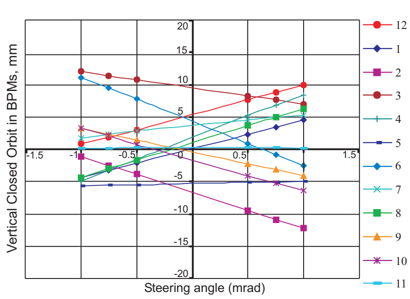

Figure 1: Vertical CO vs. vertical steering angles.

The accuracy of a beam position measurement, defined as a statistical error of a single measurement is $\pm$ 0.5 mm in horizontal and $\pm$ 0.14 mm in vertical plane respectively. Note that the horizontal accuracy is $\sim$3 times poorer than the vertical. This accuracy is determined by the ADC resolution, thermal noise of the preamplifier and digitalization noise. Both noise sources have stochasic character, whereas the noise caused by the digitalization is dominant. The thermal noise is reduced by using the narrowband analysis. The averaging over many turns increases the accuracy of the position measurement, but it can be used only as long as the beam parameters remain constant. Otherwise, the changes of the beam parameters enter as systematic error in the position measurement accuracy. The ORM is given by the linear fit to the closed orbit vs. the steerer angle. The slopes of the lines in Fig. 1 form the ORM and their standard deviations form the rms uncertainty matrix σ_{ij}. The best accuracy of the ORM achieved in the fit is 0.02 mm/mrad. The maximum deviation from the linear fit is 0.9 mm/mrad.

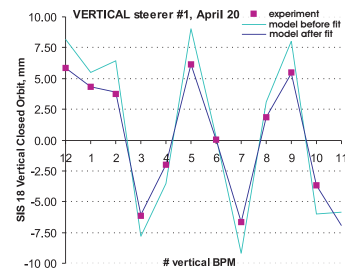

Figure 2: Measured (April 20) and model response at 12 BPMs for the vertical steerer #1, BPM #11 is missing.

We applied the above algorithm to calibrate 24 quadrupole gradients (12 vertical focusing and 12 horizontal focusing), 24 BPMs with 12 in each plane and 18 steerers (6 horizontal and 12 vertical) of the SIS18. The degeneracy [1] of BPM and steerer calibrations was avoided by assuming one horizontal (#6) and one (#12) vertical steerer were calibrated correctly (have calibration factor 1). In the ORM modelling these two steerer strengths are kept fixed, and all the other steerers and BPMs were calibrated relative to these two steerers. Fig. 2 shows the 'model before' obtained with MAD for the vertical steerer #1 before ORM fit applied, the 'model after' is a MAD simulation for the same steerer using parameters derived from the fit. The simplest way to find out how much the fitted parameters vary due to random errors in the measurements is to analyze each of three data sets separately, and see how much the fitted parameters vary [1]. The minimum χ^2 normalized by the number of data points N is taken as a criteria of the fit quality and defined as follows [1] $\chi^2_{\text{min per data point}} = \chi^2_{\text{min}}/N_{\text{data points}} \simeq 1$, indicates that errors in the fitting are of the order of random errors [1, 5]. In Table 1 the minimum χ^2s per data point and the total numbers of data points of the three ORM measurements are presented. The vertical minimum χ^2 per data point of the February 28 measurement is large compared to the April measurements, for which the minimum χ^2s per data point are centered around 1.0. The value 13.98, several standard deviations above 1.0, could be explained as the fact that orbit measurement errors were not normally distributed and there is a contribution of systematic errors to the February 28 result. However, in the results of February 28 the vertical data fit agree well with the other two data fits, see Fig. 3. The agreement of all three vertical data fits within rms deviation of several % indicates the uniqueness of the retrieved vertical model. Fig. 3 a) shows that SIS18 vertical

Table 1: χ^2 statistics from three ORM data fits.

Exp.	vert. $\chi^2_{\text{min per data point}}$	hor. $\chi^2_{\text{min per data point}}$
Feb. 28	13.98, $N = 120$	3.30, $N = 60$
Apr. 8	1.19, $N = 132$	19.02, $N = 72$
Apr. 20	0.74, $N = 132$	23.17, $N = 72$

BPM calibrations are centered not around 1.0, but around 0.7. The horizontal BPM calibrations are lowered in $\sim$10-15% as well. In Ref. [2] is noticed that the scaling pick-up constants for the BPMs obtained in the simulations are 20% larger than the experimental values measured with the test set up, so that agrees with ORM simulation. As the CO correction by local bump method was possible, all steerers were expected in general to be well calibrated. The vertical steerer calibrations retrieved from the ORM model are centered around 1.0 except for the steerer #11, see Fig. 3 b). The ORM modelling gives 33% lower calibration factor for vertical steerer #11 (S11KM2DV). For this steerer it was found that its magnetic field is $\sim$30% weaker than expected, as predicted by ORM analysis. The reason for that is not a device malfunction, but a construction different design: the steerer has a larger aperture. This steerer needs a correction in the control program, that its effect becomes equal to those of the other steerers.

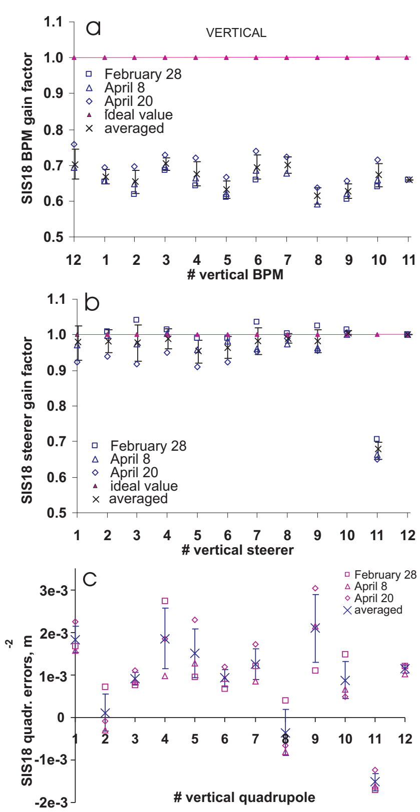

Figure 3: Vertical BPM a) and steerer b) calibration factors, quadrupole gradient errors c).

The horizontal minimum χ^2s per data point all are not very satisfactory, see Table 1. The present low accuracy of the horizontal beam position led to high systematic and random errors in the measured horizontal response matrices. The experimental horizontal ORM has gotten many small eigenvalues that correspond to noise in the data. In the horizontal data fit the SVD method with tolerance threshold equal to eigenvalue $\sim$0.05 has been applied. The present accuracy of the horizontal measurements does not allow to uniquely define all 24 horizontal focusing quadrupole errors in the ring and brings large error bars in the fit of other parameters. It was possible to define uniquely only 12 horizontal focusing quadrupole errors.

The the currently used values of quadrupole gradients of the SIS18 lattice model [6] are proper adjusted. Therefore, the quadrupole errors are expected to be not too high in average $\sim$0.1-0.3% of the nominal values. It is well known at GSI that the SIS18 tune has a systematic tune shift from the one set in SISMODI. Simulations made with the retrieved quadrupole gradient errors bring the model tune closer to the measured value.

The crosscheck wherever the fitted model parameters are correct is made by comparing them with the SIS18 data that were not used in the model fitting. As ORM measurement on April 20 was done for the tune $Q = (4.17, 3.35)$ the following test was performed: the ORM model deriving from two measurements for the tune $(4.29; 3.29)$ is used to predict the measurement for the tune $(4.17; 3.35)$. The quadrupole gradient errors were retrieved from two measurements for the tune $(4.29; 3.29)$ and then modelled in MAD. The model MAD matrix multiplied by BPM and steerer calibrations retrieved from the same two measurements is able to fit the measured matrix for the tune $(4.17; 3.35)$. Using the model parameters derived from two measurements the initial vertical χ^2 normalized by number of data points has been reduced from 942.03 to 40.89.

CONCLUSION

Three ORM were fitted and provided the detailed information on the SIS18 linear optics. The fitted values of the 24 quadrupole gradients, 24 BPM and 18 steerer calibrations provide the optimized lattice model as well as hints of device malfunctions. The 6 horizontal steerers used at the moment are too few parameters for the horizontal ORM modelling. To better resolve the horizontal data at least 12 steerers are needed: one in each period. The dispersion was not measured in the SIS18 ORM experiments here discussed. The dispersion measurement would be desirable in the future.

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