WARM-FLUID EQUILIBRIUM THEORY OF AN INTENSE CHARGED-PARTICLE BEAM PROPAGATING THROUGH A PERIODIC SOLENOIDAL FOCUSING CHANNEL*

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Abstract

A warm-fluid theory of a thermal equilibrium for a rotating charged–particle beam in a periodic solenoidal focusing magnetic field is presented. The warm-fluid equilibrium equations are solved in the paraxial approximation. It is shown that the flow velocity for the thermal equilibrium corresponds to periodic rotation and radial pulsation. The equation of state for the thermal equilibrium is adiabatic. The beam envelope equation and self-consistent Poisson's equation are derived. The comparison between analytically computed density profiles and the recent experimental results from University of Maryland Electron Ring (UMER) is presented. Temperature effects in the beam equilibria are investigated. The radial confinement of the beam is discussed.

INTRODUCTION

Many charged-particle beam experiments and applications, such as particle accelerators, spallation neutron sources, high-power microwave sources and high-energy colliders, use high-intensity beams of charged particles. For such systems, beams of high quality (i.e., low emittance, high current, small energy spread, and low beam loss) are desired. The processes of emittance growth and beam losses are related to the evolution of particle beams in non-equilibrium states. It is crucial to find and study equilibrium states of charged-particle beams to prevent beam losses, to preserve beam emittance, to provide operational stability, and to control chaotic particle motion and halo formation.

Although several kinetic equilibria have been discovered for periodically focused intense charged-particle beams [1-3], periodically focused thermal beam equilibrium has not been reported until our present work, which includes both a kinetic treatment presented elsewhere [4, 5] and a warm-fluid treatment presented in this paper.

In this paper, we present a warm-fluid equilibrium theory of a new thermal charged-particle beam in a periodic solenoidal focusing field. Solving the warm-fluid equations in the paraxial approximation, we obtain the beam density and flow velocity. We derive the self-consistent rms beam envelope equation and the self-consistent Poisson equation, governing the beam density and potential distributions. For such thermal beam equilibria, temperature effects are found to play an important role. Due to temperature effects, the beam profile is bell-shaped, which is a more realistic representation of the beam density than the uniform density profile in previous theories (see, for example, Ref. 1-3). We present the comparison between our analytical calculation and recent experimental results from UMER [6]. Finally, we discuss the radial confinement of the beam.

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WARM-FLUID BEAM EQUILIBRIUM

We consider a thin, continuous, axisymmetric, single-species charged-particle beam, propagating with constant axial velocity $\beta_b c\hat{\mathbf{e}}_z$ through an applied periodic solenoidal magnetic focusing field. The applied periodic solenoidal focusing field inside the beam can be approximated by

$$\mathbf{B}^{ext}(r,s) = -\frac{1}{2}B_z'(s)\,r\,\hat{\mathbf{e}}_r + B_z(s)\hat{\mathbf{e}}_z\ , \qquad (1)$$

where $z = s$ is the axial coordinate, r is the radial distance from the beam axis, prime denotes the derivative with respect to s, and $B_z(s) = B_z(s+S)$ is the axial magnetic field, which is periodic along the $z-$axis with periodicity length S.

We solve the warm-fluid equilibrium ($\partial/\partial t = 0$) equations in the paraxial approximation with the adiabatic equation of state [7, 8],

$$T_\perp(s)\,r_{brms}^2(s) = const\ , \qquad (2)$$

where $T_\perp(s)$ is the transverse beam temperature which remains constant across the cross-section of the beam, and $r_{brms}(s)$ is the root-mean-square (rms) radius of the beam, defined by $r_{brms}^2(s) = N_b^{-1}\,2\pi\int_0^\infty dr\,r^3\,n_b(r,s)$.

The equilibrium flow velocity profile has the form [7, 8]

$$\mathbf{V}_\perp(r,s) = r\,\frac{r_{brms}'(s)}{r_{brms}(s)}\beta_b c\,\hat{\mathbf{e}}_r + r\,\Omega_b(s)\hat{\mathbf{e}}_\theta\ , \qquad (3)$$

which corresponds to a beam undergoing radial pulsation and angular rotation with the angular frequency

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$\Omega_b(s) = -\Omega_c(s)/2 + \omega_b r_{b0}^2 / r_{brms}^2(s)$. Here, $\Omega_c(s)$ is the cyclotron frequency, ω_b and r_{b0} are constants.

We derive the self-consistent beam equilibrium density distribution from the radial force balance equation [7, 8],

$$n_b(r,s) = \frac{C}{r_{brms}^2(s)} \exp\left\{ -\frac{r^2}{4\varepsilon_{th}^2}\left[\frac{K}{2} + \frac{4\varepsilon_{th}^2}{r_{brms}^2(s)}\right] - \frac{q\phi^{self}(r,s)}{\gamma_b^2 k_B T_\perp(s)} \right\}, \quad (4)$$

where C is a constant in the paraxial approximation and assures that the total number of particles per unit axial length is conserved, $K \equiv 2N_b q^2 / \gamma_b^3 m \beta_b^2 c^2$ is the self-field perveance and $\varepsilon_{th}^2 = k_B T_\perp(s) r_{brms}^2(s) / 2m\gamma_b \beta_b^2 c^2$ is the rms thermal emittance of the beam, which is a constant.

In Eq. (4), the scalar potential for the self-electric field $\phi^{self}(r,s)$ is determined self-consistently from the Poisson equation

$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\phi^{self}(r,s)\right] \\ &= -\frac{4\pi q C}{r_{brms}^2(s)} \exp\left\{ -\frac{r^2}{4\varepsilon_{th}^2}\left[\frac{K}{2} + \frac{4\varepsilon_{th}^2}{r_{brms}^2(s)}\right] - \frac{q\phi^{self}(r,s)}{\gamma_b^2 k_B T_\perp(s)} \right\}. \end{aligned} \quad (5)$$

In Eqs. (4) and (5), the rms beam radius $r_{brms}(s)$ is determined using the envelope equation for the evolution of the rms beam radius [7, 8],

$$r_{brms}''(s) + \left[\kappa_z(s) - \frac{\omega_b^2 r_{b0}^4}{r_{brms}^4(s)\beta_b^2 c^2}\right] r_{brms}(s) - \frac{K}{2r_{brms}(s)} = \frac{4\varepsilon_{th}^2}{r_{brms}^3(s)}, \quad (6)$$

where $\sqrt{\kappa_z(s)} \equiv qB_z(s)/2\gamma_b\beta_b mc^2$ is the focusing parameter.

WARM-FLUID BEAM EQUILIBRIA CALCULATIONS

In this section we present a comparison between our analytical results and recent experimental results from UMER experiment [6] and discuss the temperature effects on the beam density distribution. We also show that thermal beam equilibria exist for a wide range of parameters and discuss the radial confinement of the beam.

Comparison with UMER experiment

In a recent UMER experiment [6], a 5 keV electron beam was focused by a short solenoid. The electron beam was generated by a gridded gun and exited the gun through an anode aperture at $s = 0$. Bell-shaped beam density profiles were imaged by a fluorescent screen.

Using our thermal equilibrium theory, we have calculated transverse beam density profiles of the UMER 5 keV, 6.5 mA electron beam at three axial distances: $s = 6.4$ cm, 11.2 cm, and 17.2 cm, as shown in solid curves in Fig. 1. The dashed curves are the equivalent KV beam density profiles [2, 5]. The calculated beam density profiles are in good agreement with the experimental measurements (dotted curves) [6].

As the beam radius increases, the beam density profile approaches the KV (uniform) beam density distribution, because the beam temperature must decrease in order to keep $T_\perp(s) r_{brms}^2(s)$ a constant. Here, the Debye length $\lambda_D \equiv \sqrt{\gamma_b^2 k_B T_\perp(s) / 4\pi q^2 n_b(0,s)}$ is 0.54 mm. The warm-fluid beam density profile is nearly uniform up to the beam edge where it falls rapidly within a few Debye lengths.

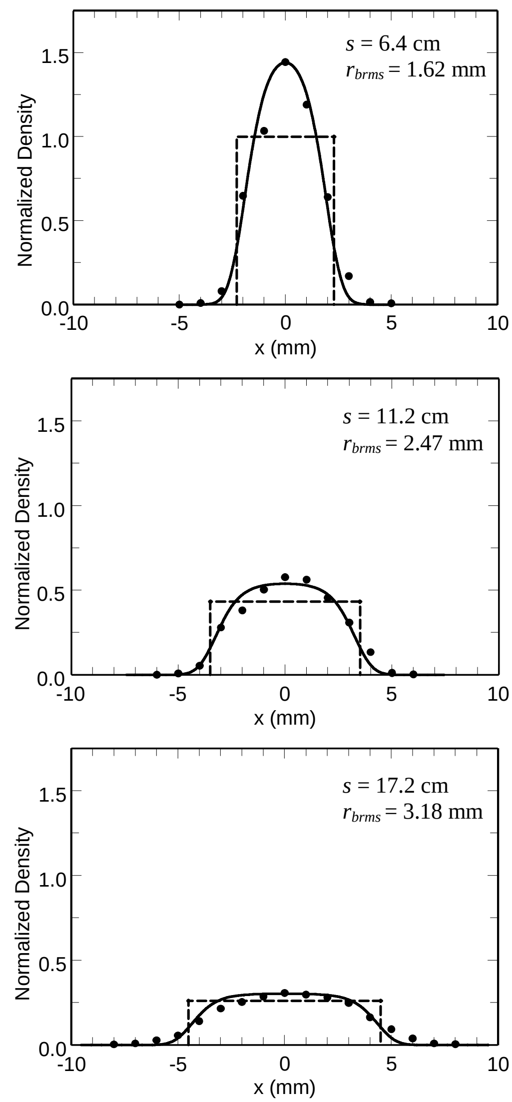

Figure 1. Normalized beam transverse density profiles of a 5 keV, 6.5 mA ($4\varepsilon_{th} = 30$ mm-mrad) electron beams at three axial distances: $s = 6.4$ cm, 11.2 cm, and 17.2 cm. Solid curves are analytical results; dotted curves, experimental measurements; dashed lines, equivalent KV beam density distributions. The densities are normalized to the equivalent KV beam density at $s = 6.4$ cm.

Radial confinement condition for the beam

There is a wide range of parameters for which the warm-fluid beam equilibrium exists in a periodic focusing channel. For practical purposes, it is useful to determine the radial confinement in an average sense. In Fig. 2, we plot the normalized angular frequency of beam rotation in the Larmor frame, $(S/\sigma_v \beta_b c)\langle \Omega_b(s) + \Omega_c(s)/2 \rangle$, as a function of the effective self-field parameter $\langle s_e \rangle \equiv S^2 \langle \omega_{pb}^2(s) \rangle / 2\gamma_b^2 \sigma_v^2 \beta_b^2 c^2$ for $\hat{K} = 0.1$, 0.2, 1, and 10. The beam propagates in the periodic solenoidal focusing field with $S\sqrt{\kappa_z(s)} = a_0 + a_1 \cos(2\pi s/S)$ and $a_0 = a_1 = 1.14$. The beam current is kept the same while the rms thermal emittance of the beam ε_{th} decreases. Here, σ_v is the vacuum phase advance (see, for example, Ref. [3]), $\omega_{pb}(s) \equiv \left(4\pi q^2 n_b(0,s)/\gamma_b m\right)^{1/2}$ is the plasma frequency, and $\langle \cdots \rangle$ denotes the average over one axial period S.

While Fig. 2 is computed for the specific periodic solenoidal focusing field, we observe no change in the Fig. 2 if we vary periodic solenoidal focusing field, provided that the vacuum phase advance σ_v of the magnetic field does not change. For any value of the effective self-field parameter $\langle s_e \rangle$ below a critical value, a confined beam can rotate at two angular frequencies, either positive or negative relative to the Larmor frame. For each value of $\hat{K}$, the maximum (critical) value of the effective self-field parameter for a confined beam is reached when the beam does not rotate relative to the Larmor frame. In Fig. 3, the critical effective self-field parameter $\langle s_e \rangle$ is plotted as a function of $\hat{K}$. The parameter space for radial beam confinement is indicated by the shaded region in Fig. 3.

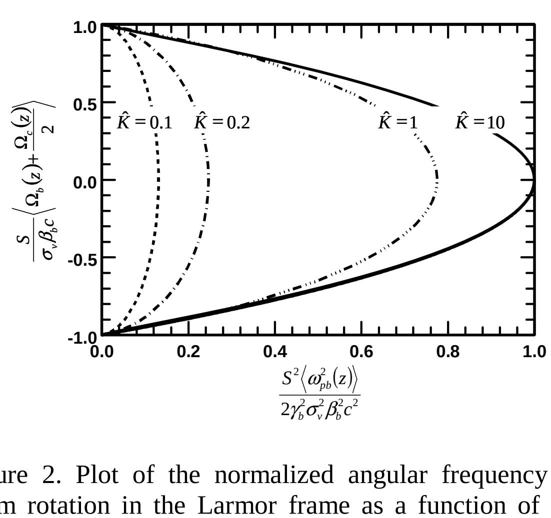

Figure 2. Plot of the normalized angular frequency of beam rotation in the Larmor frame as a function of the effective self-field parameter for normalized perveances $\hat{K} = 0.1$, 0.2, 1, and 10.

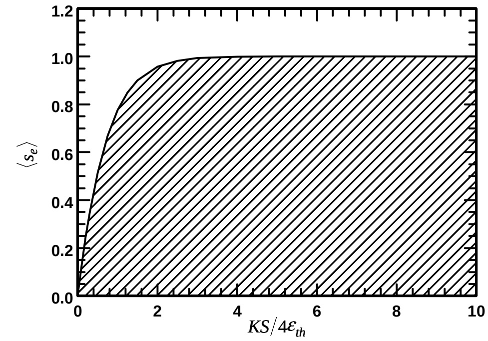

Figure 3. Plot of the critical effective self-field parameter $\langle s_e \rangle$ as a function of $\hat{K} \equiv KS/4\varepsilon_{th}$. The shaded region gives the parameter space for radial beam confinement.

CONCLUSIONS

We presented a warm-fluid equilibrium beam theory of a new thermal charged-particle beam propagating through a periodic solenoidal focusing field. We presented the rms beam envelope equation and the self-consistent Poisson equation, governing the beam density and potential distributions. We found good agreement between the adiabatic thermal equilibrium theory and a recent UMER experiment. The thermal beam equilibrium density profile has a bell-shaped density profile and a uniform temperature profile across the beam cross-section. Finally, we discussed the radial confinement of the beam

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