A MULTISLICE APPROACH FOR ELECTROMAGNETIC GREEN'S FUNCTION BASED BEAM SIMULATIONS*

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Abstract

We present a multislice approach for modeling the space-charge fields of bunched electron beams that are emitted from a metallic cathode using electromagnetic Green's function techniques. The multislice approach approximates a local region of the beam density and current with a slice of zero longitudinal thickness. We show examples of how the multislice approach can be used to accurately compute the space-charge fields for electron bunch lengths in the regime of photocathode sources, i.e. (<10 ps).

INTRODUCTION

Novel methods for simulating electron sources using time-dependent Green's functions have recently been introduced, and resulted in the development of an rf photocathode source simulation code at the Indiana University Cyclotron Facility called IRPSS [1,2]. Unlike, grid based field solvers such as Yee/PIC algorithms [3], Green's function based codes use analytical methods for calculating the electromagnetic fields in the presence of conductor boundaries to extremely good accuracy. The advantage of using the Green's function approach over a Yee algorithm is two-fold. First, since the electromagnetic fields in a Green's function based code are not calculated on a grid, the code does not suffer from the effects of numerical grid dispersion and the associated numerical Cherenkov radiation [4]. Secondly, since Green's functions are generated by point like (delta function) distributions, it is possible to model arbitrarily small bunch density variations to extremely high accuracy (<1% field error).

In order to utilize a Green's function approach, it is required that the charge and current densities of the beam are computed at every point in space. If one uses the "simple" approach of modeling the charge and current densities with point particles, i.e. the Klimontovich distribution, one would need to calculate the force of each particle on every other particle resulting in N_p^2 calculations, where N_p is the number of macroparticles in the system. A more prudent approach models the charge and current densities using a set of smooth distribution functions instead of point charges.

This paper outlines our recent efforts to model the fields of a bunched electron beam within the IRPSS code using a multislice approach. It addresses the computational requirements that a multislice scheme must satisfy in order to accurately model the electromagnetic fields of a bunched electron beam. Our paper is organized as follows. In Sec. 2, we review the theoretical framework upon which IRPSS is based. In Sec. 3, we show the results of multislice simulations for a uniformly moving bunch of with a bunch length of 9 ps and an accelerating bunch corresponding to the parameters of the ANL Argonne Wakefield Accelerator 1.3 GHz photocathode source [5]. In Sec. 4, we give a summary of our paper.

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THEORETICAL FRAMEWORK

At present, IRPSS can simulate the electromagnetic space-charge fields of an electron bunch in a geometry consisting of an outer conductor pipe (parallel to the *z*-axis) and a flat conductor cathode (*z = 0*). For this paper, the conductor geometry is assumed to have a circular cross-section, but other cross-sections, such as rectangular, can be modeled. We have shown in previous papers [1,2], that the electromagnetic fields can be expressed in terms of time dependent Green's functions. In particular, for a given beam charge and current density, $\rho(\mathbf{r},t)$ and $\mathbf{J}(\mathbf{r},t)=J_z(\mathbf{r},t)\mathbf{e}_z$ which satisfies the continuity equation, the transverse electric and magnetic space-charge fields are given by

$$\mathbf{E}_\perp(\mathbf{r},t)=-\frac{1}{\varepsilon_0}\nabla_\perp\int_{-\infty}^{t}\int G_-\rho(\mathbf{r}',t')d^3\mathbf{r}'dt' \qquad (1)$$

$$\mathbf{B}_\perp(\mathbf{r},t)=-\mu_0\mathbf{e}_z\times\nabla_\perp\int_{-\infty}^{t}\int G_+J_z(\mathbf{r}',t')d^3\mathbf{r}'dt', \qquad (2)$$

and the longitudinal electric field is given by

$$E_z(\mathbf{r},t)=-\frac{1}{\varepsilon_0}\int_{-\infty}^{t}\int\left[\frac{\partial G_-}{\partial z}\rho+\frac{1}{c^2}\frac{\partial G_+}{\partial t}J_z\right]d^3\mathbf{r}'dt', \qquad (3)$$

where $G_\pm(\mathbf{r},t;\mathbf{r}',t)$ are Green's functions which satisfy non-homogeneous wave equations with delta function sources and appropriate boundary conditions to ensure that $\mathbf{E}_{\|}\big|_{surface}=0$ and $\mathbf{B}_\perp\big|_{surface}=0$ at the conductor surface.

In order to implement this approach numerically, it is essential to compute $\rho(\mathbf{r},t)$ and $J_z(\mathbf{r},t)$. One method for accomplishing this is to represent $\rho(\mathbf{r},t)$ and $J_z(\mathbf{r},t)$ as a sum over zero-thickness charged slices, i.e. for a cylindrically symmetric beam

$$\rho(\mathbf{r},t)=\sum_{i=1}^{N}\sigma_i(r,t)\delta(z-z_i''(t)) \qquad (4a)$$

and

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$$J_z(\mathbf{r},t)=\sum_{i=1}^{N}\sigma_i(r,t)\frac{dz_i''}{dt}\delta(z-z_i''(t)), \qquad (4b)$$

where $\sigma_i(r,t)$ is the charge per area of the ith slice, $z_i''(t)$ is the longitudinal location of the ith slice, and *N* is the number of slices on the simulation. The benefit of choosing a zero-thickness slice representation is that the integration over z' in Eqs. (1)-(3) can be performed analytically. However, in order to construct a physically relevant model using zero-thickness slices, each $\sigma_i(r,t)$ must go to zero at the beam edge to prevent an unphysical divergence of the electric field.

The computational speed of a multi-slice algorithm is dramatically improved when the choice of $\sigma_i(r,t)$ is such that the radial integrals in Eqs. (1)-(3) can be performed analytically. The simplest choice for $\sigma_i(r,t)$ which is zero at the beam edge and can be integrated analytically is

$$\sigma_i(r,t)=\left(2Q_i/\pi r_{bi}^2\right)\theta(r_{bi}-r)\left(1-r^2/r_{bi}^2\right), \qquad (5)$$

where r_{bi} and Q_i are the beam edge radius and total charge of the i[th] slice, respectively. Improvements of Eq. (5) are possible by modeling higher order radial density variations at each slice with annuli that are represented by fourth-order polynomials.

One of the key issues in constructing a multi-slice model of an electron bunch is determining the appropriate number of slices, *N*, with which to use. It is possible to make a lower bound estimate of *N* based on a few important parameters of the electron bunch. In particular, suppose that a beam bunch has a uniform radius r_b, a bunch length *L*, and is uniformly moving with a relativistic energy factor, γ. Since the bunch is undergoing uniform motion, one finds that the electromagnetic potentials as a function of *z* due to each slice are maximized at the slice location and have a characteristic width of order r_b/γ in *z*-space. In order to compute the electric fields accurately, it is necessary that the number of slices times the characteristic width of the potential is much larger than *L*, or $Nr_b/\gamma >> L$. In practice, one finds that in order to achieve (<1% field error near electric field extremum) for typical electron bunches which are found in photocathode source experiments,

$$N \sim 50L\gamma/r_b \ . \qquad (6)$$

MULTISLICE SIMULATIONS

Figs. 1(a) and 1(b) show plots of the radial electric field versus *r/a* and longitudinal electric field versus *z/a* at a time *ct/a=0.05*, for slice numbers corresponding to 1, 61, 101, 151, and 301. The bunch is assumed to have a uniform velocity *V=0.9c*, and the head of the bunch is emitted at time *t=0*, where *a* is the conductor pipe radius. The bunch is assumed to have uniform density in the longitudinal direction with a bunch length of 9.1ps (2.5 mm) and a transverse density profile corresponding to Eq. (5) with a beam edge radius of 1 mm. The slices in the model are identical and are equal spaced throughout the bunch. The first noticeable feature of Figs. 1(a) and 1(b) is that the single slice model gives an enormous over-

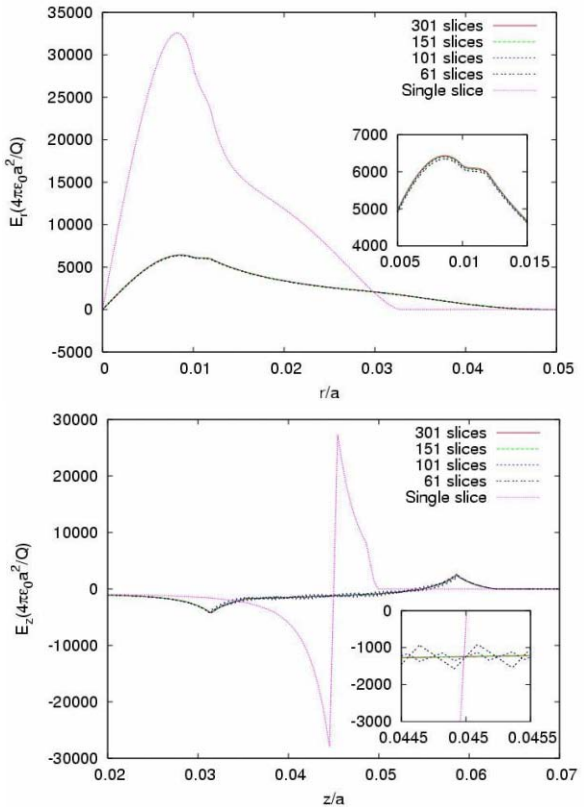

Figure 1: (a) Plot of the normalized radial electric field as a function of *r/a* and (b) plot of the normalized longitudinal electric field as function of *z/a* for 1, 61, 101, 151, and 301 slices.

estimate of the electric fields. For the higher slice numbers, we see good agreement in the radial electric field. However in the case of the 61 slice model, we see a 20% error in the longitudinal electric field. In general, we find that the error in the longitudinal electric field will typically be much greater than the error in the radial electric field for low slice numbers. This is due to the fact that each slice is contributing a longitudinal electric field with a sign that is opposite to the field of the adjacent slice causing large field cancellations. According to the estimate in Eq. (6), the 1% field error should be achieved at around 281 slices. This is in agreement with the observation of extremely small fluctuations in the 301 slice case. We are currently investigating promising methods of removing the effects of electric field errors when slice numbers are low, including filtering techniques [6].

Figs. 2(a), (b), and (c), show the results of space-charge electric field calculations using similar parameters as the ANL AWA 1.3 GHz photocathode gun [5]. Based on the parameters of the maximum electric field and rf phase injection for the experiment, we determined the trajectory of the head of the bunch (Fig. 2(a) in red) which is

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assumed to be emitted at time *t=0*. Also, plotted in Fig. 2(a), is the lightline (in blue) *z=ct*. We assumed that the

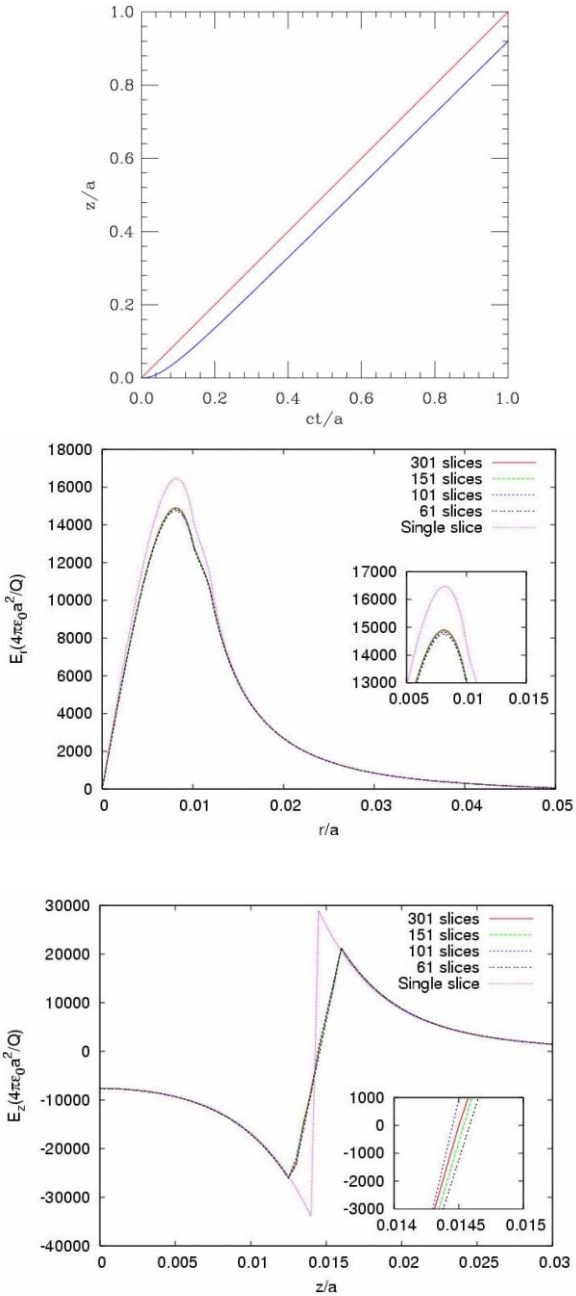

Figure 2: (a) Plot of the head slice trajectory (red) and lightline (blue) versus *ct/a*, (b) plot of normalized radial electric field as a function of *r/a* and (c) plot of the normalized longitudinal electric field as function of *z/a* for 1, 61, 101, 151, and 301 slices for an accelerating beam.

bunch had a transverse cross-section given by Eq. (5) with a beam edge radius of 1 mm, which corresponds to the experimental laser spot size. The beam is assumed to have uniform density in the longitudinal direction with subsequent slices following the same trajectory as the head slice, but displaced in time by equal amounts over a 1.8 ps bunch emission. The radial and longitudinal electric fields in Figs. 2(b) and 2(c) correspond to a time of *ct/a=0.05*. At this chosen time, one finds that the head and tail of the electron bunch have relativistic factors of $\gamma_{head} = 1.18$ and $\gamma_{tail} = 1.14$. The bunch length is approximately 0.28 mm, which according to Eq. (6) implies that low field error should occur when a minimum of 17 slices is used. It is clear from Figs 2(b) and 2(c), that the electric fields for the cases of 61, 101, 151, and 301 slices, are all in excellent agreement.

SUMMARY

In summary, we have demonstrated the computational requirements for using a multislice approach in conjunction with a Green's function formalism to accurately resolve the space-charge electromagnetic fields. The number of slices which are necessary for precision modeling of the electromagnetic fields depends on the bunch length, bunch radius, and beam energy. In general, the dominant numerical errors for a multislice model are within the longitudinal electric field due to the large scale cancellation of electric fields between adjacent slices. We are actively pursuing techniques, such as filtering methods [6], which will drastically reduce the number of slices that are necessary for high-accuracy field calculations.

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